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Robust Regulation of Heave-Induced Pressure Oscillations in Offshore MPD ⋆

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Abstract: This paper presents a modification to L_1 adaptive control that allows for disturbances entering at the output and application of this control strategy to Managed Pressure Drilling with harmonic disturbances representing the heave motion of a floating drilling rig. By incorporating the disturbance at the output into the reference model, it is shown that the L_1 adaptive control structure can be left unchanged while the original transient performance bounds are preserved. It is further shown that rejection of the output disturbance can be taken care of entirely in the filter design step of L_1 adaptive control using the internal model principle. A systematic filter design procedure based on LMIs is provided, that requires only one tuning parameter to be adjusted by the designer. The control design is applied to disturbance attenuation and set-point tracking in the so-called heave problem in oil well drilling. We present experimental results of control system tests in a medium size test facility emulating a $900m$ long well.

Keywords: Managed Pressure Drilling (MPD), Active Heave Compensation, Robust Output Regulation, Drilling and Well Technology

1. INTRODUCTION

Deepwater offshore drilling operations are very complex. This is especially true in regions such as the Gulf of Mexico, where wells are drilled in water depths of up to 3 kilometers, drilling depths can exceed 6 kilometers, and geologic formation pressures can exceed 1400 bar Mac (2011).

In drilling operations, a fluid called mud is pumped into the drill string, flows through the drill string and out of nozzles in the bit. It then flows up the well in the annular volume around the drill string carrying away cutting debris (see Figure 1). In addition to carrying cuttings, the drilling mud is used to control pressures inside the wellbore. To avoid fracturing, collapse of the well, or influx of fluids from the reservoir surrounding the well, it is crucial to control the pressure in the open part of the well within a certain operating window. In conventional drilling, this is done by mixing a mud of appropriate density and adjusting mud pump flow rate. In Managed Pressure Drilling (MPD), the well is sealed and the mud exits through a controlled choke, allowing for faster and more precise control of the pressure in the well. In automatic MPD systems, the choke is controlled by an automatic control system which manages the well pressure to be within specified upper and lower limits. Several disturbances and possible faults affect a drilling system. These include packoff, stuck pipe, washout, twist-off, delay in measurements and control, well ballooning (wellbore breathing), fluid loss, hole cleaning problems because of gumbo shale, equipment failure, kick-loss scenarios, wellbore instability, tripping, and the heave motion of the drilling rig while making connections Hannegan (2012); Skalle and Podio (1998); Mahdianfar et al. (2012a). Moreover, uncertainty in hydraulic parameters (such density, rheology of the drilling mud, temperature distribution in the well, frictional pressure loss for the pipe flow and the annular flow in the well, effective bulk modulus, well geometry Lohne et al. (2008); Florence and Iversen (2010); Mahdianfar et al. (2013)) makes drilling even more challenging. Therefore, when designing MPD control systems, one should take into account various operational procedures, uncertainties and disturbances that affect the pressure inside the well.

The study presented in this paper is motivated by the heave disturbance challenge in offshore drilling. When drilling from a floating rig, the rig moves vertically with the waves, referred to as heave motion. When drilling ahead with weight on bit, a heave compensation system is in effect that isolates the drill string from the heave motion

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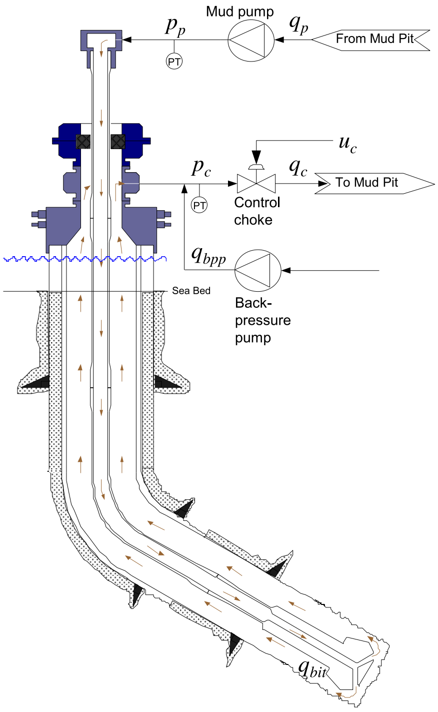

Fig. 1. Schematic of an MPD system. Shown by courtesy of Statoil.

of the rig. As drilling proceeds, the drill string needs to be extended with new sections. Thus, every couple of hours or so, drilling is stopped to add a new segment of about 27 meters to the drill string, referred to as a connection. During connections, the pump is stopped and the drill-string is disconnected from the heave compensation mechanism and put into slips rigidly attached to the rig. The drill string then moves vertically with the heave motion of the floating rig, and acts like a piston on the mud in the well. The heave motion can be as large as 3 meters in amplitude with a period of 10-20 seconds, Godhavn (2010), resulting in severe pressure fluctuations in the bottom of the well. Pressure fluctuations have been observed to be an order of magnitude higher than the standard limits for pressure regulation accuracy in MPD Pavlov et al. (2010). Downward movement of the drill string into the well gives pressure increase (surging), and upward movement gives pressure decrease (swabbing). Excessive surge and swab pressures can lead to mud loss resulting from high pressure fracturing the formation, or a kick-sequence (uncontrolled influx from the reservoir) that can potentially grow into a blowout, as a consequence of low pressure. The first attempt at handling this problem by automatic control of the top-side choke was presented in Pavlov et al. (2010). Two nonlinear control algorithms based on a lumped model were designed, and their performance were tested on a full-scale drilling rig. The controllers demonstrated good performance for slowly varying vertical drillstring movements, but failed in case of fast varying drillstring movements. The identified cause of this was the distributed nature of the flow, which was not taken into account in those controllers, Landet et al. (2012a). A series of papers followed addressing the distributed nature of the system by using a distributed parameters model of a hydraulic flow line. In Landet et al. (2012c,b) two controllers based on a finite difference discretization of the flow line are presented, and show in simulations to have significant attenuating effect on downhole pressure oscillations. In Mahdianfar et al. (2012a) an infinite-dimensional observer that estimates the heave disturbance is designed for a simplified distributed model that ignores friction. An observer-based controller that rejects the effect of the disturbance on the down-hole pressure is designed, and a model reduction technique based on the Laguerre series representation of a transfer function is used to reduce the controller to a simple, rational transfer function. In Mahdianfar et al. (2012b) the results are extended by incorporating friction partially into the model, and in Aamo (2013) they are extended by rigorously incorporating friction. In Anfinsen and Aamo (2014) the results are further refined to allow for attenuation of the effect of the disturbance at any point in the well, for instance at the casing shoe. A constrained model predictive controller is designed in Nikoofard et al. (2013, 2014) for handling heave disturbance and output regulation constraints. MPC is also used in Albert et al. (2014), which was the first successful result from the so-called Heave Lab [1].

Common for all previous works on the heave problem, is that uncertainty in model parameters is not taken rigorously into account in the control design, although several parameters in the well are uncertain during drilling operations Lohne et al. (2008); Mahdianfar et al. (2013). This fact, along with the complexity of some of the proposed methods such as MPC, motivate the present work in which adaptive control theory is employed. Robust adaptive control has been an active research topic for decades Ioannou and Sun (1996); Hovakimyan and Cao (2010); Lavretsky and Wise (2013). In this paper we employ L_1 adaptive output control architecture from Cao and Hovakimyan (2008). A comprehensive overview of the L_1 adaptive control theory can be found in Hovakimyan and Cao (2010). Several successful applications of L_1 adaptive control, especially in aerospace and flight control systems, are reported in Hovakimyan et al. (2011); Xargay et al. (2012). A key feature of the L_1 adaptive control theory is that its architectures decouple the control loop from the estimation loop, which allows for employing fast estimation rates without sacrificing robustness. Furthermore, the versatility of L_1 adaptive control architecture allows for various modifications to its basic blocks, as observed in Kharisov et al. (2011); Kharisov and Hovakimyan (2011). The filtering structure of L_1 adaptive controllers allows for development of systematic methods for robustness/performance tuning, and also for optimization of the positive invariant set in the presence of saturation, Li et al. (2013). Similar to Kharisov et al. (2011); Kharisov and Hovakimyan (2011), but yet a different and novel modification of the main structure of L_1 adaptive output feedback controller is considered here to accommodate the disturbance entering at the system output.

In Section 2 of the paper it is shown that by incorporating the disturbance at the output into the reference model, the L_1 adaptive control structure can be left unchanged while preserving the usual transient performance bounds. It is

[1] The Heave Lab is a medium scale (900m well) experimental facility at NTNU, tailor made for testing control strategies for the heave problem

further shown that rejection of the output disturbance can be taken care of in the filter design step of L_1 adaptive control using the internal model principle. A systematic filter design procedure based on LMIs is provided in Section 3, that requires only one tuning parameter to be adjusted by the designer. The control design is successfully applied to the heave problem described above and tested in the Heave Lab. Experimental results are reported in Section 4.

Notation. Throughout this paper subscripts n and d denote the numerator and denominator of SISO transfer functions, respectively. For a signal $\xi(t) \in \mathbb{R}, t \geq 0$, its truncated $\mathcal{L}_\infty$ and $\mathcal{L}_\infty$ norms are $\|\xi_t\|_{\mathcal{L}_\infty} = \sup_{0\leq\tau\leq t}|\xi(\tau)|$, $\|\xi\|_{\mathcal{L}_\infty} = \sup_{\tau\geq 0}|\xi(\tau)|$. The $\mathcal{L}_1$ gain of a bounded-input-bounded-output (BIBO) stable proper SISO system is defined by $\|H(s)\|_{\mathcal{L}_1} = \int_0^\infty |h(t)|\mathrm{d}t$, where $h(t)$ is the impulse response of $H(s)$. Denoting by $u(t)$ and $y(t)$ the input and output of system H, respectively, the bound $\|y_t\|_{\mathcal{L}_\infty} \leq \|H(s)\|_{\mathcal{L}_1}\|u_t\|_{\mathcal{L}_\infty}$ will be used frequently.

2. $\mathcal{L}_1$ ADAPTIVE CONTROL DESIGN

The main difference between derivations in this part and the results in Cao and Hovakimyan (2008) is inclusion and treatment of unmatched periodic disturbances at the plant output, hence the name L_1 adaptive output regulator. This results in new asymptotic output regulation constraints on the underlying filter in L_1 adaptive control. We consider systems in the form

$$y(s) = A(s)(u(s) + d(s)) + D_o(s)d_o(s), \quad (1)$$

where $u(t) \in \mathbb{R}$ and $y(t) \in \mathbb{R}$ are the input and output of the system, $A(s)$ is a strictly proper tranfer function, $D_o(s)$ is a strictly proper and stable transfer function, and $d(s)$ and $d_o(s)$ are disturbances at the input and output, respectively. The system is the same as the one in Cao and Hovakimyan (2008), with the exception of the output disturbance term $D_o(s)d_o(s)$, which satisfies the following assumption.

Assumption 1. There exists a constant $\gamma_d > 0$ such that

$$|d_o(t)| \leq \gamma_d, |\dot{d}_o(t)| \leq \gamma_d. \quad (2)$$

Furthermore, $d_o(s)$ has known structure defined by the disturbance generating polynomial $d_n(s)$. That is,

$$d_n(s)d_o(s) = d_0(0, s), \quad (3)$$

where $d_0(0, s)$ is a polynomial in s arising from initial conditions of the disturbance $d_o(t)$ ($d_o(0)$, $\dot{d}_o(0)$, $\ddot{d}_o(0)$, etc.).

In other words, $d_n(s)$ contains as zeros the poles of $d_o(s)$. Please note that conditions (2) and (3) imply that the polynomial $d_n(s)$ can only have zeros in the left-half plane or simple zeros on the imaginary axis. In particular, in the application to Managed Pressure Drilling we are interested in the case when (3) describes a harmonic oscillator, heave disturbance oscillations, with frequencies determined by $d_n(s)$ and initial conditions by $d_0(0, s)$.

$d(s)$ is the Laplace transform of unknown disturbances matched with plant input, $d(t) = f(t, y(t))$, where f is an unknown map that satisfies the following assumptions Cao and Hovakimyan (2008).

Assumption 2. There exist constants $L > 0$ and $L_0 > 0$ such that for all $y_1, y_2 \in \mathbb{R}$ and $t \in \mathbb{R}^+$,

$$|f(t, y_1) - f(t, y_2)| \leq L|y_1 - y_2|, \quad (4)$$

$$|f(t, y)| \leq L|y| + L_0. \quad (5)$$

Assumption 3. There exist constants $L_1 > 0$, $L_2 > 0$ and $L_3 > 0$ such that for all $t \in \mathbb{R}^+$,

$$|\dot{d}(t)| \leq L_1|\dot{y}(t)| + L_2|y(t)| + L_3. \quad (6)$$

The control objective is the same as in Cao and Hovakimyan (2008), namely to design an adaptive output feedback controller that makes the output $y(t)$ track the output of a reference model, that is

$$y(s) \approx M(s)r(s), \quad (7)$$

where $r(t) \in \mathbb{R}$ is a bounded and continuous reference signal. As in Cao and Hovakimyan (2008), we consider $M(s) = m/(s + m)$ with $m > 0$. Following Cao and Hovakimyan (2008), we rewrite system (1) as

$$y(s) = M(s)(u(s) + \sigma(s)), \quad (8)$$

$$\sigma(s) = \frac{(A(s) - M(s))u(s) + A(s)d(s) + D_o(s)d_o(s)}{M(s)}, \quad (9)$$

and define the closed-loop reference model

$$y_{ref}(s) = M(s)(u_{ref}(s) + \sigma_{ref}(s)), \quad (10)$$

$$\sigma_{ref}(s) = \frac{(A(s) - M(s))u_{ref}(s) + A(s)d_{ref}(s) + D_o(s)d_o(s)}{M(s)},$$

$$u_{ref}(s) = C(s)(r(s) - \sigma_{ref}(s)), \quad (11)$$

where $d_{ref}(t) = f(t, y_{ref}(t))$ and

$$C(s) = C_n(s)/C_d(s) = (C_d(s) - C_0(s)d_n(s))/C_d(s). \quad (12)$$

Remark 4. Notice that the reference system (10)-(11) is not implementable, since it depends on the unknown system uncertainties and disturbances, and it is used only for analysis purposes.

The polynomials $C_0(s)$ and $C_d(s)$ must be designed such that $C(s)$ is strictly proper,

$$H(s) = \frac{A(s)M(s)}{C(s)A(s) + (1 - C(s))M(s)} \quad (13)$$

is BIBO stable and

$$\|G(s)\|_{\mathcal{L}_1}L \leq 1, G(s) = H(s)(1 - C(s)). \quad (14)$$

The problem of designing $C(s)$ is handled in Section 3. Enforcing the structure (12) on $C(s)$, ensures that the disturbance at the output is attenuated asymptotically with time.

Lemma 5. Consider the closed-loop reference model in (10)–(11), and suppose the filter $C(s)$ has structure (12) and satisfies (13). Then the effect of $d_o(s)$ on the closed-loop reference model output y_{ref} is rejected asymptotically with time.

Proof. is given in Mahdianfar et al. (2015). ■

Consider the $\mathcal{L}_1$ adaptive controller from Cao and Hovakimyan (2008), given as

$$\dot{\hat{y}} = -m\hat{y}(t) + m(u(t) + \hat{\sigma}(t)), \hat{y}(0) = 0, \quad (15)$$

$$\dot{\hat{\sigma}} = \Gamma_c\text{Proj}(\hat{\sigma}(\tau), -mP(\hat{y}(t) - y(t))), \hat{\sigma}(0) = 0, \quad (16)$$

$$u(s) = C(s)(r(s) - \hat{\sigma}(s)), \quad (17)$$

where m is the time constant of the desired reference model (7), $P > 0$, $\Gamma_c > 0$ and the projection operator confines $\hat{\sigma}(t)$ to $|\hat{\sigma}(t)| \leq \Delta$. While P is arbitrary, Γ_c and Δ must be chosen sufficiently large. Block Diagram of the system and controller is shown in Figure 2. The following result holds under the stated conditions.

Theorem 6. Signals of the closed loop consisting of system (1) and controller (15)–(17) satisfy

$$\|\hat{y} - y\|_{\mathcal{L}_\infty} \leq \gamma_0, \tag{18}$$

$$\|y - y_{ref}\|_{\mathcal{L}_\infty} \leq \gamma_1, \tag{19}$$

$$\|u - u_{ref}\|_{\mathcal{L}_\infty} \leq \gamma_2, \tag{20}$$

where γ_0, γ_1 and γ_2 are constants inversely proportional to $\sqrt{\Gamma_c}$.

Proof. The proof follows the steps of the proof of Theorem 1 in Cao and Hovakimyan (2008) and can be found in Mahdianfar et al. (2015). ■

Remark 7. Precise conditions on Γ_c and Δ are provided in Mahdianfar et al. (2015). They are different from the ones provided in Cao and Hovakimyan (2008) due to the disturbance entering at the output.

Remark 8. The bounds (18)–(20) are qualitatively the same as in standard L_1 adaptive control. Notice that (19) implies in addition that the controller attenuates the effect of the output disturbance in view of Lemma 4.

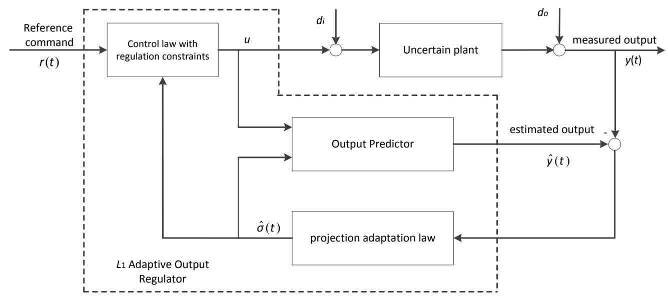

Fig. 2. L_1 Adaptive Output Regulator.

3. SYSTEMATIC FILTER DESIGN

Linear matrix inequalities (LMIs) have been used for design and analysis of robust adaptive controllers in Li et al. (2008); Peaucellea and Fradkov (2008); Peaucelle et al. (2009). In the present work, the choice of $C(s)$ is critical to stability and performance of the regulator presented in the previous section. Here, we present a design methodology based on LMIs for selecting $C(s)$ to meet the design objectives. To this end, insert

$$u(s) = K_I(s)v(s) \tag{21}$$

into (1), where $K_I(s)$ is an internal model compensator that will be specified shortly. Disregarding the disturbance at the output, $D_o(s)d_o(s)$, which has no bearing on what follows, we get

$$y(s) = A(s)(K_I(s)v(s) + d(s)). \tag{22}$$

We now seek to find the compensator

$$v(s) = -K(s)y(s) \tag{23}$$

that minimizes the peak-to-peak gain from $d(s)$ to $y(s)$. Inserting (23) into (22) we get

$$y(s) = \frac{A(s)}{1 + A(s)K_I(s)K(s)}d(s). \tag{24}$$

So, we want to find the $K(s)$ that minimizes

$$\left\| \frac{A(s)}{1 + A(s)K_I(s)K(s)} \right\|_{\mathcal{L}_1}. \tag{25}$$

The solution to this optimal control problem leads to irrational compensators Dahleh and Pearson (1987), but sub-optimal rational compensators can be found along the lines of Boyd et al. (1994); Abedor et al. (1998); Scherer et al. (1997). We will use the procedure for computing $K(s)$ and ζ, the upper bound on the norm in (25), provided in Scherer et al. (1997). Let

$$\dot{x} = Ax + B_1 v + B_2 d, \tag{26}$$

$$y = Cx, \tag{27}$$

be a minimal realization of (22) and

$$\dot{x}_{cl} = \mathcal{A}x_{cl} + \mathcal{B}d, \tag{28}$$

$$y = \mathcal{C}x_{cl}, \tag{29}$$

a minimal realization of the closed-loop (24). From Scherer et al. (1997), we have that for fixed $\lambda > 0$, if X, Y, $\hat{A}$, $\hat{B}$, $\hat{C}$, $\hat{D}$, μ, ζ satisfy the LMIs

$$\begin{pmatrix} \lambda X & \lambda I & 0 & * \\ \lambda I & \lambda Y & 0 & * \\ 0 & 0 & (\zeta - \mu)I & * \\ CX & C & 0 & \zeta I \end{pmatrix} > 0 \tag{30}$$

$$\begin{pmatrix} AX+XA^T+B_1\hat{C}+(B_1\hat{C})^T+\lambda X & * & * \\ \hat{A}+(A+B_1\hat{D}C)^T+\lambda I & A^TY+YA+\hat{B}C+(\hat{B}C)^T+\lambda Y & * \\ B_2^T & YB_2 & -\mu I \end{pmatrix} \tag{31}$$

then the closed-loop system is stable and

$$\left\| \frac{A(s)}{1 + A(s)K_I(s)K(s)} \right\|_{\mathcal{L}_1} \leq \zeta(\lambda). \tag{32}$$

To optimize the bound, a line search over $\lambda > 0$ can be carried out to obtain

$$\zeta^* = \inf_{\lambda > 0} \zeta(\lambda). \tag{33}$$

After solving the LMIs, the controller construction proceeds by finding nonsingular matrices $\mathcal{M}$ and $\mathcal{N}$ such that

$$\mathcal{M}\mathcal{N}^T = I - XY. \tag{34}$$

A state-space realization of $K(s)$ is then given by

$$D_K = \hat{D} \tag{35}$$

$$C_K = (\hat{C} - D_K CX)\mathcal{M}^{-T} \tag{36}$$

$$B_K = \mathcal{N}^{-1}(\hat{B} - YB_1 D_K) \tag{37}$$

$$A_K = \mathcal{N}^{-1}\Big(\hat{A} - \mathcal{N}B_K CX - YB_1 C_K \mathcal{M}^T \tag{38}$$

$$-Y(A + B_1 D_K C)X\Big)\mathcal{M}^{-T}. \tag{39}$$

The following result specifies the internal model compentsator $K_I(s)$ and provides the filter $C(s)$ of the previous section with the desired properties.

Theorem 9. Let $K_I(s)$ be a proper transfer function with poles given by the polynomial $d_n(s)$ (i.e. $K_{I_d}(s) = d_n(s)$). If

$$\frac{A(s)}{1 + A(s)K_I(s)K(s)} \tag{40}$$

is BIBO stable with

$$\left\| \frac{A(s)}{1 + A(s)K_I(s)K(s)} \right\|_{\mathcal{L}_1} < \zeta(\lambda), \tag{41}$$

then the filter

$$C(s) = \frac{K_I(s)K(s)M(s)}{1 + K_I(s)K(s)M(s)} \tag{42}$$

has the form (12), renders $H(s)$ as defined in (13) BIBO stable, and ensures $\|G(s)\|_{\mathcal{L}_1} L \leq 1$ for $G(s)$ as defined in (14), with $L = 1/\zeta(\lambda)$.

Proof. is given in Mahdianfar et al. (2015). ■

Remark 10. The narrow margins between pore pressure and fracture gradient in offshore MPD operations leave little margin for safe drilling and completion. In MPD systems, there are standard limits for pressure regulation accuracy. These constraints are expressed in time-domain and therefore application of the proposed method for designing a controller to minimize error signal peak value subject to a peak value constraint on the control is very well motivated.

4. APPLICATION TO THE HEAVE PROBLEM IN MANAGED PRESSURE DRILLING

4.1 The Heave Lab

In this section, the theory from the previous sections is applied to the heave problem described in the introduction. A medium scale lab has been built at NTNU, tailor-made for testing control strategies for the heave problem. Figure 3 shows the photograph of the lab. It consists of a choke, a back-pressure pump, 900 meters copper pipe, a piston connected to an AC motor, 12 pressure transmitters, 3 flow-meters, DAQ cards and a computer. This experimental setup represents a well with MPD equipment as shown in Figure 1. The copper pipe models the annular volume of the well around the drill pipe. Since there is no flow through the drill pipe during connections, the drill pipe and mud pump are not represented in the lab facility. The bottom-hole-assembly is represented by a piston, which is controlled by the AC motor to generate heave-like disturbances. The piston moves inside a transparent plastic pipe connected to one end of the copper pipe. The choke and the back-pressure pump are connected at the other end of the copper pipe. Figure 4 shows a schematic of the lab.

4.2 System Identification

A model for the heave lab in the form (1) is developed using standard system identification techniques in MATLAB Ljung (2013). The inputs are choke opening, $u(t)$, and piston velocity, $v(t)$, while the output is down-hole pressure, $y(t)$ (pressure transmitter P2 in Figure 4). The transfer functions from choke and piston velocity to down-hole pressure are identified as

$$A(s) = \frac{0.15s - 0.29}{0.67s^2 + 0.8s + 1} \tag{43}$$

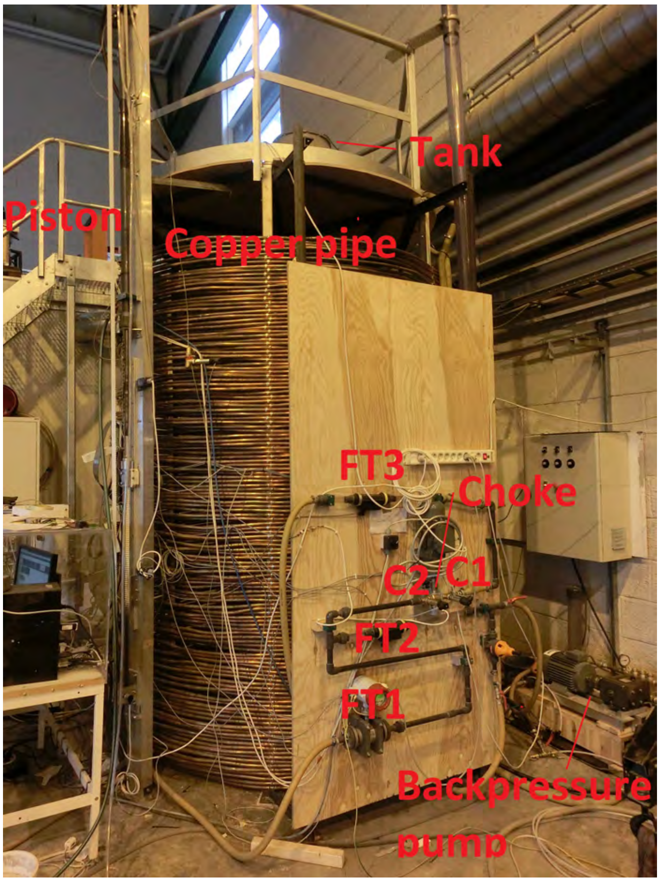

Fig. 3. MPD Heave Lab Albert et al. (2014).

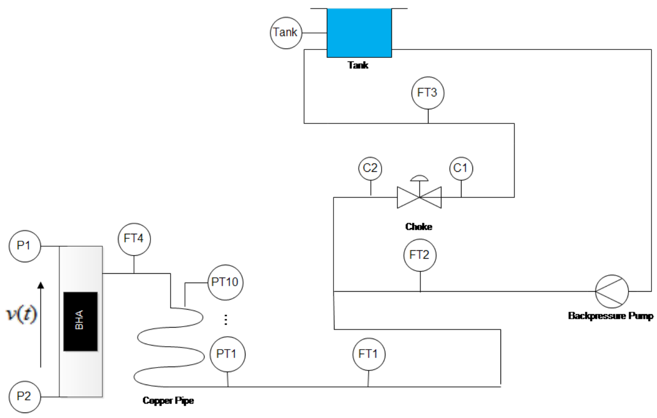

Fig. 4. MPD Heave Lab schematic Albert et al. (2014).

and

$$D(s) = -\frac{1.15s + 3.39}{1.31s + 1}, \tag{44}$$

respectively. Two data sets were used for system identification and verification. Fit to estimation and verification data are 73.11% and 61.8% in terms of normalized root mean square error (NRMSE) measure, respectively. Time domain response and comparison with the identified model is shown in Figure 5. Choke input and piston velocity spectrum of the identification data are computed using FFT. They indicate the model is accurate for the frequency interval $0 - 0.5Hz$, which is above our practical requirements. Choke input frequency spectrum, used for system identification, is illustrated in Figure 6.

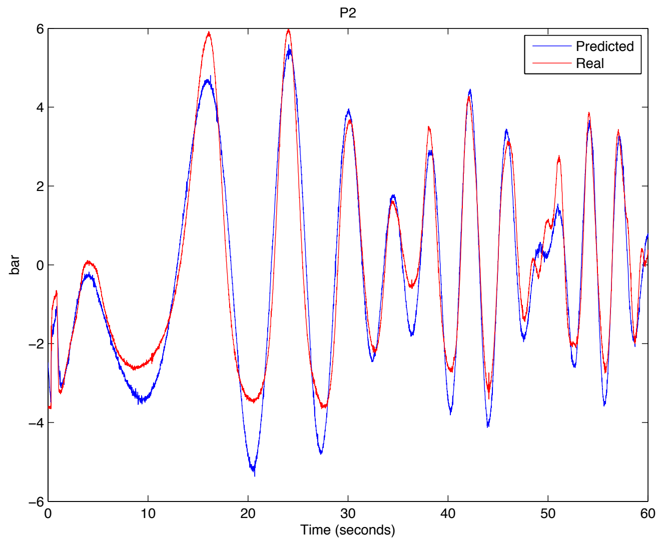

Fig. 5. Predicted and real measurements.

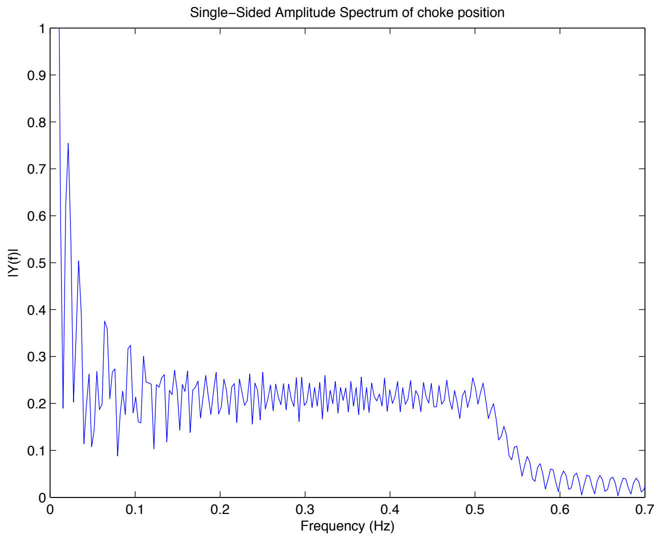

Fig. 6. Choke input frequency spectrum.

4.3 Control Design

Wave elevation of regular waves in deep water can be modeled by a single sinusoid Perez (2005). Therefore, the speed of the drill string $v(t)$ can be considered to be the output of a harmonic oscillator with frequency ω. A more thorough analysis of the effect of sea waves on floating drilling rigs is done in Nikoofard et al. (2013, 2014). Please note that during connection operations drilling is stopped and therefore the angular velocity of drill-string is zero. Robust adaptive regulators are designed and tested successfully for the scenarios of pure harmonic heave disturbance with periods $t = 3, 5, 10s$ and a constant offset, that is

$$v(t) = c_1 + c_2 \sin(\omega t + \phi) \tag{45}$$

for some constants c_1, c_2 and ϕ. For these cases, the output disturbance in (1) takes the form

$$D_o(s) = -\frac{1}{1.31s + 1}, \tag{46}$$

$$d_o(s) = \frac{1.15s + 3.39}{s(s^2 + \omega^2)} \tag{47}$$

for $\omega \in \{2\pi/3, 2\pi/5, 2\pi/10\}$. One of the constraints imposed on $D_o(s)$ is strict properness, which is the reason why the zero of $D(s)$ in (44) is placed in $d_o(s)$ rather than $D_o(s)$ in the definitions (46)–(47). This is possible since the resulting $d_o(s)$ has a well defined Laplace inverse which is bounded. The denominator of $d_o(s)$, which comes from the form of $v(t)$ in (45), defines the disturbance generating polynomial to be included in the filter design as $d_n(s) = s(s^2 + \omega^2)$. The reference model is selected as $M(s) = \frac{m}{s+m}$, $m = 0.2$, so that it gives the desired settling time of about 20 seconds. The desired adaptation rate, projection bound, and projection tolerance bound are set at $\Gamma_c = 1200$ and $\Delta = 50$, $\epsilon = 0.1$ respectively. Based on the chosen reference model $M(s)$, and the identified transfer function $A(s)$ from (43), using the filter design procedure in Section 3 the filters are derived. LMIs (30)-(31) are solved using YALMIP Lofberg (2004) as the interface and SeDuMi 1.3 Sturm (1999) as the solver. To optimize the bound in (32), a line search over λ is performed between [0.03, 1.5], 50 steps, LMIs (30)-(31) are solved and the corresponding ζ values are computed. The Bode plots of filters are shown in Figure 7.

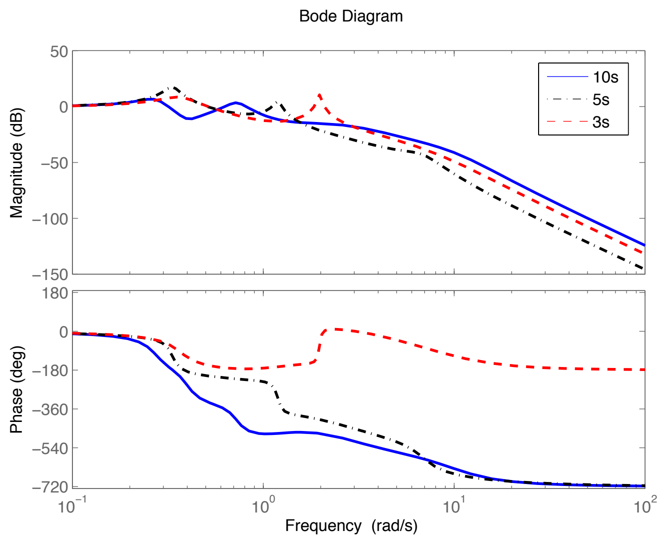

Fig. 7. Bode plots of designed filters.

4.4 Experimental Results

The set-point for the desired bottom-hole pressure is $r = 5\,bar$. For the scenarios of heave disturbance with 5, and 10 seconds period, the bottom-hole pressure, and the choke control signals are illustrated in Figures 8, and 9 respectively. In these experiments at about $t = 75s$ and $t = 120s$ the closed-loop control is switched to constant choke opening, and as a result surge and swab pressures are not compensated afterwards. Experimental results for the case of heave disturbance with period of 3 seconds, the most challenging scenario, are shown in Figure 10. Clearly the proposed regulator has been successful in regulating the output to the desired set-point and attenuating the effect of heave disturbance.

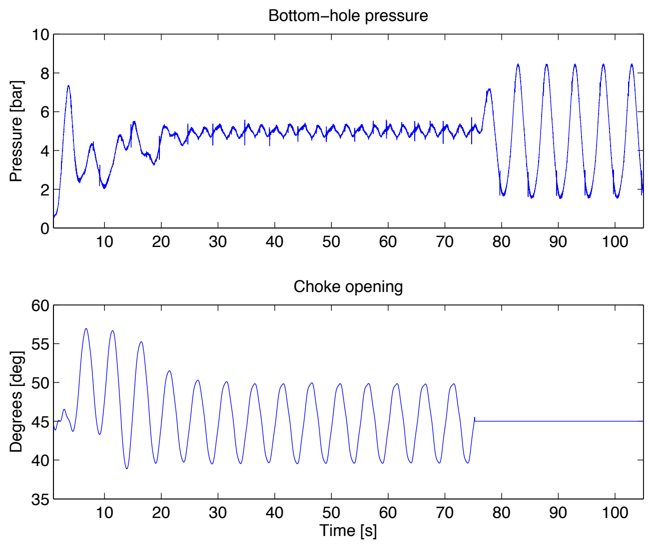

Fig. 8. Bottom-hole pressure and choke opening, heave disturbance with 5 seconds period.

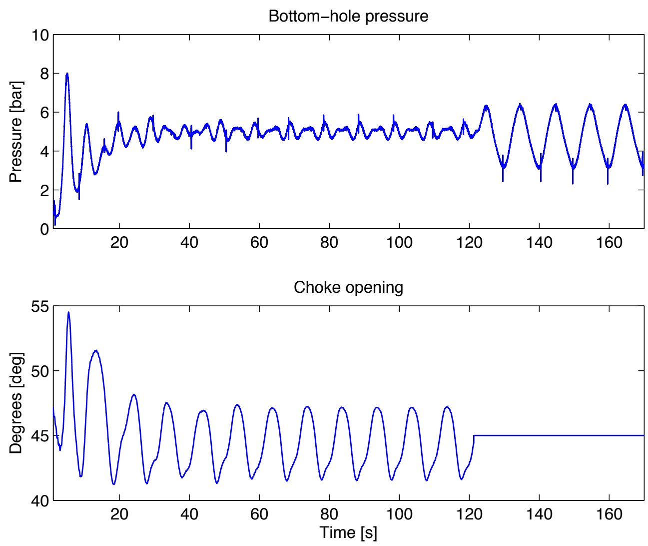

Fig. 9. Bottom-hole pressure and choke opening, heave disturbance with 10 seconds period.

5. CONCLUSIONS

In this paper, a control system design approach is proposed and applied for disturbance attenuation and set-point regulation in the so-called heave problem in oil well drilling. Furthermore it is tested in a medium size experimental test facility. The results demonstrate that the proposed regulator efficiently regulates the down-hole pressure to the desired set-point, with significant attenuation of periodic disturbances. The control system methodology is a modification to L_1 adaptive control that allows for disturbances entering at the plant output. By incorporating the disturbance at the output into the reference model, it is shown that the L_1 adaptive control structure can be left unchanged while the original transient performance bounds are preserved. It is further shown that rejection of the output disturbance can be taken care of entirely in the filter design step of L_1 adaptive control using the internal model principle. A systematic filter design procedure

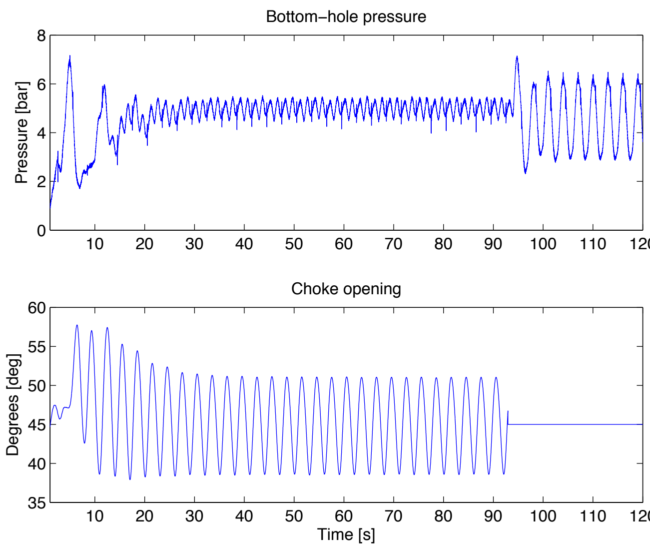

Fig. 10. Bottom-hole pressure and choke opening, heave disturbance with 3 seconds period.

based on LMIs is provided, that requires only one tuning parameter to be adjusted by the designer.

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