2nd IFAC Workshop on Automatic Control in Offshore Oil and Gas Production,
May 27-29, 2015, Florianópolis, Brazil

# Systemwide Optimal Control of Offshore Oil Production Networks with Time Dependent Constraints

**Marco Aurélio Aguiar** * **Andres Codas** **
**Eduardo Camponogara** *

* *Department of Automation and Systems Engineering, Federal University of Santa Catarina, Florianópolis, SC 88040-900 Brazil (e-mails: marcoaaguiar@gmail.com, eduardo.camponogara@ufsc.br)*[1]
** *Department of Engineering Cybernetics, Norwegian University of Science and Technology, 7491 Trondheim, Norway, E-mail: andres.codas@itk.ntnu.no.*

**Abstract:** Operating policies for oil production systems based purely on static behavior can incur excessive gas flaring and potentially violate environmental regulations, specially when the system undergoes transients in response to predictable and unanticipated events. On the other hand, optimal control strategies can reach an optimal steady state by accounting for the system dynamics, while handling the transients triggered by such events. This paper develops optimal control strategies for a representative class of offshore oil production systems that consist of a gas-lift injection system, subsea equipment, and surface processing units. The optimal control problems are solved with the collocation method, which discretizes time and uses polynomial interpolation to find a continuous-time solution. The collocation method results in a large, but sparse nonlinear programming problem which is solved with state-of-the-art algorithms for dynamic optimization. We show an application where the proposed method can be used to plan and schedule processing equipment maintenance.

*Keywords:* Optimal Control, Production System Modeling, System-wide Dynamic Optimization, Plantwide Control

## 1. INTRODUCTION

Oil production optimization is usually carried out disregarding system dynamics [Codas et al. 2012]. In gas-lifted oil fields, the production optimization problem is typically set up to maximize oil production in steady-state, which is achieved by injecting high rates of gas into the wells. The gas-lift rates that would induce maximum production may exceed the capacity of the compressors, which therefore impose constraints on the optimization problem.

Field engineers are often reluctant to implement new operating setpoints because wells and risers may be difficult to control [Eikrem et al. 2008, Jahanshahi and Skogestad 2011]. The implementation of optimized new setpoints can force wells to undergo transients, potentially violating the operating limits and ultimately incurring production loss. Therefore, it is desired to analyze the consequences of switching strategies on dynamic models before their application. In order to optimize production during transients, time-depend rather than steady-state models must be considered.

The contribution of this work is twofold. First, the paper proposes a system-wide dynamic model for oil and gas production networks, which arises from the interconnection of dynamic unit-process models that are available in the literature, such as production wells, pipeline-risers, and separators inspired in the works of Codas et al. [2013], Jahanshahi et al. [2012], Jahanshahi and Skogestad [2011], and Sayda and Taylor [2007]. Second, an optimal control strategy is developed to mitigate the side effects on oil production that are caused by dynamic perturbations. The perturbations arise from time-dependent constraints, such as compressor scheduled-maintenance which is considered in this work.

Three optimal control strategies are applied to the compressor scheduled-maintenance problem (CSM), which consists in optimizing oil production over a prediction horizon that includes a reduction in compressor capacity. The first control strategy is based on tracking the operating point that results from steady-state optimization. The second strategy maximizes production during the prediction horizon based on dynamic optimization. The third control strategy also tracks a steady-state operating point, however high penalization is imposed to avoid gas flaring.

As explained so far, the CSM problem can be stated as a dynamic optimization problem. In the last decades, algorithms and software tools have been developed to tackle large dynamic optimization problems that enable practical applications. Today, nonlinear programming solvers are

[1] This work was funded in part by CNPq under grant 471978/2013-2.

available off-the-shelf such as IPOPT [Wächter and Biegler 2005] which, combined with automatic differentiation tools like CasADi [Andersson et al. 2012], can solve large and sparse problems arising from the discretization of dynamic optimization problems. Together with a high-level modeling language known as Modelica/Optimica, these tools are conveniently integrated in an open-source platform called JModelica.org [Åkesson et al. 2010].

## 2. MODELING

From the reservoir to the exportation line, the oil field model is composed of gas-lift wells, subsea manifolds, pipelines, risers, separators, compressors, and an exportation line, as shown in Figure 1. This section presents the unit-process models, in the form of differential algebraic equations (DAE), which are later combined into a system-wide model of the production network.

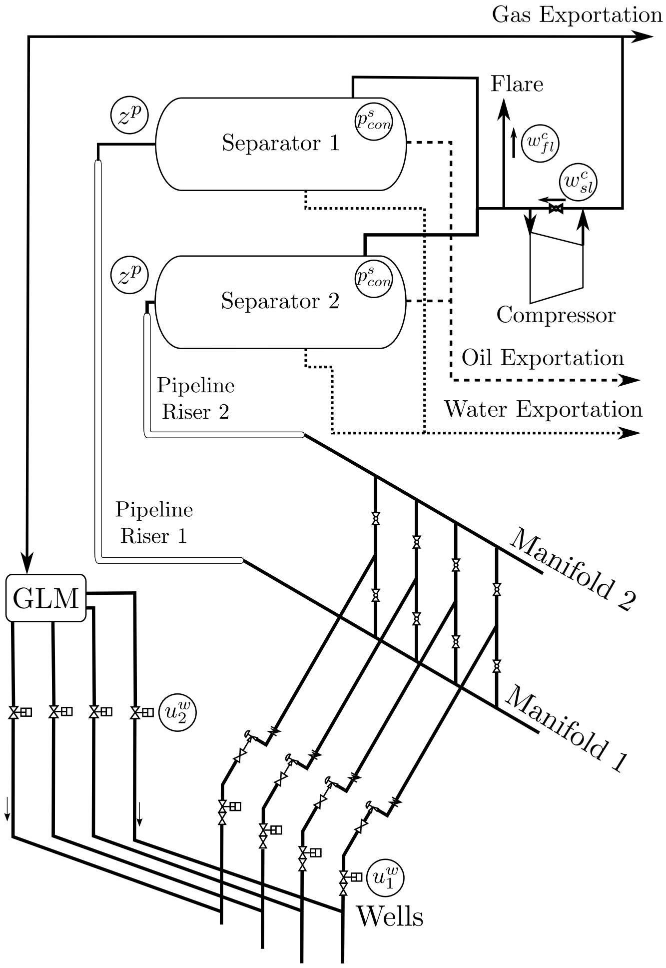


Fig. 1. Production network illustration. GLM = Gas-Lift Manifold.

### 2.1 Well

The wells are the elements that draw hydrocarbons from the reservoir. In gas-lifted wells, high pressure gas is injected at the bottom of the production tubing to increase the pressure gradient from reservoir to wellhead, thereby increasing the production flow.

A well can be split in two distinct parts: the annulus and the production tubing. The annulus is the space between the casing and the production tubing, within which flows the high pressured gas from the gas-lift choke to the injection valve at the bottom of the well. The streams of gas, oil, and water emanating from the reservoir flow inside the production tubing to reach the wellhead.

The model of Jahanshahi et al. [2012] was chosen to represent the dynamics involved in gas-lift injection and flow production of the wells. Despite being defined in a reduced space, the well dynamics are consistent with more complex models that are described by partial differential equations [Plucenio et al. 2012].

The model state is represented by the vector $x_w \in \mathbb{R}^3$ that has the mass of gas in the annulus, the mass of gas in the tubing, and the mass of liquid in the tubing. The vector $y_w$ contains several algebraic variables for mass flow, density, and pressure. The boundary conditions are the inlet pressure $p_{in}$ (at the point where the well connects to the gas-lift manifold) and the outlet pressure $p_{out}$ (at the point where it connects to the production manifold). The control variables are the production-choke opening $u_1$ and the gas-lift choke opening $u_2$. The model output is the gas-lift mass rate $w_{gl}$ transferred from the gas-lift manifold into the annulus, and the mass flow-rate vector $w_{out}$ containing the gas, oil, and water that goes through the production choke.

The well model of Jahanshahi et al. [2012] follows the same principles of the model of Eikrem et al. [2008], however the former improves upon the latter by accounting for the pressure loss due to friction and the calculation of phase fractions. The interested reader may refer to [Jahanshahi et al. 2012] for a full description of the well model.

### 2.2 Manifolds

Manifolds are the components that connect different elements, merging and splitting flows. The production manifold gathers the streams coming from various wells into a single stream that is directed to a single pipeline-riser, meaning that routing is fixed. The gas-lift manifold distributes the gas coming from the compressor to the production wells.

For the production manifold, which has a many-to-one design, the pressure of the $i \in \{1, \dots, N\}$ inlet is equal to the outlet pressure ($p_{i,in} = p_{out}$), being $N$ the total number of inlets. The outlet flow is equal to the sum of the inlet flows ($w_{out} = \sum_{i=1}^{N} w_{i,in}$). These algebraic variables are represented by a vector $y_m$. For the gas-lift manifold, which has a one-to-many design, the pressure of the $j \in \{1, \dots, M\}$ outlet is equal to the inlet, and the sum of outlet flow is equal to the inlet ($w_{in} = \sum_{j=1}^{M} w_{j,out}$).

### 2.3 Pipeline and Riser

The flow of each production manifold is transferred to the platform by a pipeline that has a horizontal and a vertical segment, with the latter typically referred as riser. The flow in the horizontal pipeline section is stratified, whereas in the riser a bubble-flow pattern prevails.

The combined pipeline-riser model in this work was developed by Jahanshahi and Skogestad [2011]. The model uses four state variables, two for the mass of gas and liquid in the pipeline and two for the mass of gas and liquid in the riser. The pipeline and the riser models are connected through a virtual valve, whose flow depends on the pressure drop and liquid level at connecting point. The model takes as input the incoming three-phase flow $w_{in}$ (with constant water cut), the outlet pressure $p_{out}$, and the choke opening $z$. This choke is located at the topside of the riser where it connects to the separator. The pipeline-riser outputs are the pressure at the pipeline inlet $p_{in}$ and the three-phase flow at the riser outlet $w_{out}$.

The model provides a reasonable trade-off between dynamics representation and the number of variables. In [Jahanshahi and Skogestad 2011], a comparison with the state-of-the-art Schlumberger's multiphase flow simulator Olga shows that they have compatible results for pressures and flows. The referred paper also compares the Jahanshahi's model with others available in the literature.

### *2.4 Separator*

The gravitational separator is the first stage of the topside processing unit. It splits the incoming three-phase flow into three outlet flows: gas, oil, and water. The separation efficiency depends on the separator pressure and the flow being processed. Because separator dynamics is considerably faster than the well and pipeline-riser dynamics, the model proposed by Sayda and Taylor [2007] was simplified in order to disconsider the dynamics of masses inside the separator. The model adaptation captures the steady state behavior of the separator by implementing an instantaneous input-output response.

The separator have one inlet that takes a homogeneous flow ($w_{in}$) of gas, oil, and water. Each phase has one outlet: gas ($w_{g,out}$), oil ($w_{o,out}$), and water ($w_{w,out}$). The inlet and outlet pressure are equal to $p_{con}$, which is the reference of a regulatory controller responsible for pressure maintenance inside of the separator. The separation performance is a function of the flows and pressures in the separator.

For the purposes herein, this work develops a simplification of the model of Sayda and Taylor [2007] with a varying separation efficiency, assuming a cylindrical separator and using basic geometry and chemical principles. The simplified separator model is presented in Appendix A.

### *2.5 Compressor*

The compressor dynamics is considerably faster than the dynamics of the other processes. Therefore, modeling the operational limits of the compressor is more relevant than modeling its dynamics. Such limits are obtained from the compressor map which is obtained from data sheets provided by the manufacturer, as exemplified in Figure 2.

The compressor map relates the inlet volumetric flow and compressor speed with the pressure gain from the inlet and outlet. The continuous lines of the map in Figure 2 represent the compressor behavior at fixed operational speeds; the dashed lines indicate the compressor safety limits. The map is bounded at the top and the bottom by the maximum and minimum operational speed, respectively, which are established for safety and technical reasons. The dashed line on the left corresponds to the surge line, which defines the boundary of the unstable region caused by a low inlet flow rate. The dashed line on the right corresponds to the choke line, which establishes the inoperable region where the inlet flow rate is too high to be handled by the compressor.

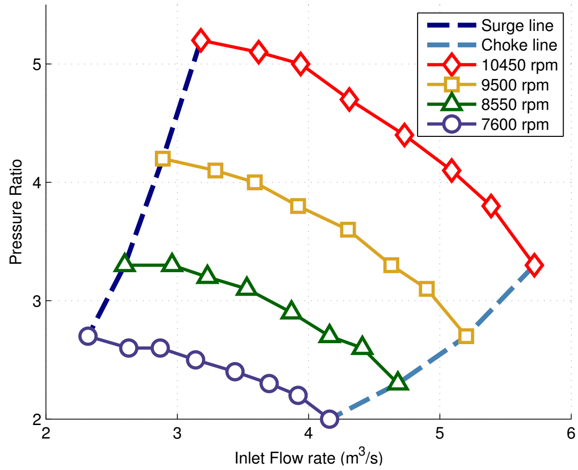

Fig. 2. Example of compressor map

To avoid compressor surge, an anti-surge line is usually installed connecting the compressor outlet to its inlet. By recirculating part of the compressor outflow, it is possible to increase the inflow. To avoid flows beyond the choke line, an exhaustion valve prior to the inlet allows to direct part of the incoming flow to the gas flare.

A mathematical model was developed by approximating each of the boundaries by a 2[nd] order polynomial. The parameters of the polynomials were obtained by solving a least-squares problem using the compressor map data.

The inputs to the model are the inlet flow $w_{in}$, the inlet pressure $p_{in}$, and the desired outlet pressure $p_{out}$. The control variables are the gas mass flow sent to the flare $w_{fl}$ and the amount of gas recirculated in the anti-surge line $w_{sl}$. The output is the gas mass flow at the outlet $w_{out}$. It is assumed that a regulatory controller actuates on the compressor speed to track the reference for the outlet pressure.

The model developed in this work does not account for the energy required for compressor operation. The outlet flow is split in two stream, one directed to the exportation line and the other sent to the gas-lift manifold for injection in the wells.

## 3. NETWORK COMPOSITION

After instantiating the models above for each physical unit, their inputs and outputs are connected in order to assemble the production system. The model inputs that remain free correspond the control variables that will be subject to optimization, such as the openings of the gas-lift choke valves.

 

### 3.1 Component Connection

Let $N_w$ be number of wells, $N_m$ be the number of manifolds, $N_p$ be the number of pipeline-risers, $N_s$ be the number of separators. Moreover, let the function $\mathcal{R}_{\text{w,m}}(w)$ return the $m$ manifold connected to the well $w$, the function $\mathcal{R}_{\text{m,p}}(m)$ give the pipeline $p$ connected to manifold $m$, and the function $\mathcal{R}_{\text{p,s}}(p)$ return the separator $s$ that is connected to pipeline $p$. Assume that the superscript $w$ is a reference to well $w \in \{1,\ldots,N_w\}$, $m$ is a reference to manifold $m \in \{1,\ldots,N_m\}$, $p$ is a reference to the pipeline $p \in \{1,\ldots,N_p\}$, $s$ is a reference to the separator $s \in \{1,\ldots,N_s\}$, $c$ is a reference to the compressor, and $glm$ is a reference to the gas-lift manifold. Likewise, the superscript $exp$ is a reference to the exportation lines of gas ($w^{exp}_{gas}$), oil ($w^{exp}_{oil}$), and water ($w^{exp}_{water}$).

Based on the models described in Section 2 and the definitions given above, the system is obtained by imposing mass balances and pressure equalities between the inlets and outlets of the components. For every well $w \in \{1\ldots,N_w\}$ and the corresponding manifold $m = \mathcal{R}_{\text{w,m}}(w)$, the equations that connect the gas-lift manifold to the well and the well to the production manifold are:

$$w^w_{gl} = w^{glm}_{w,out}, \qquad p^w_{in} = p^{glm}_{w,out} \tag{1a}$$
$$w^m_{w,in} = w^w_{out}, \qquad p^m_{w,in} = p^w_{out} \tag{1b}$$

For each manifold $m \in \{1\ldots,N_m\}$ and the corresponding pipeline $p = \mathcal{R}_{\text{m,p}}(m)$, the equations that connect the manifold to the pipeline are:

$$w^m_{out} = w^p_{in}, \qquad p^m_{out} = p^p_{in} \tag{2}$$

For each pipeline $p \in \{1,\ldots,N_p\}$ and the connected separator $s = \mathcal{R}_{\text{p,s}}(p)$, the equations that connect the pipelines to the separators are:

$$w^p_{out} = w^s_{in}, \qquad p^p_{out} = p^s_{in} \tag{3}$$

Since a separator has three outlets, the connections are handled differently:

$$\sum_{s=1}^{N_s} w^s_{g,out} = w^c_{in}, \qquad p^s_{out} = p^c_{in} \quad \forall s \in \{1,\ldots,N_s\} \tag{4a}$$
$$w^{exp}_{oil} = \sum_{s=1}^{N_s} w^s_{o,out}, \qquad w^{exp}_{water} = \sum_{s=1}^{N_s} w^s_{w,out} \tag{4b}$$

Finally, the gas compressor is connected to the gas exportation line and the gas-lift manifold:

$$p^c_{out} = p^{exp} = p^{glm}_{in}, \qquad w^{exp}_{gas} = w^c_{out} - w^{glm}_{in} \tag{5}$$

where $p^{exp}$ is a specified pressure that should be kept constant. It is not unusual to have a single compression station serving the exportation line and gas-lift injection system [Rasmussen and Kurz 2009]. Therefore, the pressure of the gas-lift manifold is considered equal to the exportation line, disconsidering the pressure variation in the connecting line.

The set of equations above leave some of the inputs not assigned, namely $u^w_1$ and $u^w_2$ for every well $w$, the choke opening $z^p$ of every pipeline $p$, the controlled pressure $p^s_{con}$ of the separators, the gas flared $w^c_{fl}$ and the gas in the anti-surge line $w^c_{sl}$ of the compressor. Let vector $u_{sys}$ aggregate all of the free variables.

### 3.2 Optimal Control Problem

After describing the dynamic behavior and the algebraic relations of the units, and further establishing how the units interact with one another, a generic optimal control problem is formulated for the production network. Let $\psi(\cdot)$ be the objective function, which will be specified later. Then, given a time horizon $t \in [t_0, t_f]$ for control, the optimal control problem (OCP) is defined as:

$$\min_{u^{sys}} \quad \psi(x_{sys}, y_{sys}, u_{sys}, t_0, t_f) \tag{6a}$$
$$\text{s.t.:} \quad \dot{x}_{sys} = f_{sys}(x_{sys}, y_{sys}, u_{sys}, t) \tag{6b}$$
$$g_{sys}(x_{sys}, y_{sys}, u_{sys}, t) = 0 \tag{6c}$$
$$g_{con}(x_{sys}, y_{sys}, u_{sys}, t) = 0 \tag{6d}$$
$$c_{sys}(x_{sys}, y_{sys}, u_{sys}, t) \leq 0 \tag{6e}$$
$$x_{sys}(t_0) = x_{sys,t_0} \tag{6f}$$

where the vector $x_{sys}$ contains the state variables of the wells and pipelines and $x_{sys,t_0}$ has their initial conditions, the vector $y_{sys}$ contains the algebraic variables of all elements in the network, and the vector $u_{sys}$ contains the control variables. Further, the function $f_{sys}$ defines the dynamic response of the state variables $x_{sys}$, the function $g_{sys}$ defines the algebraic variables $y_{sys}$, the function $g_{con}$ establishes the relationships between input and output of the components, and the function $c_{sys}$ represents the constraints over the state, algebraic, and control variables [2].

## 4. COMPRESSOR SCHEDULED MAINTENANCE

Rasmussen and Kurz [2009] describe the use of gas compressors in the gas and oil industry and the need of routine compressor maintenance. For this reason, it is common to have more than one compressor sharing the load, in such way that if one of the compressors goes down the entire production system keeps producing with minor side effects. The compressors can be arranged either in series or in parallel. With the series configuration, gas compression is performed in two stages, whereby each compressor yields half of the pressure gain while sharing the same flow rate. With a parallel configuration, each compressor handles half of the flow while the pressure gain is the same. Although both configurations are valid, the latter is widely used by the industry for being more resilient.

Because the compressors are identical, under normal operating conditions they can be modeled as a single unit by multiplying the volumetric flow that enters the compression system ($q_v$) by a factor $f_{c,0} = 1/N_c$, where $N_c$ is the number of parallel units in the compression station. In the case of a compressor shutdown, the factor can be raised to $f_{c,f} = 1/(N_c - 1)$. Because an abrupt change causes a major disturbance in the system, this work considers a smooth change implemented with a sigmoid function for being economically advantageous. Let $f_c(t) = f_{c,0} + (f_{c,f} - f_{c,0})\theta(t - t_t)$ be the time dependent factor and $\theta(t)$ be the sigmoid function, which is mathematically defined as:

$$\theta(t - t_t) \approx \frac{1}{1 + e^{-2\tau(t-t_t)}} \tag{7}$$

[2] Such constraints might be problematic for some solution methods (*e.g.*: indirect methods, single-shooting method).

where $t_t$ is the time at which the transition will be half way through (triggering time) and $\tau$ the transition time constant.

As the flow through the compressor rises it can go towards the choke line in the compressor map (see Fig. 2), being necessary to flare part of the incoming gas, which implies in the operation settings being no longer valid. In order to optimize the transient response of the system during a scheduled compressor maintenance, an optimal control problem of the form (6) can be formulated considering different objective functions as detailed below.

### *4.1 Naive Approach*

A common strategy on process optimization consist in using two layers of optimization. In the top layer, often referred as real-time optimization (RTO), a problem that uses a steady-state model is solved to define the references that will maximize an economic objective. In the lower layer, a dynamic optimization problem is solved considering a quadratic objective function to track the reference provided by the RTO. The naive approach follows this concept to emulate the behavior of an operator that does not have a new reference for the compressor (during the maintenance interval) and uses the flare and the anti-surge lines as a relief to keep the production at the reference values.

Because steady-state RTO and dynamic models are intrinsically different, the same control profile values will induce different responses with respect to masses, flows, and pressures. A possible way to implement the RTO results in the lower layer is by tracking the input variables. In this way, the controller tries to keep the free variables as close as possible to their reference without violating any constraint. The objective is put mathematically as:

$$\min \ \psi = \int_{t_0}^{t_f} (u_{sys}^{ref} - u_{sys})^T R_u (u_{sys}^{ref} - u_{sys})\, dt \qquad (8)$$

where $R_u$ is a weighting matrix, with the penalty entries for the control variables corresponding to the flare and anti-surge line being considerably smaller than the penalties on the other variables.

Rather than solving an RTO problem, a heuristic was used to obtain the references. The heuristic solves an optimal control problem that defines time-invariant control inputs to be implemented during the entire prediction horizon, which is sufficiently long for the system to settle in a steady state. Other alternatives to obtain the static operating point can be considered, such as strategies purely based on static models that can yield a globally optimal operating point [Codas et al. 2012, Aguiar et al. 2014].

### *4.2 Production Maximization Approach*

An alternative to the two-layer strategy is the dynamic real-time optimization (DRTO), by means of which dynamic optimization maximizes an economic objective. Two drawbacks of this strategy are the computational cost which can be impractical for large systems and the need of high-fidelity dynamic models. However, such an approach can bring out economic gains during the transients, not only at the steady-state.

The production maximization approach deals with compressor scheduled maintenance by optimizing oil production. In addition, a penalization is imposed on flaring in order to meet environmental regulations and to prevent financial losses. This strategy will not only reach an optimal steady-state, but also minimize losses during the transients by considering the dynamics induced by a scheduled compressor shutdown. Formally the economic objective is

$$\max \ \psi = \int_{t_0}^{t_f} \left( k_{oil} w_{oil}^{\exp} - k_{\mathrm{fl}} w_{\mathrm{fl}}^{c} - k_{\mathrm{gl}} w_{in}^{\mathrm{glm}} \right) dt \qquad (9)$$

where the parameter $k_{\mathrm{oil}}$ is the monetary gain from selling oil, whereas the parameters $k_{\mathrm{fl}}$ and $k_{\mathrm{gl}}$ are the cost for flaring and injecting gas-lift, respectively.

### *4.3 Smart Tracking Approach*

Similar to the naive approach, smart tracking applies the formulation given in Eq. (8) to track the static operating set-point, however it imposes a high penalty on the flaring gas flow $w_{fl}^{c}$. While the naive approach tries to be as close as possible to the set-point, this approach further reduces flaring due to its high cost.

## 5. COMPUTATIONAL RESULTS

Computational experiments were performed to assess how the different approaches solve the compressor scheduled maintenance problem.

The oil production network was specified using the Modelica dynamic modeling language and the three optimal control approaches were formulated using Optimica, a Modelica extension for dynamic optimization. The code was compiled using the JModelica.org 1.14 environment, which has an algorithm to transform the continuous-time nonlinear optimal control problem into a nonlinear programming problem according with the direct collocation method. The JModelica.org environment also provides some of the most advanced tools for dynamic optimization, including the Sundials simulation package, the CasADi automatic differentiation tool, a Python scripting interface, and the large-scale nonlinear optimization solver IPOPT. The computational experiments were carried out in a computer equipped with an Intel i5-4440QM @ 3.10GHz and 8 GB of RAM.

The direct collocation method used 80 finite elements, each with a 2$^{\text{nd}}$ order polynomial, resulting in a configuration with a finite element of 150 seconds.

### *5.1 Oil Production Network*

A synthetic oil field was put together by combining subsystem components with properties of typical real-world systems. The oil field is composed of 4 wells, 2 subsea manifolds, 2 pipeline-risers, 2 separators, 2 compressors, and a gas-lift manifold. The oil production network has the same set-up of the network depicted in Figure 1.

The wells are arranged in 2 clusters, with 2 wells each. All wells have a tubing with an inner diameter of $12.4\,cm$, constant temperature of $348\,K$, and the injection point located at $75\,m$ from the bottom hole; the annulus has a hydraulic inner diameter of $16\,cm$. The parameters that vary depending on the well are in the Table 1, being $p_r$ the reservoir pressure, $GOR$ the gas-oil ratio, and $PI$ the reservoir production index.

Table 1. Wells parameters

| Well No. | Depth ($m$) | $p_r$ (bar) | $GOR$ ($sm^3/sm^3$) | PI ($sm^3/d/bar$) |
|---|---|---|---|---|
| 1 | 1500 | 200 | 90 | 10.3 |
| 2 | 1900 | 220 | 120 | 11.5 |
| 3 | 1300 | 190 | 50 | 8.0 |
| 4 | 1800 | 210 | 160 | 9.2 |

The production of each cluster flows to a subsea manifold, which routes the flow to a pipeline-riser that elevates the production to a surface separator. The section that lays in the seabed has an inner diameter of $12\,cm$ and length of $4300\,m$ in the Pipeline-Riser 1 and $5100\,m$ and in the Pipeline-Riser 2. The vertical section has an inner diameter of $10\,cm$, the Pipeline-Riser 1 has a height of $700\,m$ and the Pipeline-Riser 2 has $900\,m$.

The separator splits the flow in three phases, with the oil and water phases going out of the system, whereas the gas is directed to a compressor for well gas-lift injection and exportation. A safety valve is installed upstream the compressor to release gas to the flare when the inlet flow is above the compressor capacity. The separator pressure must be at least 50 bar and not above 60 bar. The prediction horizon has 12000 seconds, which is about 3 hours and 20 minutes. The compressor has initially 2 parallel units, the maintenance will occur after 6000 seconds of simulation, and the time constant is equal to 1/500. The network is not allowed to drain gas from the exportation line to be injected, thus $w_{gas}^{exp} \geq 0$.

Initially, the system is operated with gas-lift choke opening at 20%, production choke at 100%, riser choke opening at 100%, and separator pressure at 55 bar.

### *5.2 Experiments*

Figure 3 illustrates the results of the proposed approaches, showing values for oil production ($w_{oil}^{exp}$), gas flaring rate ($w_{fl}^{c}$), and gas-lift choke opening ($u_2^w$) for well 1. The compressor constraints are all satisfied, and the gas exportation is greater than zero before the compressor maintenance and equal to zero afterwards, meaning that all the gas produced is injected in the wells.

It can be notice from the results that, by tracking the control reference, the naive approach is forced to burn a considerable rate of gas in the flare. The oil production rate suffers only a small reduction of 0.8 % when compared to the production plateau, but at the cost of flaring $0.6kg/s$ of gas. Moreover, after the compressor shutdown the opening of the gas-lift valve is reduced to ensure that the gas injection is not higher than its production.

The oil production maximization approach incurs a minor reduction of 1.7 % in relation to the production plateau, while no flaring is caused. The gas-lift choke opening is reduced to induce a lower production of gas and thereby prevent gas flaring, which would be caused by the limited capacity of compression. However, oscillations in the oil flow and controlled variables occur towards the end of the

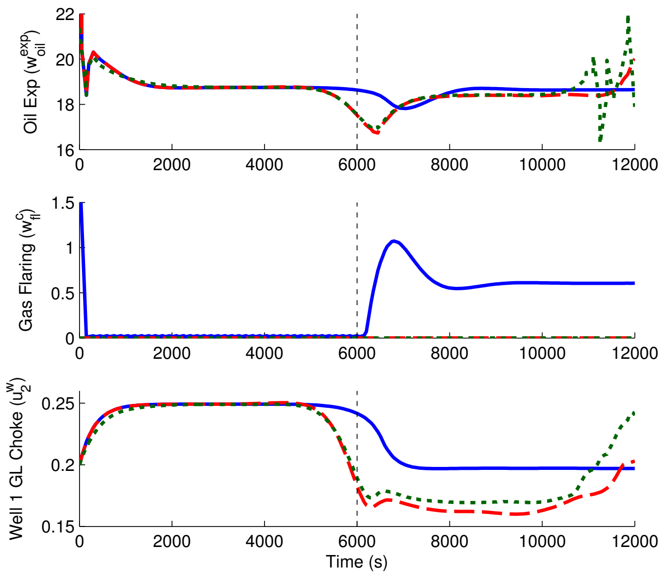


Fig. 3. Optimization Results. The continuous line is the naive appr., the short dashed line is production maximization, and the long dashed line is smart tracking.

prediction horizon. Some techniques could be implemented to avoid this undesirable behavior, *e.g.:* penalization of the oil production derivative and the enforcement of constant control profiles at the end of prediction horizon.

With the smart tracking approach the system suffers only a 1.8 % decrease in the oil production, about the same underproduction induced by the production maximization approach. Further, smart tracking and production maximization have similar behavior considering gas flaring and choke openings: Both strategies do not cause gas flaring and induce system oscillations near the end of the prediction horizon, which can be mitigated by the strategies suggested above.

Since the developed nonlinear programming (NLP) formulation is complex, the solving time and algorithm performance depend on the initial guess and the parametrization of the problem and the solver. For the instances used, the NLP problems were solved in the range from 20 seconds to 300 seconds for all control strategies. The wide variation on the solution time is a consequence of the solution space that have 45,000 variables and moderate nonlinearity (*e.g.:* square-root and logarithmic functions). The initial guess is also decisive factor, if implemented using a rolling horizon the solution time should decrease with appliance of a warm start technique.

It is noticed that after compressor shutdown the smart tracking and naive approach try to follow an unreachable reference, in which case the obtained solution strives to minimize the distance to the reference. This sort of situation is better studied by Rawlings et al. [2008], who developed the unreachable setpoint MPC for which some properties were derived.

The results shows that the control solutions based on the two-layer scheme (*i.e.*, RTO and dynamic optimization) are sensitive to the operating conditions. For the case of tracking an infeasible set point, undesirable behavior and gas flaring may result from an inadequate parametrization of the control-weighting matrices. On the other hand, the single-layer scheme (DRTO) followed by the production maximization approach is computationally expensive, but more reliable regarding the economic performance.

## 6. CONCLUSION

This work integrated unit models from the literature into a systemwide dynamic model for control and optimization of oil production networks. The developments and analyses considered the dynamic optimization problem that arises from time-dependent constraints, in particular a scheduled compressor maintenance. The dynamic optimization problem was converted into a nonlinear programming problem by the direct collocation method and subsequently solved with efficient optimization algorithms.

Most of the technical literature focuses on the control of a single unit, with only a few works addressing the systemwide control of the production network. In [Willersrud et al. 2013], a network is modeled considering production and injection wells, pipelines, separator and compressor; this work compares an unreachable setpoint method with the oil production maximization. However, the work of Willersrud et al. [2013] relies on Cybernetica's in-house model library to compose the production network, whereas our model is based on models available in the literature.

We proposed three solution methods for the problem of compressor scheduled maintenance. First, a naive approach shows how a simple tracking technique with no penalization on undesired dynamics leads to flaring and loss of performance. The use of the oil production approach gives good results, suppressing the flare and with a minor reduction in the oil production. The downside of this approach is that it relies on an accurate modeling of the dynamic network. Smart tracking also avoids gas flaring, yielding an overall oil production rate comparable to what is obtained with oil production maximization approach. The good performance of smart tracking can be attributed to the combination of RTO, which is concerned with steady-state behavior, with tracking which is dynamically following the reference provided by RTO.

The modular structure of the systemwide dynamic model emerges from the distributed nature of oil production networks. Such a structure enables the change of a unit model with an alternative model, possibly more accurate, without reformulation of the whole system model. Further, a given systemwide dynamic model can be used for other control and optimization problems, such as the problem of optimizing an economic gain while considering rerouting of wells to manifolds.

Future research will be geared towards the synthesis of a system state observer and the implementation of a closed-loop nonlinear optimal control. Further, the modular structure of the dynamic model suggests the development of a distributed optimization algorithm, instead of a centralized approach, akin to what has been developed in the literature on distributed model predictive control [Camponogara and Scherer 2011].

## REFERENCES

Aguiar, M.A., Camponogara, E., and Silva, T. (2014). A mixed-integer convex formulation for production optimization of gas-lifted oil fields with routing and pressure constraints. *Brazilian Journal of Chemical Engineering*, 31(2), 439–455.

Åkesson, J., Årzén, K.E., Gäfvert, M., Bergdahl, T., and Tummescheit, H. (2010). Modeling and optimization with Optimica and JModelica.org – languages and tools for solving large-scale dynamic optimization problems. *Computers & Chemical Engineering*, 34(11), 1737–1749.

Andersson, J., Åkesson, J., and Diehl, M. (2012). CasADi – a symbolic package for automatic differentiation and optimal control. In *Recent Advances in Algorithmic Differentiation*, volume 87 of *Lecture Notes in Computational Science and Engineering*, 297–307.

Camponogara, E. and Scherer, H.F. (2011). Distributed optimization for model predictive control of linear dynamic networks with control-input and output constraints. *IEEE Transactions on Automation Science and Engineering*, 8(1), 233–242.

Codas, A., Aguiar, M.A., Nalum, K., and Foss, B. (2013). Differentiation tool efficiency comparison for nonlinear model predictive control applied to oil gathering systems. In *9th IFAC Symposium on Nonlinear Control Systems*, 821–826.

Codas, A., Campos, S., Camponogara, E., Gunnerud, V., and Sunjerga, S. (2012). Integrated production optimization of oil fields with pressure and routing constraints: The Urucu field. *Computers & Chemical Engineering*, 46, 178–189.

Eikrem, G., Aamo, O., and Foss, B. (2008). On instability in gas lift wells and schemes for stabilization by automatic control. *SPE Production & Operations*, 23(2), 268–279.

Jahanshahi, E. and Skogestad, S. (2011). Simplified dynamical models for control of severe slugging in multiphase risers. In *18th IFAC World Congress*, 1634–1639.

Jahanshahi, E., Skogestad, S., and Hansen, H. (2012). Control structure design for stabilizing unstable gas-lift oil wells. In *8th IFAC Advanced Control of Chemical Processes*, 93–100.

Plucenio, A., Ganzaroli, C., and Pagano, D.J. (2012). Stabilizing gas-lift well dynamics with free operating point. In *IFAC Workshop on Automatic Control in Offshore Oil and Gas Production*, 95–100.

Rasmussen, P. and Kurz, R. (2009). Centrifugal compressor applications - upstream and midstream. In *38th Turbomachinery Symposium*, 169–186.

Rawlings, J.B., Bonné, D., Jørgensen, J.B., Venkat, A.N., and Jørgensen, S.B. (2008). Unreachable setpoints in model predictive control. *IEEE Transactions on Automatic Control*, 53(9), 2209–2215.

Sayda, A. and Taylor, J. (2007). Modeling and control of three-phase gravity separators in oil production facilities. In *American Control Conference*, 4847–4853.

Wächter, A. and Biegler, L.T. (2005). On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. *Mathematical Programming*, 106(1), 25–57.

Willersrud, A., Imsland, L., Hauger, S., and Kittilsen, P. (2013). Short-term production optimization of offshore oil and gas production using nonlinear model predictive control. *Journal of Process Control*, 23(2), 215–223.

## Appendix A. MODELS DESCRIPTION

### A.1 Separator Equations

The average speed of a hydrocarbon (gas and oil solution) droplet inside the mixture has two components. The vertical component is given by the Stoke's law:

$$v_v = \frac{g(\rho_o - \rho_w)d_m^2}{18\mu} \tag{A.1}$$

where $\rho_o$ and $\rho_w$ are oil and water density, $g$ is the gravitational acceleration, $d_m$ is the average water droplet diameter, and $\mu$ is the dynamic viscosity. The horizontal component velocity is estimated using the aqueous phase retention $v_h = L/\tau$, where $L$ is the separator length and $\tau = w_w/m_w$ is the retention time, $w_w$ is the water mass flow, and $m_w$ is the mass of water in the separator.

As the incoming flow goes through the separator, the water and the mixture make a separation surface that has an angle of $\Phi_v$, shown in Figure A.1, which is given by the velocity of the hydrocarbon droplets, being $\Phi_v = \tan^{-1}(v_v/v_h)$. For the maximum separation of water and hydrocarbon, the projection of $\Phi_v$ should reach the oil surface inside the separator, as depicted by the dotted line projected by $\Phi$ in Figure A.1. The separation efficiency is compromised if a virtual extension of the separator is needed to have the intersection, as for $\Phi_v$ in Figure A.1. The angle $\Phi = \tan^{-1}(h/L)$ with the separator length $L$ and mixture height $h$, which are specified parameters. If the virtual extension is needed ($\Phi_v \leq \Phi$), the virtual length is given by $L_v = h\cot(\Phi_v)$ and the water level at the end of the separator is $h_v = L\tan(\Phi_v)$.

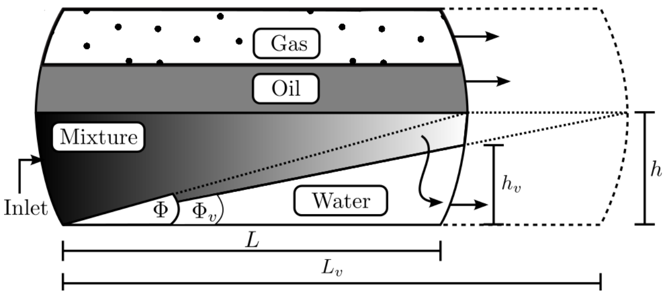


Fig. A.1. Separator Representation

Since the separator is cylindrical, taking the cross section at end of the separator a circular segment (⌓) is formed by the water phase. The circular segment height is equal to the height $h_v$ and the central angle is $2\theta_v$, with $\theta_v = \cos^{-1}(1 - h_v/r)$ being $r$ the separator radius. The volume of water phase $V_{w,s}$ is obtained by integrating the circular segment along the separator. The volume of the mixture $V_{m,s}$ is the volume of the separator above the oil phase less the water phase volume:

$$V_{w,s} = r^2 L\left[\frac{3\sin\theta_v - 3\theta\cos\theta_v - \sin^3\theta_v}{3(1-\cos\theta_v)}\right] \tag{A.2a}$$

$$v_{m,s} = r^2 L\big[\theta_v - 0.5\sin(2\theta_v)\big] - V_{w,s} \tag{A.2b}$$

Similarly, at the end of the virtual extension the cross section have a circular segment with height $h$ and the central angle $\theta = \cos^{-1}(1 - h/r)$. The water phase volume accounting the virtual extensions $V_{w,s}$ and the volume of mixture $V_{m,v}$ are given by:

$$V_{w,v} = r^2 L_v\left[\frac{3\sin\theta - 3\theta\cos\theta - \sin^3\theta}{3(1-\cos\theta)}\right] \tag{A.3a}$$

$$V_{m,v} = r^2 L_v\big[\theta - 0.5\sin(2\theta)\big] - V_{w,v} \tag{A.3b}$$

The water-hydrocarbon separation efficiency $\varepsilon$ is given by the ratio of mixture volume in the separator and in the virtual separator, which is mathematically defined as:

$$\varepsilon = \begin{cases} \dfrac{V_{m,s}}{V_{m,v}} & \text{if } \Phi_v \leq \Phi \\ 1 & \text{otherwise} \end{cases} = \min\left\{1, \frac{V_{m,s}}{V_{m,v}}\right\} \tag{A.4}$$

The three phase flow through the water outlet is defined by:

$$w_{w,out}^g = (1-\varepsilon)w_{in}^g, \qquad w_{w,out}^o = (1-\varepsilon)w_{in}^o \tag{A.5a}$$

$$w_{w,out}^w = w_{in}^w \tag{A.5b}$$

The gas-oil separation efficiency comes from the gas flashing calculation. Assuming that the system has an ideal phase equilibrium, the Raoult's law can be used to assess the amount of gas dissolved in the oil. The Raoult's law stands that the molar fraction of substance in the liquid phase ($\alpha$) is the ratio of the system pressure ($p_{con}$) and the substance vapor pressure ($P_v$), mathematically $\alpha = \frac{p_{con}}{P_v}$. Hence, the gas and oil flowing through the oil outlet is given by $w_{o,out}^g = \varepsilon\, w_{in}^g$ and $w_{o,out}^o = x\,\varepsilon\, w_{in}^o$. The remaining gas flows through the gas outlet $w_{g,out}^g = \varepsilon(1-x)w_{in}^g$.

### A.2 Compressor Model

Let $w_c = w_{in} + w_{sl} - w_{fl}$ be the gas mass flow that runs through the compressor. Hence the volumetric flow in standard conditions $q_v$ is given by:

$$q_v = \frac{w_c R T_{std}}{M_g p_{std}} \tag{A.6}$$

where $R$ is the ideal gas constant, $T_{std}$ and $p_{std}$ are the standard temperature and pressure, $M_g$ is gas molar mass. Let the $r_p = p_{in}/p_{out}$ be the pressure ratio.

Let the subscript letters $M$, $m$, $s$, and $c$ indicates maximum speed, minimum speed, surge line, and choke line, respectively. Then for each $i \in \{M, m, s, c\}$ there is a function $f_i(q_v)$ such that $f_i(q_v) = a_i q_v^2 + b_i q_v + c_v$.

Given a point $(q_v, r_p)$, it is possible to determine if the compressor operating-point is in the feasible area of the compressor by checking whether or not the operating point is within the boundaries of the map. Let $\delta_M$ and $\delta_m$ be the surplus of $q_v$ from the maximum and minimum speed bounds, which are given by $\delta_M = f_M(q_v) - r_p$ and $\delta_m = r_p - f_m(q_v)$. The surplus variables for the surge and choke lines, $\delta_s$ and $\delta_c$, are calculated differently. The surplus is calculated out of $q_v$, for $i \in \{s, c\}$ the solution of the second order equation is explicitly given:

$$\delta_i = q_i - \left[\frac{-b_i + \sqrt{b_i^2 - 4a_i(c_i - r_p)}}{2a_i}\right] \tag{A.7}$$

To assure that the operation is within bounds, it is necessary to add to the OCP the constraint that ensures non-negative surplus ($\delta_i \geq 0$ for $i \in \{M, m, s, c\}$).