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Modelling of System Failures in Gas Turbine Engines on Offshore Platforms

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Abstract: The system reliability of gas turbine engines on offshore platforms, maintained (i.e. repaired) upon process failures, is considered in this study. A set of condition monitoring (CM) data (i.e. failure events) of a selected gas turbine engine is considered, where the system maintenance actions with minimum repair conditions (i.e. that should not disturb the failure rate intensity) are assumed. A nonhomogeneous Poisson process is used to model the age dependent reliability conditions of a gas turbine engine and maximum likelihood estimation (MLE) for calculating the same model parameters is implemented. Finally, a summary on the system behavior under failure intensity, mission reliability and mean time between failures (MTBF) is also presented in this study.

Keywords: System reliability, failure rates, failure intensity, gas turbine engines, maximum likelihood estimation, nonhomogeneous Poisson process.

1. INTRODUCTION

Industrial power plants are life critical systems in offshore platforms and their operational behavior (i.e. failure rates) can be used to encounter their diagnostic and prognostic challenges. Since these power plants play a crucial operational role in the oil and gas industry, this study proposes to understand the system failure behavior under aging conditions and that can also be used to formulate optimal maintenance policies. In general, these power plants consist of various engine-power configurations (i.e. reciprocating engine, gas-turbines, etc.) to satisfy the power requirements of offshore platforms. These engines are operating under harsh ocean environmental conditions; therefore condition monitoring (CM) and conditions based maintenance (CBM) applications should be implemented to overcome the respective degradation conditions.

This study is based on an offshore power plant with four industrial gas turbine engines/generators and that is located in a floating production, storage and offloading (FPSO) unit. The offshore platform is located in Campos Basin in Rio de Janeiro and an additional study on the same platform is done in (Machado *et al.,* 2014). These FPSO units have often been used by the offshore industry to receive, to process and to store the hydrocarbons produced from nearby offshore platforms and sub-sea production systems. This system consists of 4 turbo-generators consisting of aero-derivative gas turbine engine with normal capacity of 25000 (kW) coupled with electric generators with normal capacity of 28750 (kVA). The required grid load of the offshore platform approximately 35-45 (MW) and each generator is rated for approximately 12-15 (MW). Therefore, at least 3 generators in the isochronous mode should be operated to satisfy the requirements of the offshore platform.

In general, gas turbines have been used under open cycle and combined-cycle application in various power plants. In combined cycle approach, the exhaust gas temperature can be used to run steam generators as an energy recovery approach. As the power plant consists of several gas turbine engines, the system reliability measures on a selected gas turbine engine is considered in this study. Therefore, the system failure intensity of a gas turbine has been considered to model the overall power plant behavior. Furthermore, it is important to identify the system failure situations in these power plants ahead of time; therefore the optimal maintenance policy should be implemented to minimize the operational cost. That has been done by analyzing the CM data from the respective gas turbine engine.

2. SYSTEM RELIABILITY

Complex systems can often be repaired after failure events and those system failures can be modeled as stochastic processes. A system operational period (i.e. system age) that starts at $t=0$ and continues until $t=T$ with a number of failures $N(T)$ is considered in this model. Furthermore, these failure events are recovered by a same number of repairs with negligible time periods. The time periods for those failures from $t=0$ can be considered as $X_1, X_2, \ldots, X_N$. The $i-th$ successive operational period between two failures events can be considered as $X_i - X_{i-1}$ where $i = 1,2,\ldots,N$. These failure events are often been considered as an independent, identically distributed (IID) random variable that can be molded as a Poisson process (HPP) with the respective failure rate (λ). One should note that these repairable systems have often been modelled as Poisson process models and the inter-occurrence times (i.e. functioning time failures) in those events are independent events with exponential behavior, in which can be presented in system failure rates.

However, the system failure rate with an increasing (i.e. deteriorating), constant (i.e. neither deteriorating nor improving) or decreasing (i.e. improving) trends can be observed by the Laplace trend test (LTT). Hence, the LTT test statistics can be written as (Kim et al., 2004):

$$U_L = \frac{\sum_{i=1}^{n} X_i}{N} - \frac{T}{2} \bigg/ T\sqrt{\frac{1}{12N}} \tag{1}$$

When the LTT value is greater than zero, the system has an increasing trend (i.e. decreasing reliability) and the Laplace trend test value is less than zero, the system has a decreasing trend (i.e. increasing reliability) can be concluded. This test statistics approximate a standard normal distribution, therefore the significant level of the results can also be observed from the standard normal table. Therefore, this test has been considered as the first step in this CM data analysis.

However, a Poisson process model with a constant failure rate (i.e. homogenous Poisson process) cannot capture the system reliability throughout its life cycle. Therefore, that has often been limited to a section of the system life cycle. Hence, the system operational considerations such as mission reliability, reliability growth or deterioration, and maintenance polices cannot be included in these models (i.e. constant failure rates). Therefore, a nonhomogeneous Poisson process for modelling of the system failure events in a gas turbine engine is also considered. One should note that the time intervals between two respective failures in a nonhomogeneous Poisson process cannot be IID, because the system age has effected on the system failure rate. Hence, the system failure rate intensity of a system can be written as (Crow, 1990):

$$\mu(t) = \lambda \beta t^{\beta - 1} \tag{2}$$

where $\lambda > 0$ and $\beta > 0$ are system parameters and t is the age of the system. One should not that when $\beta < 1$, $\mu(t)$ is decreasing (i.e. the phase of infant mortality), when $\beta = 1$, $\mu(t)$ is a constant (i.e. the phase of useful life) and when $\beta > 1$, $\mu(t)$ is increasing (i.e. the phase of wear-out). It is assumed that the system has restored to its previous conditions after each failure with "minimal repair", where the intensity of the system failures has not been disturbed (Crow, 1975). Therefore, this behavior can also be described under the famous "bath-tub curve" for a system life cycle (Klutke *et al.,* 2003). Similarly, the power laws mean value function (i.e. the expected number of failures,) for a nonhomogeneous Poisson process with the failure intensity in (2), the expected number of failures for the same system during the system life time of $(t_{i-1} \quad t_i]$, can be written as:

$$\mathrm{E}[N(t_{i-1} \quad t_i) = n_i] = \int_{t_{i-1}}^{t_i} \mu(t) dt = \lambda t_i^{\beta} - \lambda t_{i-1}^{\beta} \tag{3}$$

where $N(t_{i-1} \quad t_i) = n_i$ is the number of failures that are experienced during the same system life time. One should note that (3) represents the expected number of failures (i.e. mean value) during the same system life time. Hence, the probability of encountering n_i failures during the same system life time can be written as:

$$\mathrm{P}[N(t_{i-1} \quad t_i) = n_i] = \frac{\mathrm{E}[N(t_{i-1} \quad t_i)]^{n_i} e^{-\mathrm{E}[N(t_{i-1} \quad t_i)]}}{n_i!} = \frac{(\lambda t_i^{\beta} - \lambda t_{i-1}^{\beta})^{n_i} e^{-(\lambda t_i^{\beta} - \lambda t_{i-1}^{\beta})}}{n_i!} \tag{4}$$

Therefore, the mission reliability (i.e. the probability that the system operational conditions that are satisfied without any failures) of the system for the same system life time can be written as:

$$R(t) = e^{-\int_{t_{i-1}}^{t_i} \mu(t) dt} = e^{-\int_{t_{i-1}}^{t_i} \lambda \beta T^{\beta - 1} dt} = e^{-(\lambda t_i^{\beta} - \lambda t_{i-1}^{\beta})} \tag{5}$$

However, to calculate the conditions derived in (3), (4) and (5), the parameters for the nonhomogeneous Poisson process model in (2) should be estimated. Hence, maximum likelihood estimation (MLE) is proposed to estimate those parameters and there are several optimal properties of MLE can be identified with respect to other parameter estimation methods (Myung, 2003). Considering the failure events in (4), the likelihood function can be written as (Smith and Oren, 1980):

$$\begin{aligned} L(\lambda, \beta) &= \prod_{i=1}^{N} \mathrm{P}(N(t_{i-1}, t_i) = n_i) = \prod_{i=1}^{N} \frac{(\lambda t_i^{\beta} - \lambda t_{i-1}^{\beta})^{n_i} e^{-(\lambda t_i^{\beta} - \lambda t_{i-1}^{\beta})}}{n_i!} \\ &= \prod_{i=1}^{N} e^{-(\lambda t_i^{\beta} - \lambda t_{i-1}^{\beta})} \prod_{i=1}^{N} \frac{(\lambda t_i^{\beta} - \lambda t_{i-1}^{\beta})^{n_i}}{n_i!} = e^{-\lambda T^{\beta}} \prod_{i=1}^{N} \frac{(\lambda t_i^{\beta} - \lambda t_{i-1}^{\beta})^{n_i}}{n_i!} \end{aligned} \tag{6}$$

Considering (6), the log likelihood function can be written as:

$$\log L(\lambda, \beta) = -\lambda T^{\beta} + \sum_{i=1}^{N} n_i \left(\log \lambda + \log(t_i^{\beta} - t_{i-1}^{\beta}) \right) + \log n_i! \tag{7}$$

The partial derivatives of both parameters, λ and β, should be considered in (7) to calculate the maximum likelihood values for the respective parameters and that can be written as:

$$\begin{aligned} \frac{\partial}{\partial \lambda} \log L(\lambda, \beta) &= -T^{\beta} + \frac{1}{\lambda} \sum_{i=1}^{N} n_i = 0 \\ \frac{\partial}{\partial \beta} \log L(\lambda, \beta) &= -\lambda T^{\beta} \log T + \sum_{i=1}^{N} n_i \frac{(t_i^{\beta} \log t_i - t_{i-1}^{\beta} \log t_{i-1})}{(t_i^{\beta} - t_{i-1}^{\beta})} = 0 \end{aligned} \tag{8}$$

Hence, the maximum likelihood values of λ and β in (8) satisfy the following conditions:

$$\begin{aligned} \hat{\lambda} &= \frac{\sum_{i=1}^{N} n_i}{T^{\beta}} = \frac{N}{T^{\beta}} \\ -\lambda T^{\beta} \log T &+ \sum_{i=1}^{N} n_i \frac{(t_i^{\beta} \log t_i - t_{i-1}^{\beta} \log t_{i-1})}{(t_i^{\beta} - t_{i-1}^{\beta})} = 0 \end{aligned} \tag{9}$$

One should note that (9) should be solved iteratively and that has a unique solution for the parameters of λ and β. However, the solution can calculate under time truncated and failure truncated situations. A situation with the observations that are truncated after a prefixed time for a respective number of failures (i.e. the number of failures is a random variable) is considered as time truncated. A situation with the observations that are truncated after a prefixed number of failures for a respective time interval (i.e. the time interval is a random variable) is considered as failure truncated. However, a time truncated situation with respect to the

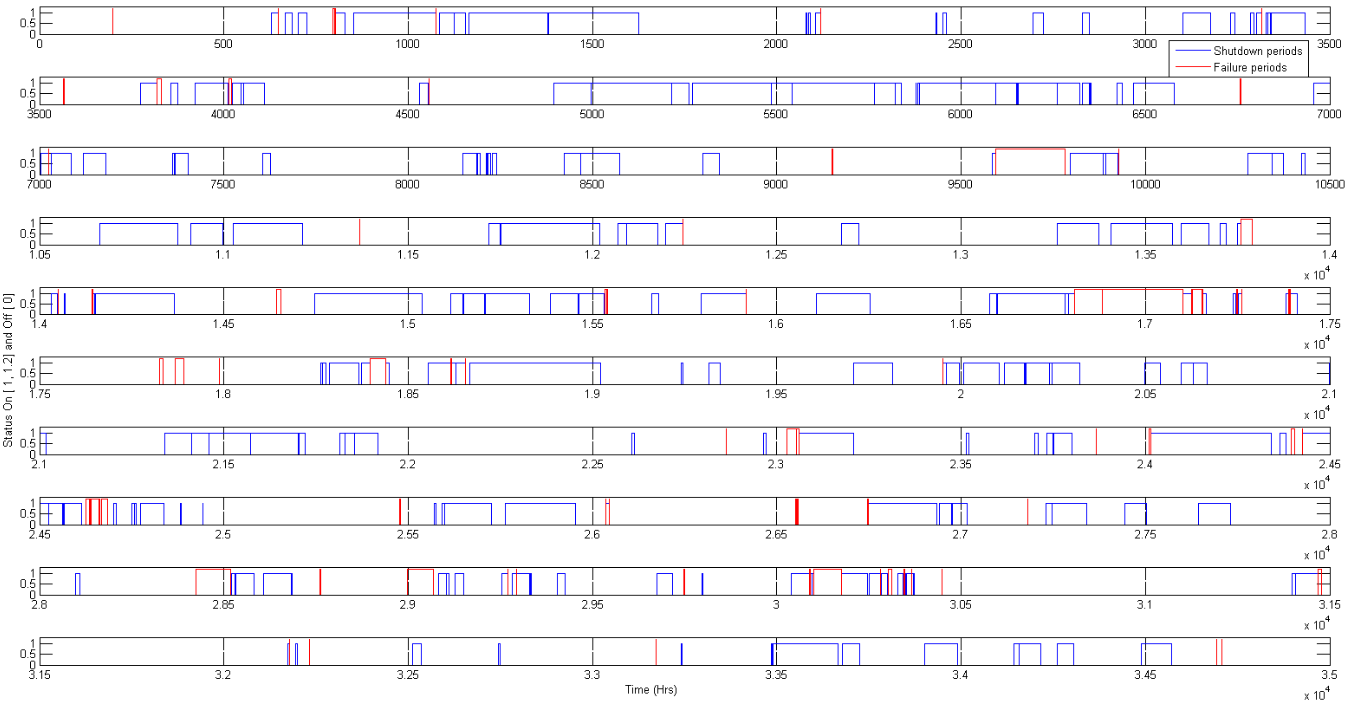

Fig. 1: Shutdown periods and Failure periods for TG-A

system age in a gas turbine engine, is considered in this study. Hence, (9) can be derived as (Crow, 1974):

$$\hat{\lambda} = \frac{N}{T^{\beta}}, \quad \hat{\beta} = \frac{N}{\sum_{i=1}^{N} \log\left(\frac{T}{X_i}\right)} \tag{10}$$

Hence, the unbiased estimator for the variable, β can be written as (Crow, 1975):

$$\bar{\beta} = \frac{N-1}{N}\hat{\beta} \tag{11}$$

As the next step, the confidence bounds for the parameters of λ and β should be derived. Considering the parameter, $\hat{\beta}$, the confidence bounds for hypotheses testing the true value of β are derived by using a Chi-square distribution with $2M$ degrees of freedom (Crow, 1975):

$$\chi^2 = \frac{2M\beta}{\hat{\beta}} \tag{12}$$

(12) can be used to test hypotheses on β. By considering M is moderate, the statistics of ω can be written as (Crow, 1975):

$$\omega = \sqrt{M}\left(\frac{\beta}{\hat{\beta}} - 1\right) \tag{13}$$

where (13) is distributed approximately with mean *0* and variance *1*. Hence, the approximate confidence bounds for β, were the $(1-\alpha)\cdot 100$ percent lower and upper confidence bounds can be written as:

$$\beta_{LB} = \hat{\beta}\left(1 - \frac{P_{\pi}}{\sqrt{M}}\right), \quad \beta_{UB} = \hat{\beta}\left(1 + \frac{P_{\pi}}{\sqrt{M}}\right) \tag{14}$$

where P_{π} is the π-th $\approx 1 - \alpha/2$ percentile for a normal distribution with mean *0* and variance *1*. Hence, the $(1-\gamma)\cdot 100$ percent lower and upper confidence bounds for λ can be written as:

$$\lambda_{LB}(\beta_{UB}) = \frac{\chi^2\left(\frac{\gamma}{2}, 2N\right)}{2T^{\beta_{UB}}}, \quad \lambda_{UB}(\beta_{LB}) = \frac{\chi^2\left(1-\frac{\gamma}{2}, 2N+2\right)}{2T^{\beta_{LB}}} \tag{15}$$

One should note that (14) and (15) have often been categorized as the conservative simultaneous confidence bounds on the parameters of λ and β with $(1-\alpha)(1-\gamma)\cdot 100$ percent. As the next step of this study, the estimated and actual system failure events should be compared to observe the goodness for the proposed model. Considering a situation, where the actual system failure events are known, Cramer-Von Mises goodness statistics can be used to test the null hypothesis. Hence, the proposed NHPP model with the estimated parameter values and its capabilities to appropriately capture the actual system failure behaviour can be inspected. Hence, the Cramer-Von Mises goodness test can be written as (Crow, 1974):

$$C_M^2 = \frac{1}{12N} + \sum_{i=1}^{N}\left(\left(\frac{X_i}{T}\right)^{\bar{\beta}} - \frac{2i-1}{2N}\right)^2 \tag{16}$$

Considering the hypothesis H_1 for the system failures are following a $\bar{\beta} = 1$, where the constant failure rate is associated. Hence, the hypothesis H_2 can be presented as that the failure rate follows a nonhomogeneous passion process with the proposed intensity function and $\bar{\beta}$ is unspecified. If the hypothesis H_2 has proven to be accepted, the parameters estimated for λ and β are acceptable. Hence, if C_M^2 is greater than the selected critical value, the hypothesis H_2 is rejected and if C_M^2 is less than the selected

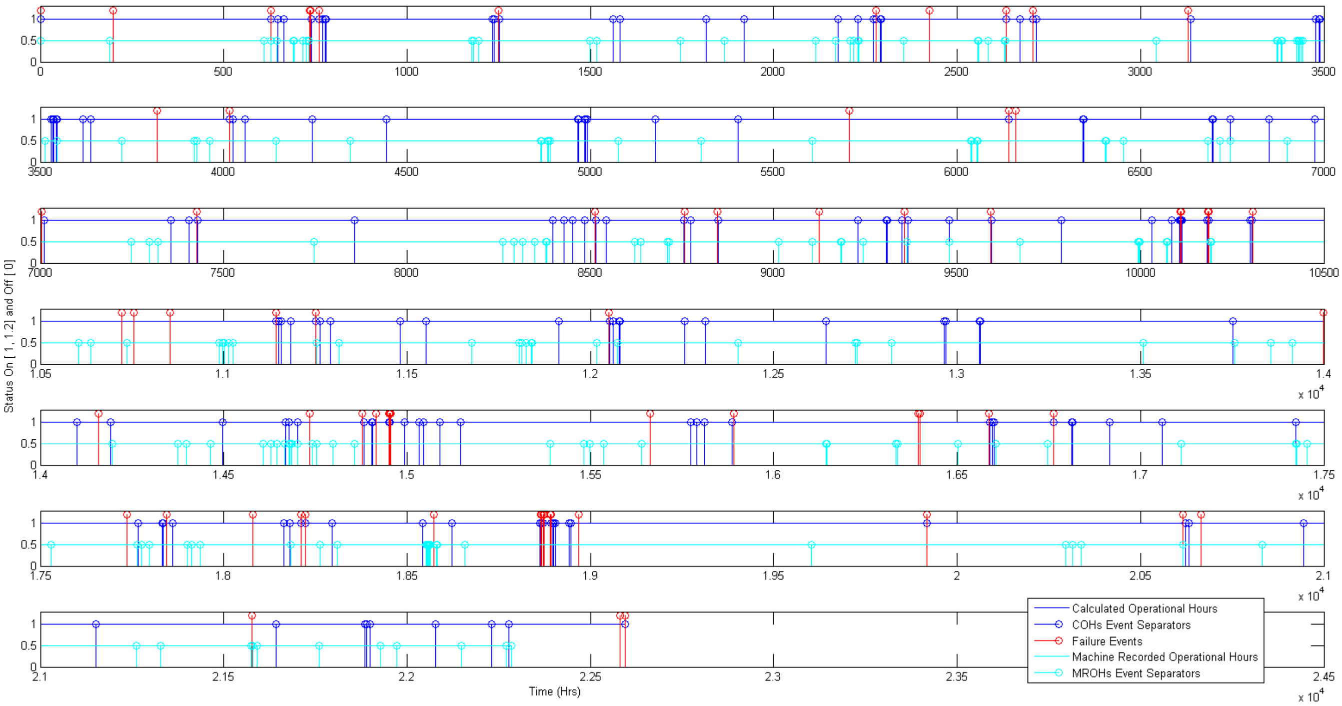

Fig. 2: Operational hours and Failure events for TG-A

critical value, the hypothesis H_2 is accepted at the respective significant levels.

Finally, the mean time between failures (MTBF) is calculated for the proposed model. Hence, the instantaneous MTBF can be written as:

$$\hat{M}(t)=(\hat{\mu}(t))^{-1}=\left(\hat{\lambda}\hat{\beta}t^{\hat{\beta}-1}\right)^{-1} \tag{17}$$

The confidence interval for $\hat{M}(t)$ (i.e. estimated value of $M(t)$) provides a measure of the uncertainty around the calculated value. The two sided $(1-\alpha)100$ percent confidence intervals on $M(t)$ can be written as (Crow, L. H., 1977):

$$\Pi_1\hat{M}(t)\le\hat{M}(t)\le\Pi_2\hat{M}(t) \tag{18}$$

where Π_1 and Π_2 can be obtained from the available data tables in (Crow, 1977) for the $(1-\alpha)\cdot 100$ percent lower and upper confidence bounds.

3. PARAMETER ESTIMATION

The system shutdown and failure events for a selected gas turbine engine (TG-A) for the last 4 year period (i.e. the total operational period) is presented in Figure 1. This CM data consist of a total monitoring period of 34708 (Hrs) of the shutdown and failure periods of the selected gas turbine engine. That has been divided into 3500 (Hrs) operational intervals under 10 plots in the same figure, in which has the improved visibility. The shutdown periods are represented under blue color blocks and the system failure periods are represented under red color blocks (see Figure 1).

Considering the CM data of the gas turbine engine, the cumulative non-shutdown period for the same gas turbine engine is derived, where the shutdown periods have been removed and the non-shutdown period are combined to calculate a cumulative total operational period. One should note that the combination point of two non-shutdown periods is considered as an event separator. These event separators are introduced to keep track of the non-shutdown periods. Furthermore, the respective failure periods are adjusted in accordance with the removal of the non-shutdown periods. Therefore, the cumulative non-shutdown period has reduced to 23649 (Hrs) from the total operational period of 34708 (Hrs). Then, the lengths of the failure periods are removed from the cumulative non-shutdown period and the failure events are introduced. Therefore, this approach has reduced the operational period (i.e. system age) for the respective gas turbine engine, where the total operational hours has reduced approximately to 22596 (Hrs) (i.e. $T = 22596\,(\text{Hrs})$) from the cumulative non-shutdown period of 23649 (Hrs).

The system operational period (i.e. calculated operational hours (COHs)), the event separators, and failure events are presented in the Figure 2. Furthermore, the machine recorded operational hours (MROHs) with the separators that were recorded by the gas turbine engine itself are also plotted in the same figure. It is expected that the MROHs should overlay within the COHs and the failure events. However, the MROHs have been shifted left with respect to the COHs and the failure events as presented in the figure. One should note that the MROHs consist of fewer hours than the COHs of the same gas turbine engine.

The number of failures for the same gas turbine engine with respect to the system age is presented in plot (a) of Figure 3. Furthermore, the respective MROHs and COHs values with respect to each failure are presented in plot (b) of Figure 3

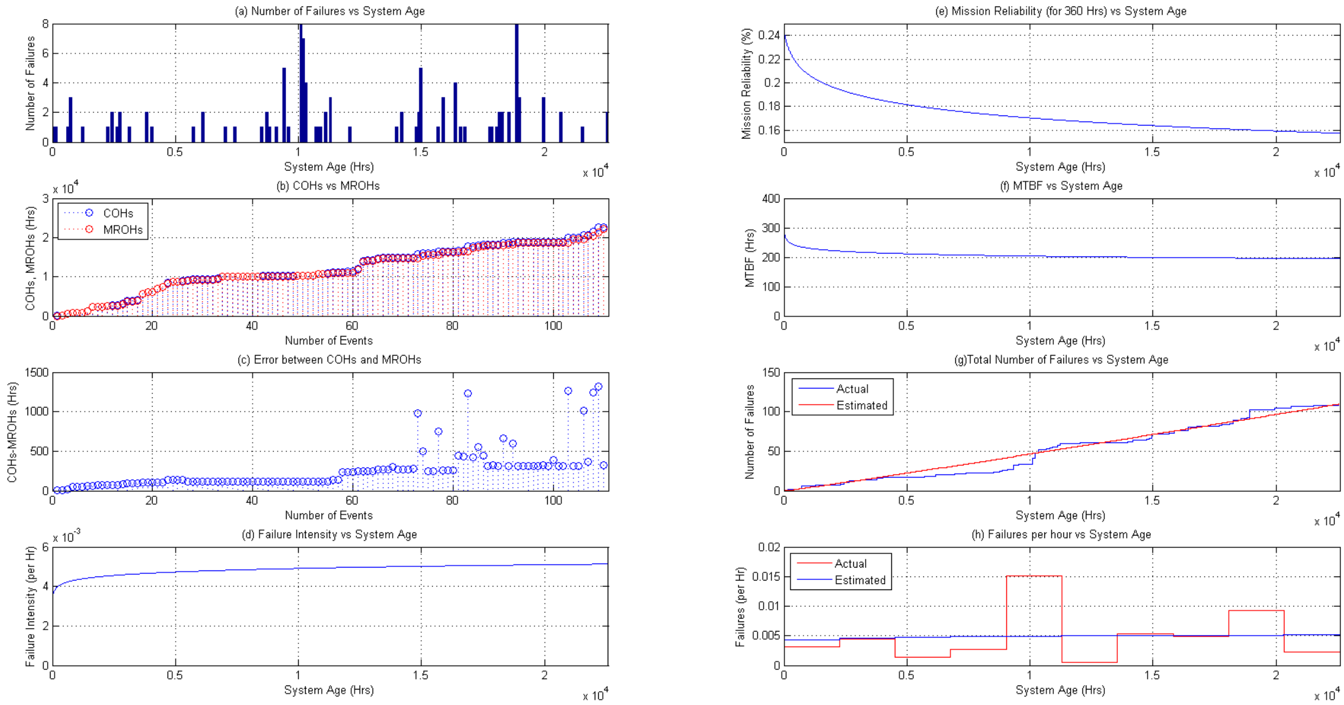

Fig. 3: System event data for TG-A

and the respective errors between the MROHs and COHs values are presented in plot (c) of Figure 3.

Even though, it is expected that the MROHs and COHs may have same values for respective failures, there are some deviations. In some situations, the error value is increased to a larger value initially and that is decreased to a much lower error value. This results show that the MROH values may have some time delay on recording the data with compare to the COH values. Furthermore, it is noted that the system start-up and shutdown hours have not been recoded under the MROHs due to the fact that it may not operate the MROH counting system during the start-up and shutdown periods. Therefore, the MROHs consist of fewer hours than the COHs and the errors between those two values are increasing (see plot (c) of Figure 3). However, the COHs data have been used for the parameter estimations process, the calculation of the failure intensity function with respect to the system age. Furthermore, it is concluded that the same machine has been used throughout the entire period (i.e. machine has not been replaced with another machine) by considering the MROHs. Then, Laplace trend test in (1) has been used to evaluate the behavior of the system failure rate and the calculated value can be written as:

$$U_L = 1.1894 \tag{19}$$

The Laplace trend test value is greater than zero, therefore the gas turbine engine with slight increasing failure rate (i.e. decreasing reliability) can be concluded. The significant level (from the standard normal table) of the results in can be further analyzed, where $|U_L| = 1.1894$ can approximate the statistical significance to 88%. Therefore, the proposed nonhomogeneous Poisson process model in (2) is a suitable candidate (within the respective statistical significance) for modeling the system reliability in a gas turbine engine. As the next step, the estimated values for the failure intensity function are calculated by considering (10) and that can be written as:

$$\hat{\lambda} = 0.0028\,,\ \hat{\beta} = 1.0542 \tag{20}$$

Therefore, the gas turbine engine characterization slightly under the wear-out phase can be concluded. The respective failure intensity function for the selected gas turbine engine with respect to the system age is presented in plot (d) of Figure 3. Hence, unbiased estimator for β in (11) can be calculated as:

$$\overline{\beta} = 1.0447 \tag{21}$$

The approximate confidence bounds for β, where 90% lower and upper confidence bounds in (14) can be calculated as:

$$\beta_L = 0.9254\,,\ \beta_U = 1.1831 \tag{22}$$

The lower and upper confidence bounds for λ can be written as:

$$\lambda_L = 6.5879 \cdot 10^{-4}\,,\ \lambda_U = 0.0121 \tag{23}$$

Hence, 81% conservative simultaneous confidence bounds on the parameters λ and β are presented in (22) and (23) equations, where $\alpha = .1$ and $\gamma = .1$. The mission reliability for 15 day (i.e. 360 (Hrs) intervals is presented in plot (e) of Figure 3. The mission reliability represents the probability that the selected gas turbine engine survives without any failures within the next 15 days (i.e. 360 (Hrs) with respect to its age. As presented in the figure, the mission reliability is decreasing with respect to the system age despite the present maintenance actions.

Furthermore, the expected number of failure with next 15 days (i.e. 360 (Hrs) at the end of operational hours of 22596 (Hrs) has also been calculated by using (3): the calculated value is 1.8476 failure events per 15 days (i.e. MTBF is approximately 194.85 (Hrs). Therefore, it is concluded that every 16 days 2 system failures in average can be observed under the present operational conditions. The instantaneous MTBF for the same gas turbine engine under the same operational period is presented in plot (f) of Figure 3. One should note that the MTBF value has reduced to 194.85 (Hrs) at the end of the operational period and that can be approximated as 8.12 (days). Therefore, it can be concluded that a system failure can occur approximately every 8 day in average for this gas turbine engine. Hence, the estimated lower and upper bound for the MTBF with the two-sided 90% confidence interval can be written as:

$$156.27\,(\text{Hrs}) \le M(T = 22596(Hrs)) \le 246.48\,(\text{Hrs}) \quad (24)$$

where the table values can be extrapolated as (AMSAA, 2000) $\Pi_1 = 0.802$ and $\Pi_2 = 1.265$. Therefore, the system can face a failure in approximately 6.51 (days) to 10.27 (days). The actual failures (Actual) of the gas turbine engine and the predicted failures (Estimated) with respect to the system age are presented in plot (g) of Figure 6. The average failure rate for actual and estimated situations using for 2256 (Hrs) by considering 20 intervals are presented in plot (h) of Figure 3. These results present an increasing failure rate with respect to the system age. Finally, the Cramer-Von Mises goodness-of-fit for the derived parameters λ and β is conducted. The requirement of Cramer-Von Mises goodness-of-fit to be N=110, $\alpha = 0.01$ then $C_T{}^2 \approx 0.34$ (AMSAA, 2000). So the model to be accepted for the same significant level is $C_M{}^2 < C_T{}^2$. A Cramer-Von Mises goodness of test for this model is calculated:

$$C_M^2 = 0.4315 \quad (25)$$

Hence, the hypothesis H_2 for the presented model should be rejected due to $C_M^2 > C_T{}^2$ for the significant level of $\alpha = 0.01$.

4. CONCLUSIONS

An overview of the mathematical reliability modelling of gas turbines in offshore platforms is presented in this study. The system CM data has been analyzed and the failure intensity, the mission reliability and the MTBF conditions for the present system status have been derived. However, the hypothesis H_2 for the presented model is rejected and there are three challenges can be observed with respect to the available data. Firstly, the β value is approximately equal to 1. Secondly, the system start-up failures and operational failures are combined in this data and that should be separated. Thirdly, the differences between the actual and the predicted failures are not strongly consistence with actual failure situations (see plot (g) Figure 3). One should note that several repeated failures can be observed around 10111 (Hrs) and 18924 (Hrs) of the system age in the same figure. These two situations have influenced on the failure model in the gas turbine engines and that effect should be compensated to improve the model accuracy, in which has been proposed as the future work.

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