Dynamic Self-Optimizing Control for Oil Reservoir Waterflooding

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Abstract: Waterflooding is a common oil recovery method where water is injected into the reservoir for increased productivity. Optimal operational strategy of waterflooding processes has to consider proceeds realized from produced oil and cost of productions including both injected and produced water. This is a dynamic optimization problem. The problem could be solved through numerical algorithms based on traditional optimal control theory which can provide only open-loop control solutions and rely on an accurate process model. However, reservoir properties are extremely uncertain, and hence open-loop solutions based on a nominal model are not suitable for applications with real reservoirs. Introduction of feedback into the optimization structure to counteract the effect of uncertainties has been proposed recently. In this work, a novel feedback optimization method for optimal waterflooding operation is presented. In the approach, appropriate controlled variables as combinations of measurement histories and manipulated variables are first derived through regression based on simulation data obtained from a nominal model. Then a feedback control law was represented as a linear function of measurement histories from the controlled variables obtained. Through a case study, it was shown that the feedback control solution proposed in this work was able to achieve a near-optimal operational profit with only 0.45% worse than that achieved through the true optimal control (with system's properties assumed to be known a priori), but 95.05% better than that obtained with the open-loop solution under uncertainties.

Keywords: Reservoir waterflooding, Optimal control, Self-optimizing control, Open-loop solution, Controlled variable.

1. INTRODUCTION

The prudent search for efficient recovery methods of oil from ageing reservoirs has sprung studies on optimization techniques for reservoir waterflooding. Waterflooding is the most common type of secondary recovery methods (Adeniyi et al., 2008) which involves injection of water into the reservoir through an injection well with the aim to properly sweep the oil in place towards a production well and/or maintain the reservoir pressure (Grema and Cao, 2013).

A typical waterflooding optimization problem seeks to determine optimum injection and production settings in order to maximize a performance index such as net present value (NPV) or total oil recovery. Several works were reported to employ the traditional optimal control which provides an open-loop solution based on an off-line nominal model (Asadollahi and Naevdal, 2009; Brouwer and Jansen, 2004). Unfortunately, reservoir properties including its geometry and boundaries are uncertain (Jansen et al., 2008). There are some production behaviours that can rarely be captured well through simulation model such as well coning (Dilib and Jackson, 2013a). Therefore, for a real oil reservoir, open-loop optimal solution determined off-line from a model may be suboptimal or entirely non-optimal.

Several methods have been proposed in the literature to deal with such uncertainties. For example, in robust optimization (RO), inputs are implemented in an open-loop fashion which have to follow a predetermined profile such that system constraints are satisfied in the presence of any uncertainty or disturbance (Yeten et al., 2003; Ye et al., 2013; Gabrel et al., 2014). Because RO approaches are designed to account for all possible uncertainties, their performance is mostly conservative which hardly leads to an optimal solution. Works that reported to use such technique in the field of waterflooding include that of van Essen and others (van Essen et al., 2009). It involves use of a set of reservoir realizations with the assumption that it captures all reservoir characteristics and production behaviours, a condition which is very difficult to be met in reality. Another method developed to counteract the effects of uncertainties is parametric optimization technique (Fotiou et al., 2006). Never the less, the method is too complicated to be applicable to waterflooding processes. Stochastic optimization methods were also developed to counter the effects of systems uncertainties (Tu and Lu, 2003; Pastorino, 2007; Wu, 2012). These methods involve random search within a parameter space in which potential solutions are evaluated. (Collet and Rennard, 2007). Slow convergence and high computational power requirement is a major drawback to these methods. A practical approach, repeated learning control was developed for batch processes (Ganping and Jun, 2011; Ahn et al., 2014), unfortunately it is not applicable to processes that are not repeatable, typical of petroleum production from reservoirs.

The current practice in industries is a procedure that is commonly referred to as history matching which involves periodic updating of available reservoir models using historical data and subsequent determination of operational strategies based on the updated models. However, solutions based on history-matched models may be suboptimal or non-optimal at all because of inability of updated models to predict reality correctly.

Based on the fact that feedback is an efficient tool to deal with uncertainties, proposals have been made recently of including a direct feedback control for optimal waterflooding operations (Jansen et al., 2008; Dilib and Jackson, 2013a; Brouwer et al., 2001; Foss, B. and Jensen,J. P., 2011). But a fundamental task that has not been investigated is formulation of a simple controlled variable (CV) that should make the optimality of waterflooding process insensitive to various geological uncertainties. Recently, we have developed a robust CV based on the principle of self-optimizing control (SOC) and tested it on a system with one degree of freedom (DOF) (Grema and Cao, 2014). In that work, an optimal feedback control law was represented as a linear function of production measurements with coefficients to be determined through least square regression to approximate the gradient of the cost function against manipulated variables based on simulated data obtained from a nominal model. The whole idea is to maintain the selected CV at zero through feedback control so that the operation is automatically optimal or near optimal with an acceptable loss.

This work extended the methodology presented by (Grema and Cao, 2014) to solve multivariable waterflooding optimization problem. Results obtained were compared with the open-loop optimal control approach for cases with different uncertainties. Furthermore, true optimal control solutions where the system model was assumed to be perfect with all properties known a priori are also derived as a benchmark for the above comparison.

2. APPROACH

2.1 Dynamic Optimization for Reservoir Waterflooding using SOC

A reservoir model in a discretized form is given as

$$\boldsymbol{g}(\boldsymbol{u}^k, \boldsymbol{x}^{k+1}, \boldsymbol{x}^k) = \boldsymbol{0} \tag{1}$$

where $\boldsymbol{x}^{\mathrm{k}}$ and $\boldsymbol{u}^{\mathrm{k}}$ are the state and input vectors respectively at time-step, k. For such kind of system, an objective function, J to be optimized can be represented as

$$J = \sum_{k=1}^{N} J^k(\boldsymbol{u}^k, \boldsymbol{y}^k) \tag{2}$$

where J consists of contributions at each time step denoted by J^k, $\boldsymbol{y}^k$ is a vector of measurements at time step k, and N is the total number of time steps. From (1), it can be inferred that any change in $\boldsymbol{u}^k$ at time k will affect the states $\boldsymbol{x}^{k+1}$, which will in turn influence the outputs, $\boldsymbol{y}^{k+1}$ through some measurement functions

$$\boldsymbol{h}(\boldsymbol{x}^k, \boldsymbol{y}^k) = \boldsymbol{0} \tag{3}$$

A feedback control law is sought to maintain the gradient of the objective function with respect to control input to be zero or near zero at each time step such that the overall trajectory is optimal or near optimal, i.e. the objective function is minimum or near minimum in the presence of uncertainties. If any two or more control trajectories are perturbed, then the deviation of the cost function J, can be approximated by finite differences between two closely related trajectories, $\boldsymbol{u}_i^1, \boldsymbol{u}_i^2, \dots, \boldsymbol{u}_i^N$ and $\boldsymbol{u}_{i+1}^1, \boldsymbol{u}_{i+1}^2, \dots, \boldsymbol{u}_{i+1}^N$, if $\max\|\boldsymbol{u}_{i+1}^k - \boldsymbol{u}_i^k\| < \varepsilon$ with a sufficiently small ε. The deviation in the cost function can be written using Taylor's series expansion as

$$J_{i+1} - J_i = \sum_{j=1}^{n_u} \sum_{k=1}^{N} G_{i,j}^k (u_{i+1,j}^k - u_{i,j}^k) \tag{4}$$

where n_u is the total number of inputs and $G_{i,j}^k$ is the gradient of the objective function with respect to the input channel, j at time-step, k for the reference trajectory, i.

Generally, the analytical expression of the gradient function in (4) is difficult to obtain particularly in the presence of uncertainties. To derive an output feedback control law, $\boldsymbol{u}^k = \boldsymbol{F}(\boldsymbol{y}_i^k, \boldsymbol{y}_i^{k-1}, \dots \boldsymbol{y}_i^{k-n})$, which is equivalent to $\boldsymbol{F}(\boldsymbol{y}_i^k, \boldsymbol{y}_i^{k-1}, \dots \boldsymbol{y}_i^{k-n}) - \boldsymbol{u}^k = 0$, it is proposed to approximate these gradients by a number of measurement functions with a set of unknown parameters to be determined through regression based on simulated data. Therefore, the gradient in (4) can be replaced by a measurement function, C as

$$J_{i+1} - J_i = \sum_{j=1}^{n_u} \sum_{k=n+1}^{N} [C(\boldsymbol{\theta}_j, \boldsymbol{y}_i^k, \boldsymbol{y}_i^{k-1}, \dots \boldsymbol{y}_i^{k-n}, u_{i,j}^k) (u_{i+1,j}^k - u_{i,j}^k)] \tag{5}$$

where θ_j is a parameter vector to be determined through regression for channel j and the measurement vector includes current and past measurements with n being the number of histories, which was found to be 2 after some trial and error exercises in this study. C can be any polynomial function such that $\boldsymbol{u}^k$ can be easily obtainable, but a linear combination of measurements was adopted in this work.

For simulated data collection, the following steps are followed:

1. A control trajectory, i is found via optimal control computation given as
$$\boldsymbol{u}_i^1, \boldsymbol{u}_i^2, \dots\dots\dots\dots. \boldsymbol{u}_i^N,$$
2. The control trajectory above is used to solve the model equation in (1) where measurements and states sequences are obtained which are given respectively as:
$\boldsymbol{y}_i^0, \boldsymbol{y}_i^1, \boldsymbol{y}_i^2 \dots. \boldsymbol{y}_i^N$ and $\boldsymbol{x}_i^0, \boldsymbol{x}_i^1, \boldsymbol{x}_i^2 \dots\dots. \boldsymbol{x}_i^N$,
3. The control trajectory in step 1 is perturbed to $\boldsymbol{u}_{i+1}^1, \boldsymbol{u}_{i+1}^2, \dots. \boldsymbol{u}_{i+1}^N$ and the model is solved where perturbed measurements $\boldsymbol{y}_{i+1}^0, \boldsymbol{y}_{i+1}^1, \boldsymbol{y}_{i+1}^2 \dots\dots. \boldsymbol{y}_{i+1}^N$,

and states $\boldsymbol{x}_{i+1}^0, \boldsymbol{x}_{i+1}^1, \boldsymbol{x}_{i+1}^2 \dots\dots \boldsymbol{x}_{i+1}^N$, are obtained and J_{i+1} also calculated.

4. Input perturbations are repeatedly applied to model equations to get perturbed solutions for a predefined number of trajectories, M.

With the above data collection procedure, a data matrix of $M\ by\ N$ is obtained which is used to perform regression by minimizing the square of the residual given by

$$\min_{\theta} \sum_{i=1}^{N} \left((J_{i+1} - J_i) - q \right)^2 \tag{9}$$

Where q represents the right-hand side of (5). The goodness of fit is evaluated by R^2 statistic. For brief description of regression, the reader is referred to (Ye et al., 2013).

Once the measurement function C is obtained through regression, then the output feedback control law can be derived straightaway by solving $C = 0$. With this output feedback control law, C, and hence the gradients in (4) are all near zero, which leads to the optimality or near optimality of the feedback system.

2.2 Reservoir Model and Uncertainties

A nominal reservoir model was used to collect simulated data which is of size 20 m x 20 m x 5 m and modelled with Cartesian grid cells in the x, y and z directions of 20 x 20 x 5 respectively; therefore each cell is 1m x 1m x 1m. The reservoir has homogeneous rock and fluid properties with a permeability of 100 mD, porosity of 0.3, oil and water relative permeability Corey exponents of 2.0 each. There are two vertical injection (I1 and I2) and production (P1 and P2) wells located at the corners of the reservoir (Fig. 1). Each of the four wells is perforated at a distance of 1m vertically (five perforations for each) and is rate-constrained. The CVs were developed based on this model. In the CV formulation, uncertainty is not considered. This is because in practice, it is difficult to sample the whole space of geological uncertainties. Furthermore, the CVs developed based on this model (nominal model) can be tested for robustness against unexpected reservoir behaviours which are inevitable in real applications.

The developed CVs were first implemented on the nominal model (Case I) and then subsequently to different uncertain cases by varying one or more nominal reservoir properties so as to mimic real model/system mismatch. In Case II, the size of the real reservoir was increased to 100 m x 100 m x 10 m with five layers each of 2 m thickness and random permeability field with mean values of 200 mD, 500 mD, 350 mD, 700 mD and 250 mD from top to bottom respectively, see (Grema and Cao, 2014). For Case III, the only uncertainty introduced is in the shapes of relative permeability curves where the real exponents for oil and water were assumed to be 1.5 each. Uncertainties in reservoir geometry, size and structure were considered in Case IV (Grema and Cao, 2014). Table 1 summarises these cases.

2.3 Data Collection and Regression

With the arrangement shown in Fig. 1, the manipulative variables, MVs are injection and total production rates; but with voidage replacement assumption, the MVs were reduced to two (two DOF). To be able to implement this assumption, well pairing was employed where Injector, I1 was paired with producer, P1 and I2 with P2. So, with this setup, injection rates from I1 must equal total production rates from P1 at all time-steps and likewise with I2 – P2 pairing.

The total production time was fixed to two years (730 days) with a sampling time-step of one day. At each time-step, four measurements which include oil and water production rates from wells P1 and P2 are recorded. The measurement vector is given by

$$y = [y_{o1}\ y_{o2}\ y_{w1}\ y_{w2}]^T \tag{10}$$

where $\mathrm{y_{o1}}$ and $\mathrm{y_{w1}}$ are oil and water production rates from P1 respectively while $\mathrm{y_{o2}}$ and $\mathrm{y_{w2}}$ the respective measurements from P2. In addition to these measurements, the objective function, which is the net present value (NPV) of the process was also computed using the same economic parameters as used by Grema and Cao (2014) and shown in (11).

$$J^k = \left\{ \frac{\sum_{j=1}^{N_{prod}} \left[r_o (y_{o,j})^k - r_{wp} (y_{w,j})^k \right] - \sum_{i=1}^{N_{inj}} r_{wi} (u_{wi,i})^k}{(1+b)^{\frac{t^k}{\tau}}} \right\} \Delta t^k \tag{11}$$

where r_{wi}, r_{wp} and r_o are water injection and production costs, and oil price respectively. u_{wi}, y_w and y_o are water injection and production rates, and oil production rate respectively. N_{inj} and N_{prod} are number of injection and production wells respectively. b is a discount factor, Δt^k is time-step size, t^k is the actual time period for which NPV is computed while τ is the time unit.

Here, 500 solution trajectories were generated. For the first trajectory, the flooding process was simulated for two years using the actual optimal control solutions. The optimal controls were then slightly perturbed for subsequent trajectories. However, the controls for the first two time-steps were not perturbed because two past histories are needed (n = 2). Since there are two MVs for this system, (5) can be modified as

$$J_{i+1} - J_i = \sum_{k=n+1}^{N} C\left(\boldsymbol{\theta}_1, \boldsymbol{y}_{1,i}^k, \boldsymbol{y}_{1,i}^{k-1}, \dots \boldsymbol{y}_{1,i}^{k-n}, u_{1,i}^k\right)\left(u_{1,i+1}^k - u_{1,i}^k\right) + C\left(\boldsymbol{\theta}_2, \boldsymbol{y}_{2,i}^k, \boldsymbol{y}_{2,i}^{k-1}, \dots \boldsymbol{y}_{2,i}^{k-n}, u_{2,i}^k\right)\left(u_{2,i+1}^k - u_{2,i}^k\right) \tag{12}$$

where $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ are parameter vectors for the two CVs to be determined through regression. The vectors of the measurements, $\boldsymbol{y}_1$ and $\boldsymbol{y}_2$ are for the respective production wells P1 and P2 given as

$$\begin{aligned} y_1 &= [y_{o1}\ \ y_{w1}\ ]^T \\ y_2 &= [y_{o2}\ \ y_{w2}\ ]^T \end{aligned} \tag{13}$$

In (12), the MVs, u_1 and u_2 which are water injection rates from I1 and I2 respectively are included so that expressions can be obtained explicitly as the feedback control laws. Each of the parameter vectors in (12) has seven elements considering number of measurements with past histories. Modifying for each CV, we have

$$C_1 = \theta_1 y^k_{o1,i} + \theta_2 y^k_{w1,i} + \cdots + \theta_6 y^{k-2}_{w1,i} + \theta_7 u^k_{1,i}$$
$$C_2 = \theta_8 y^k_{o2,i} + \theta_9 y^k_{w2,i} + \cdots + \theta_{13} y^{k-2}_{w2,i} + \theta_{14} u^k_{2,i} \quad (14)$$

The feedback control law is obtained by setting C_1 and C_2 to zero in (12)

$$u^k_{1,fb} = -\theta_7^{-1}[\theta_1 y^k_{o1} + \theta_2 y^k_{w1} + \cdots + \theta_6 y^{k-2}_{w1}]$$
$$u^k_{2,fb} = -\theta_{14}^{-1}[\theta_8 y^k_{o2} + \theta_9 y^k_{w2} + \cdots + \theta_{13} y^{k-2}_{w2}] \quad (15)$$

where $u^k_{1,fb}$ and $u^k_{2,fb}$ are the two optimal settings of injection wells.

2.4 Performance Evaluation

To evaluate the performance of the developed approach, true optimal solutions of the three uncertain cases were obtained by solving the optimal control problem directly on these mismatched models (ideal solutions) to establish a benchmark (BM). The open-loop optimal solutions (OC) obtained based on the nominal model were implemented on the uncertain models (worst case). The performance of the SOC approach should therefore lie between those of BM and OC approaches. The deviation in performance of SOC and OC methods from the benchmark is evaluated as loss given by

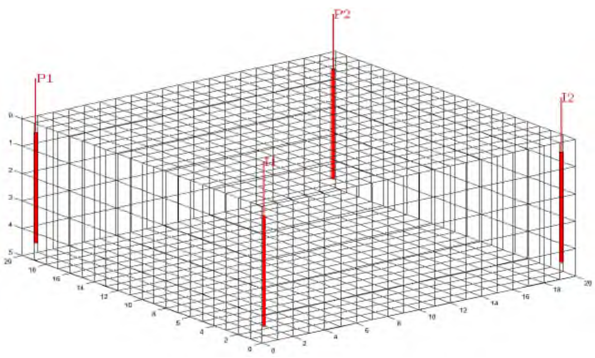

Fig. 1: Nominal Reservoir with Wells

Table I: Uncertainty Cases

Case	Property	Uncertain Case
II	Permeability and size	Log-normal distribution and 100x100x10 m^3
III	Corey exponent	1.5
IV	Geometry, Size, Grid and Structure	Corner point, 225x22.5x 10 m^3, 30 x 3 x 1, with fault

$$Loss = \frac{J_{BM} - J_{SOC/OC}}{J_{BM}} \times 100\% \quad (16)$$

The improvement of SOC strategy over OC is expressed as a gain computed by (17)

$$Gain = \frac{J_{SOC} - J_{OC}}{J_{SOC}} \times 100\% \quad (17)$$

3. RESULTS AND DISCUSSIONS

3.1 Regression

The two feedback control laws obtained are

$$u^k_{1,fb} = 0.1436 y^k_{o1} + 0.8565 y^{k-2}_{o1} + 1.0005 y^{k-2}_{w1}$$
$$u^k_{2,fb} = 0.1435 y^k_{o2} + 0.8566 y^{k-2}_{o2} + 1.0005 y^{k-2}_{w2} \quad (18)$$

It can be seen that only three measurements out of total six are relevant in the CV functions which comprises of both oil and water production rates. The immediate past measurements (*n = 1*) are irrelevant (regression parameter values of zero) but the current (*n = 0*) and past two (*n = 2*). An excellent regression performance with R-square value of 1.0 was obtained. This indicates that no higher-order polynomial or more sophisticated model is required. For an injection-production system where productions from two wells are equal, we should expect equal injection settings as suggested by (18). Results of each case are given and discussed in Sections 3.2 – 3.5

3.2 Case I: Nominal Parameters

The optimal feedback control laws, (18) obtained are implemented on the nominal model for a period of two years. This production strategy was compared to the true optimal solution (OC). The NPV recorded from SOC strategy is $128,903.70 while that from OC is $128,904.90. The loss is almost zero (0.0009593%). This shows the CVs obtained are almost perfect.

Fig. 2 shows injection settings for the two approaches. The optimal injection settings for the two wells as obtained by both approaches (OC and SOC) can be seen to be equal at each time-step. This validates the accuracy of the feedback control law given in (17). For the OC case, two regions can be identified from the injection profile; a rapidly increasing and decreasing region which spans for about 170 days from the beginning of production then followed by a constant injection regime for the remaining period. However, three distinguishing regions can be seen with SOC approach which consists of a steadily increasing phase (160 days) followed by a sharp decline phase and finally an ascending phase.

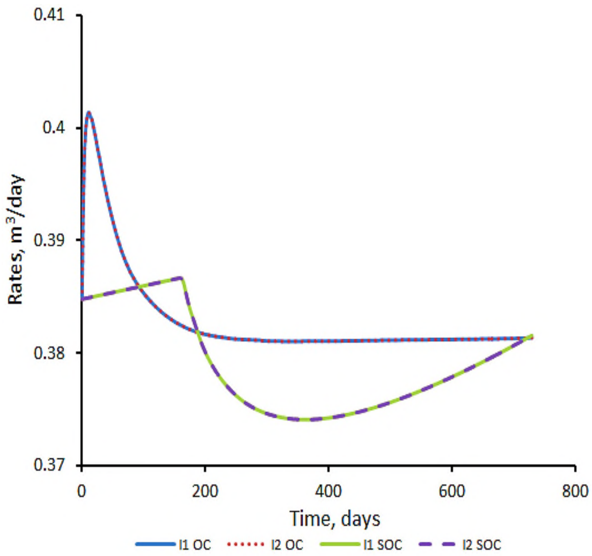

Fig. 2: Injection Settings for the Nominal Case

Another interesting feature is that the injection rates meet just at the end of production period. It is worth to know that the variability of the injection settings found by the two approaches is almost the same with respective standard deviations of 0.004287 and 0.004351 for OC and SOC.

3.3 Case II: Uncertainty in Permeability and Reservoir Size

For this case where reservoir size is increased by 97.5% with random permeability fields for each of five layers, it is obvious that the open-loop solution is non-optimal in this case with a loss of 93.21% when compared to BM case while performance of SOC is similar to that of the BM scenario. The loss here is only 0.018% with a gain of 93.21% over OC approach.

3.4 Case III: Uncertainty in the Shape of Relative Permeability Curves

The nominal values of Corey exponents for both oil and water relative permeability curves are 2.0 while the real values were considered to be 1.5 each. For this uncertainty, a loss of only 0.023% was incurred as a result of SOC implementation with a gain of 0.25%. The loss is 0.27% with OC approach.

3.5 Case IV: Uncertainty in Reservoir Size, Geometry and Structure

For this huge uncertainty consideration, open-loop solution has woefully failed to optimize the waterflooding process with a loss of 95.07%. On the other hand, the optimal feedback controls obtained based on the nominal model proved to be very robust in the presence of these uncertainties with a loss of only 0.45% when compared to the BM case that assumed perfect reservoir knowledge. The SOC approach has a gain of 95.05% over the OC case.

A summary of the results for all the four cases is given in Table 2.

Table 2: Results Summary

		NPV($)	**% Gain**	**% Loss**
Case I	OC	128,904.90	-	-
	SOC	128,903.70	-	-
Case II	BM	4,732,358.83	-	-
	OC	321,245.07	-	93.21
	SOC	4,731,512.20	93.21	0.018
Case III	BM	119,037.93	-	-
	OC	118,710.67	-	0.27
	SOC	119,010.69	0.25	0.023
Case IV	BM	6,808,782.37	-	-
	OC	335,602.48	-	95.07
	SOC	6,778,147.29	95.05	0.45

4. CONCLUSIONS AND RECOMMENDATIONS

An optimal feedback control for reservoir waterflooding operation was formulated using the principle of self-optimizing control. The CVs were derived from regression based on a nominal model. For the purpose of CV regression, injector-producer pairing was employed, where simulated measurements made up of oil and water production rates were recorded. The gradients of the objective function with respect to controls were selected as the CVs. The CVs were then approximated with linear functions of current and past measurements (typically two past histories) which were fitted to the data via least squares regression. The robustness of the CVs was tested by initially implementing it on the nominal model and then to cases with system mismatches. The performance of the SOC method was compared with open-loop solution based on optimal control theory as well as benchmark case. Findings are summarised as follows:

1. The two feedback control laws were found to have same regression coefficients, in other words, the regression resulted to symmetrical CVs.
2. Implementing the CVs on the nominal model resulted to an almost zero loss. The true optimal injection trajectories as found through optimal control theory were identical for the two injectors. This was also the case with SOC's solution.
3. A total failure of the open-loop solution was observed in two cases when the reservoir size is increased whereas SOC performed well with performance indices similar to the benchmark cases'.
4. The relative performance of SOC increases with increase in the degree of uncertainty while that of OC deteriorates in that order.
5. Although uncertainties were not sampled for CV determination due to the complexity of reservoirs, the CVs robustness is attributed to the feedback nature of SOC.
6. The work adopted a simplified reservoir system, it is therefore recommended to test the robustness of the method on a realistic reservoir.

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