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Production Optimization under Uncertainty with Constraint Handling

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Abstract: To maximize the daily production from an oil and gas field, mathematical optimization may be used to find the optimal operating point. When optimizing, a model of the system is used to predict the outcome for different operating points. The model is, however, subject to uncertainty, e.g., the gas oil ratio estimates may be imprecise. The uncertainty is often ignored, and what is known as the expected value problem is solved. Because of inherent uncertainties, there is a great chance that constraints will be violated when implemented. In this paper, we formulate the production optimization problem as a stochastic programming problem, and use Conditional Value at Risk to handle the constraints. This allows us to control the conservativeness of the solution in an efficient manner.

Keywords: Production optimization, uncertainty, stochastic programming

1. INTRODUCTION

During exploitation of hydrocarbon resources, a wide range of decisions are made on how to produce the field. They range from choosing equipment to deciding on choke positions and gas lift rates for the different wells. These decisions will affect the production and profitability of the field, and there is a growing interest in using optimization tools for decision support to increase the profitability. The term Real-Time Optimization (RTO) is used in the oil and gas industry about processes which include some sort of mathematical optimization to maximize profit. An overview of RTO within oil and gas production systems can be found in Bieker et al. (2006).

Since the production system and reservoir is a complex system, it is difficult to optimize everything simultaneously. However, the process contains parts with highly different time constants; in particular the reservoir evolves slowly compared to the dynamics of valves and pipelines. This allows for a hierarchical treatment when controlling the process. In Foss and Jensen (2011), this hierarchy is divided into the four layers Asset Management, Reservoir Management, Production Optimization, and Control and Automation. In this work, we concentrate on production optimization, however, it is closely linked to the other layers of the hierarchy, and especially reservoir management.

In most of the reported industry implementations, production optimization is done without considering the uncertainty of model parameters. Unfortunately, the quantities used in such an optimization problem are seldom known precisely. For instance, the gas oil ratio (GOR) and water cut (WC) of wells can be quite uncertain, due to sparse well tests, changing operating conditions and measurement errors. Although they are known to be uncertain, the optimization problem would typically be solved using the most likely GOR and WC, which could be the values obtained from the last well test. This leads to what is known as the expected value solution. This is an intuitive approach, however, it neglects the inherent uncertainty of the problem. It was pointed out in Bieker et al. (2007a); "The handling of model uncertainty is a key challenge for the success of RTO".

When introducing uncertainty in the optimization problem, the objective function can be expressed as a function of the decision variables and the unknown parameters. We write $J(x, \omega)$, where x is the vector of decision variables and ω is the vector of unknown parameters. ω is stochastic, hence the objective function is also stochastic. Thus, for a given decision x, we can not determine the exact outcome, because it is also dependent on the unknown parameters ω. For an unconstrained problem, the expected value solution can be obtained by solving

$$\min_{x} J(x, \mathbb{E}[\omega]) \tag{1}$$

where $E[\omega]$ denotes the expected value of ω. We denote this as the deterministic problem. However, this approach basically ignores the uncertainty in the parameters. What we are really interested in, is solving the stochastic problem, which can be expressed as

$$\min_{x} \mathbb{E}[J(x, \omega)] \tag{2}$$

when using a risk neutral preference. Note that in general, $\mathbb{E}[J(x, \omega)] \neq J(x, \mathbb{E}[\omega])$.

The stochastic problem is significantly harder to solve than the deterministic problem, since evaluating the objective function involves multidimensional integration. As a con-

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sequence of this, approximate algorithms are often used, for instance by sampling ω, we can approximate

$$\mathbb{E}[J(x,\omega)] \approx \frac{1}{N}\sum_{i=1}^{N} J(x,\omega_i) \quad (3)$$

where ω_i represents different realizations, all with the same probability. This is known as the sample average approximation (SAA).

One of the challenges when handling the uncertainty, is the need to describe, or actually model, the uncertainty. It is no longer enough to provide a reasonable estimate of the parameters, it must also be specified "how certain" they are. One way of doing this is to provide the probability distribution function of the parameters, however, they are seldom known precisely. There exists techniques for estimating the uncertainty. We will assume that such a technique is available since the focus of this work is the optimization problem. In Elgsæter et al. (2008), bootstrapping is used for obtaining parameter and uncertainty estimates.

We first give an overview of previous work in Section 2, before focussing on stochastic programming in Section 3. We then introduce a case study with results in Section 4 and 5, before a discussion and conclusion in Section 6.

2. PREVIOUS WORK

As mentioned, most of the earlier work on RTO ignore uncertainty, and thus solve what is known as the expected value problem. There are, however, a few publications treating the uncertainty explicitly, particularly in the reservoir management domain.

2.1 Work on reservoir management under uncertainty

In Aitokhuehi and Durlofsky (2005), a small number of geological realizations is used to optimize the average recovery factor in closed loop reservoir optimization. A risk term is also used in the objective.

van Essen et al. (2009) optimize the expected NPV by controlling the water injection. An ensemble of 100 realizations is used for the test case of 8 injection and 4 production wells, showing that this approach significantly improves the average NPV compared to approaches using only a single reservoir model.

Chen et al. (2009) combine an ensemble-based optimization scheme with the ensemble Kalman filter for closed loop reservoir optimization. The method uses the ensemble for estimating the gradient, eliminating the need for adjoints and thus any reservoir simulator can be used. An example where the expected NPV is optimized for a water flooding scenario is given.

In Alhuthali et al. (2010), waterflooding is optimized by minimizing the expected deviation from desired arrival time at the producers over a set of realizations. An approach minimizing the maximum deviation is also used. Chen et al. (2011) use a robust scheme to combine short and long term optimization. The long term solution is obtained by optimizing the expected NPV for an ensemble of reservoir realizations, and is used as a constraint in the short term problem. The short term problem involves a more heavily discounted expected NPV over a short time horizon, with a constraint limiting the decrease in the long term expected NPV. Operating constraints are included in a robust fashion, so all constraints must be satisfied for all realizations.

Wang et al. (2012) optimize well placement under uncertainty, using retrospective optimization to limit the number of realizations needed. The number of realizations are gradually increased in the algorithm. Li et al. (2012) optimize both the placement and operation of wells using simultaneous perturbation stochastic approximation to reduce the cost associated with gradient evaluation.

Capolei et al. (2013) compare the solution from a stochastic formulation to the certainty equivalence solution, when the model ensembles are updated based on measurements from a true model. They conclude that when updating the model ensemble, the certainty equivalence approach is superior to the stochastic solution. The comparison is, however, only done for 1 or 2 realizations, and not all of the realizations. A different choice of the true model could thus result in another conclusion.

Dilib and Jackson (2013) use an approach where the parameters of a closed loop controller is optimized to maximize the NPV of the nominal case, and their results suggest this can reduce the effect of uncertainty. The test case is, however, quite simple, and their conclusion can not be generalized.

2.2 Work on short term production optimization under uncertainty

Although there exists numerous publications for reservoir management under uncertainty, there are only a few published papers on short term production optimization under uncertainty. In Elgsæter et al. (2010), a structured approach for changing the setpoint when there is uncertainty is proposed. The uncertainty is, however, not considered in the optimization itself, only to assess the solution from the optimization. To our knowledge, the only publication where the uncertainty is explicitly handled in the optimization problem is by Bieker et al. (2007b). They propose to formulate the optimization problem as a priority list between the wells. This list represents an operational strategy the operator should follow, and whenever there is spare capacity or production must be decreased, he should follow the list. Assuming that all wells are closed, the highest priority well should be opened until it is fully open, or a constraint is hit. If there is still more capacity left, the operator should open the second highest priority well and so on.

3. STOCHASTIC PROGRAMMING

When optimizing systems containing uncertainty, it is often natural to use the expected value of the objective function by averaging over the different realizations. Often the system will also be subject to some limitations, which are modeled as constraints in the optimization problem. For the deterministic problem, they are usually expressed as $c(x) \leq 0$, but if the constraints are uncertain, they will also depend on the unknown parameters ω. There are different ways of handling the constraints, one is to use the expected value as in the objective, i.e. $\mathbb{E}[c(x, \omega)] \leq 0$. This means the constraint is satisfied on average, but this is rarely satisfactory. A stricter approach when using SAA, is requiring all realizations to satisfy the constraint

$$c(x, \omega_i) \leq 0 \; \forall \; i \in \{1 \dots N\} \tag{4}$$

This approach is used in Chen et al. (2011), and has a clear resemblance to robust optimization. However, these constraints can also make the solution overly conservative.

In stochastic programming, chance constraints or probabilistic constraints are often used, formulating the constraint in a probabilistic way

$$\Pr\left(c(x, \omega) \leq 0\right) \geq \eta \tag{5}$$

where η is a parameter, defining the confidence level of the constraint being satisfied. A typical value for η is 90%. The chance constraint is closely related to the value-at-risk (VaR), defined as

$$\mathrm{VaR}_\eta(z) = \inf\left\{t \mid \Pr(z \leq t) \geq \eta\right\} \tag{6}$$

which implies that the chance constraint in (5) is equivalent to

$$\mathrm{VaR}_\eta(c(x, \omega)) \leq 0 \tag{7}$$

When the chance constraint is linear and the uncertain parameters are normally distributed, the chance constraint can be converted into a second-order cone constraint. That is a convex deterministic problem, but unfortunately the convexity property does not hold in general for nonlinear problems.

We can, however, construct a convex conservative estimate of (5), following the derivation in (Shapiro et al., 2014, section 6.2.4). Introducing the step function

$$u(z) = \begin{cases} 1, & \text{if } z \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

For a random variable Z we have that

$$\Pr\left(Z > 0\right) = \mathbb{E}\left[u(Z)\right] \tag{9}$$

so (5) can be written as

$$\mathbb{E}\left[u(c(x, \omega))\right] \leq 1 - \eta \tag{10}$$

The step-function u is clearly a nonconvex function, and we are interested in a convex overestimate of it. We introduce the nondecreasing, nonnegative convex function $\psi\colon \mathbb{R} \to \mathbb{R}$ such that $\psi(z) \geq u(z) \; \forall \; z \; \in \mathbb{R}$. We then have

$$\mathbb{E}\left(\psi(Z)\right) \geq \mathbb{E}\left[u(Z)\right] = Pr(Z > 0) \tag{11}$$

Also note that the step function is invariant to a positive scaling of the argument, $u(tz) = u(z) \; \forall \; t > 0$, so that

$$\inf_{t>0} \mathbb{E}\left(\psi(tZ)\right) \geq \mathbb{E}\left[u(Z)\right] = Pr(Z > 0) \tag{12}$$

and we obtain a conservative approximation of the constraint by writing

$$\inf_{t>0} \mathbb{E}\left(\psi(tZ)\right) \leq 1 - \eta \tag{13}$$

It can be shown that $\psi(z) = [1 + z]_+$ where $[z]_+ = \max(0, z)$, is the best choice for ψ, and then (13) is equivalent to (Shapiro et al., 2014)

$$\inf_{t \in \mathbb{R}} \left\{ t + \frac{1}{1-\eta} \mathbb{E}\left[c(x, \omega) - t\right]_+ \right\} \leq 0 \tag{14}$$

where the expression

$$\inf_{t \in \mathbb{R}} \left\{ t + \frac{1}{1-\eta} \mathbb{E}\left[Z - t\right]_+ \right\} \tag{15}$$

is denoted as the Conditional Value at Risk (CVaR) for Z at level η. The use of CVaR for approximating chance constraints is attributed to Rockafellar and Uryasev (2000).

It can also be shown that the minimum value is $t^* = \mathrm{VaR}_\eta(Z)$, and for a continuous random variable Z

$$\mathrm{CVaR}_\eta(Z) = \mathbb{E}\left[Z | Z \geq \mathrm{VaR}_\eta(Z)\right] \tag{16}$$

A very nice property of CVaR is that it can easily be approximated by SAA, and retains convexity properties. That is, if $c(x, \omega)$ is convex for any ω, the CVaR constraint is also convex. By sampling of ω, we can add the constraint

$$t + \frac{1}{(1-\eta)N} \sum_{i=1}^{N} [c(x, \omega_i) - t]_+ \leq 0 \tag{17}$$

and the non-smooth term can be avoided by introducing

$$z_i \geq c(x, \omega_i) - t, \quad z_i \geq 0 \tag{18}$$

to obtain the constraint set

$$t + \frac{1}{(1-\eta)N} \sum_{i=1}^{N} z_i \leq 0 \tag{19}$$

$$c(x, \omega_i) - t \leq z_i, \quad z_i \geq 0 \tag{20}$$

Although CVaR can be seen as a convex approximation of VaR, using CVaR in itself to formulate the probabalistic constraint is reasonable. Because of the definition of VaR, it is insensitive to the outcome of the $1 - \eta$ worst case realizations. Furthermore, VaR is not a coherent measure of risk (Artzner et al., 1999), in contrast to CVaR. For instance, VaR of the sum of two variables can be greater than the sum of VaR of the individual variables.

In this paper we propose to use CVaR as a means to include uncertainty in the production optimization problem.

4. CASE STUDY

To illustrate the use of CVaR for constraint handling, we provide a case study with eight gas lifted platform wells. The objective is to maximize the oil production by adjusting the gas lift rates and wellhead pressure of each well. The production is initially limited by the gas processing capacity, however, later an example with both gas and water processing capacity limitations is introduced.

The GOR and WC of each well are considered uncertain. There are clearly other sources of uncertainty in the model, however, this work focuses on the above mentioned uncertainties. The method is applicable as long as realizations can be generated, representing the uncertainty. Well Performance Curves (WPC) are obtained using a steady-state multiphase flow simulator called MARLIM (Petrobras inhouse simulator) by varying the GOR and WC of each well. For all the wells, realizations are obtained by sampling from a normal distribution with the mean equal to the latest well test, and a standard deviation of 5 for GOR and 1 for WC. The WPC are obtained for a set of different wellhead pressures and gas lift rates, that is, we obtain one curve for varying wellhead pressure, and another for varying gas lift rate. The curves are linearly interpolated in the optimization using Special Ordered Sets of type 2 (SOS2). SOS2 is an ordered set of variables where at most 2 of them can be non-zero simultaneously,

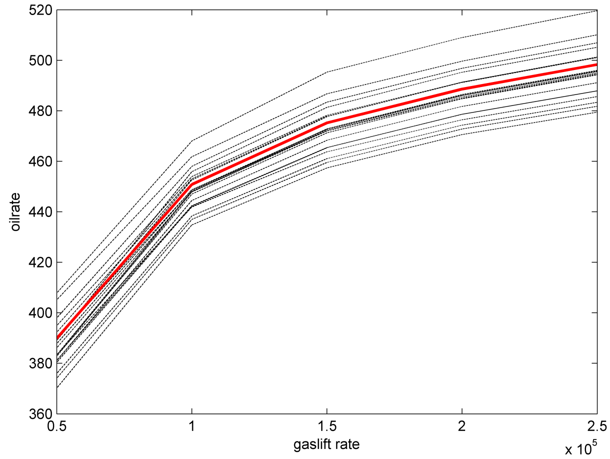

Fig. 1. Showing 20 different realizations for a single well

Table 1. Sets used in optimization model

$\mathcal{I}$	set of wells
$\mathcal{S}$	set of realizations
$\mathcal{A}(i)$	set of breakpoints when varying wellhead pressure for well i
$\mathcal{B}(i)$	set of breakpoints when varying gas lift for well i

and if there are 2, they must be consecutive variables. SOS2 is implemented by the use of binary variables.

In Figure 1 we show 20 realizations of WPC for a single well. Each of the black lines represents a realization, obtained from sampling the GOR and WC probability distributions. The red line represents the WPC using the mean of GOR and WC. The denomination in all figures are standard cubic meters per day.

4.1 Model description

In the following we give a detailed description of the model. In general, variables are in lowercase letters, while parameters are denoted by capital or Greek letters. Subscript is used for denoting index sets, and superscript for description of variables or parameters. The sets are defined in Table 1.

The binary variables y_i^{gl}, y_i^{open} are used to determine if well i is open or closed, and if gas lift is applied. Note that

$$y_i^{gl} \leq y_i^{open} \;\forall\; i \in \mathcal{I} \tag{21}$$

The piecewise linear interpolation is done using $\lambda_{i,a}^{pwh}, \lambda_{i,b}^{gl}$, which are SOS2 variables that must satisfy

$$\sum_{a\in\mathcal{A}(i)} \lambda_{i,a}^{pwh} = y_i^{open} - y_i^{gl} \;\forall\; i \in \mathcal{I} \tag{22}$$

$$\sum_{b\in\mathcal{B}(i)} \lambda_{i,b}^{gl} = y_i^{gl} \;\forall\; i \in \mathcal{I} \tag{23}$$

$$0 \leq \lambda_{i,a}^{pwh} \leq 1 \;\forall\; i \in \mathcal{I},\; a \in \mathcal{A}(i) \tag{24}$$

$$0 \leq \lambda_{i,b}^{gl} \leq 1 \;\forall\; i \in \mathcal{I},\; b \in \mathcal{B}(i) \tag{25}$$

The simulator provides the datapoints. $P_{i,a}^{wh}$ is the wellhead pressure at the sample points, and $Q_{i,a,s}^{o}$ is the corresponding oil rate for all different realizations. Similarly, $Q_{i,b}^{gl}$ is the sample points for gas lift rates, while $Q_{i,b,s}^{o}$ are oil rates. We also include that a choke controlled well can have a preset nonzero gas lift rate to ensure stability; $Q_i^{gl,whctrl}$, and the wellhead pressure when using gas lift is set to P_i^{gl}. The wellhead pressure can then be expressed as

$$p_i^{wh} = \sum_{a\in\mathcal{A}(i)} \lambda_{i,a}^{pwh} P_{i,a}^{wh} + y_i^{gl} P_i^{gl} \;\forall\; i \in \mathcal{I} \tag{26}$$

and the gas lift rate as

$$q_i^{gl} = \sum_{b\in\mathcal{B}(i)} \lambda_{i,b}^{gl} Q_{i,b}^{gl} + (y_i^{open} - y_i^{gl}) Q_i^{gl,whctrl} \;\forall\; i \in \mathcal{I} \tag{27}$$

while the oil flow rate of each well for the different realizations is expressed as

$$q_{i,s}^{o} = \sum_{a\in\mathcal{A}(i)} \lambda_{i,a}^{pwh} Q_{i,a,s}^{o} + \sum_{b\in\mathcal{B}(i)} \lambda_{i,b}^{gl} Q_{i,b,s}^{o} \;\forall\; i \in \mathcal{I}, s \in \mathcal{S} \tag{28}$$

The gas rate and water rate from the reservoir of each well can then be expressed in terms of the GOR and WC of each well for each realization, $GOR_{i,s}$ and $WC_{i,s}$ as

$$q_{i,s}^{g} = GOR_{i,s} q_{i,s}^{o} \qquad \forall\; i \in \mathcal{I}, s \in \mathcal{S} \tag{29}$$

$$q_{i,s}^{w} = \frac{WC_{i,s}}{1 - WC_{i,s}} q_{i,s}^{o} \qquad \forall\; i \in \mathcal{I}, s \in \mathcal{S} \tag{30}$$

We use box constraints on the control input; wellhead pressure is constrained by $\underline{P}_i^{wh}, \overline{P}_i^{wh}$, while gas lift rate is constrained by $\underline{Q}_i^{gl}, \overline{Q}_i^{gl}$. Because wellhead is constrained to P_i^{gl} when using gas lift, and similar for the gas lift when adjusting wellhead pressure, the constraints are formulated as

$$y_i^{gl} P_i^{gl} + (y_i^{open} - y_i^{gl}) \underline{P}_i^{wh} \leq p_i^{wh} \;\forall\; i \in \mathcal{I} \tag{31}$$

$$y_i^{gl} P_i^{gl} + (y_i^{open} - y_i^{gl}) \overline{P}_i^{wh} \geq p_i^{wh} \;\forall\; i \in \mathcal{I} \tag{32}$$

$$y_i^{gl} \underline{Q}_i^{gl} + (y_i^{open} - y_i^{gl}) Q_i^{gl,whctrl} \leq q_i^{gl} \;\forall\; i \in \mathcal{I} \tag{33}$$

$$y_i^{gl} \overline{Q}_i^{gl} + (y_i^{open} - y_i^{gl}) Q_i^{gl,whctrl} \geq q_i^{gl} \;\forall\; i \in \mathcal{I} \tag{34}$$

Robust constraints can easily be added, for instance to make the water handling capacity constraint hold for all realizations

$$\sum_{i\in\mathcal{I}} q_{i,s}^{w} \leq C^{w} \;\forall\; s \in \mathcal{S} \tag{35}$$

This set constraint can, however, be overly conservative. By using CVaR we obtain a probabilistic constraint, and we can control the conservativeness by varying the confidence parameter η. In the following, with a slightly simpler notation, we use $\alpha = 1-\eta$. We introduce the new variables t^w and $z_s^w \geq 0 \;\forall\; s \in \mathcal{S}$, and the CVaR constraint for the water handling capacity can be expressed as

$$\sum_{i\in\mathcal{I}} q_{i,s}^{w} - t^{w} \leq z_s^{w} \;\forall\; s \in \mathcal{S} \tag{36}$$

$$t^{w} + \frac{1}{\alpha^{w} N} \sum_{s\in\mathcal{S}} z_s^{w} \leq C^{w} \tag{37}$$

and similar for gas handling capacity, introduce t^g and $z_s^g \geq 0 \;\forall\; s \in \mathcal{S}$, and the constraints

$$\sum_{i\in\mathcal{I}} \left(q_{i,s}^{g} + q_i^{gl} \right) - t^{g} \leq z_s^{g} \;\forall\; s \in \mathcal{S} \tag{38}$$

$$t^{g} + \frac{1}{\alpha^{g} N} \sum_{s\in\mathcal{S}} z_s^{g} \leq C^{g} \tag{39}$$

Finally, we use the expected total oil production as objective

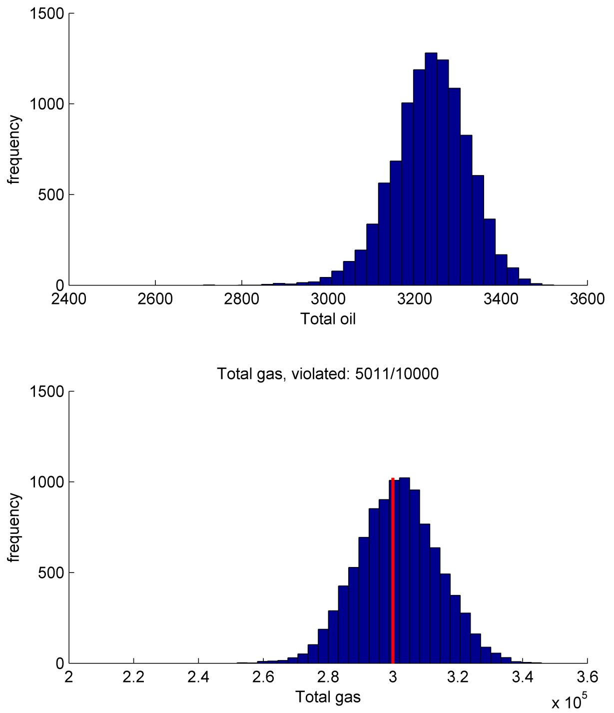

Fig. 2. Expected value solution

$$J = \frac{1}{N} \sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{I}} q^o_{i,s} \quad (40)$$

where N is the number of realizations.

5. RESULTS

The model is implemented using MATLAB and YALMIP (Löfberg, 2004), while CPLEX is used for solving the optimization problem.

We first solve the expected value problem to obtain the setpoint for all the wells in the gas capacity constrained case. This setpoint is then used for an ensemble of 10 000 realizations, to evaluate the setpoint. This evaluation is also done by linearly interpolating between the samples obtained from MARLIM. In Figure 2, we can see the histogram for the total oil and total gas using this solution. The red line represent the constraint on gas processing capacity. As expected, in roughly 50% of the realizations, the constraint is violated.

By introducing the CVaR formulation, we can control how restrictive the constraint should be. In all the stochastic problems solved, we use an ensemble of 1 000 when solving the optimization problem. The setpoint from the optimization problem is then evaluated on the ensemble of 10 000 realizations we used for evaluating the expected value solution. In Figure 3, we have used $\alpha^g = 0.1$. We see that we have obtained a conservative solution, which is far less likely of violating the constraint. By setting $\alpha^g = 0.01$, we can obtain a more conservative solution. The shape of the histogram in this case is very similar to previous ones, therefore we only give the expected oil production and number of violated constraints in Table 2.

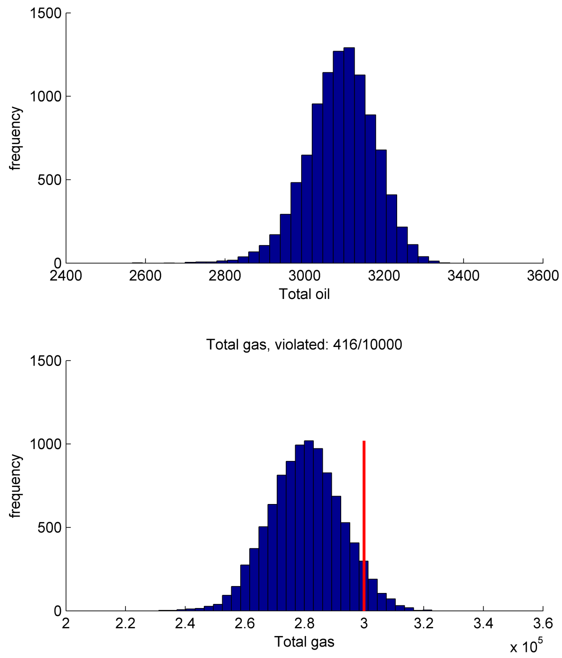

Fig. 3. Stochastic solution, $\alpha^g = 0.1$

Table 2. Summary of gas processing capacity constrained case study

	mean total oil	violated constraints
expected value solution	3222	5011
stochastic $\alpha = 0.1$	3077	416
stochastic $\alpha = 0.01$	3005	47
robust solution	2962	7

An alternative to the CVaR formulation is using the robust formulation in (35). We solve this problem using the same ensemble as for the stochastic problems, and evaluate it on the 10 000 realization ensemble. The key numbers are given in Table 2. Some of the realizations are still violated. This is because the optimization is done with a different ensemble.

The CVaR formulation can handle multiple constraints. If two constraints are to hold jointly with probability $1 - \alpha$, this can be handled by treating them individually using $1 - \alpha = (1 - \alpha_1)(1 - \alpha_2)$. This can, however, result in a very conservative constraint when the constraints are correlated. In Figure 4, we have used a constraint on both the gas and water processing capacity.

6. DISCUSSION AND CONCLUSION

It should be noted that the comparison of mean total oil rates between the solutions is not completely relevant. Although the expected value solution yields the highest value, it violates the constraint for 50% of the realizations. This means the operator in these cases can not implement the solution, and thus simply needs to disregard it. This supports the use of CVaR. For safety critical operations, however, a robust approach could be better suited.

Also note that since we are interpolating between the samples obtained from MARLIM, the results are only

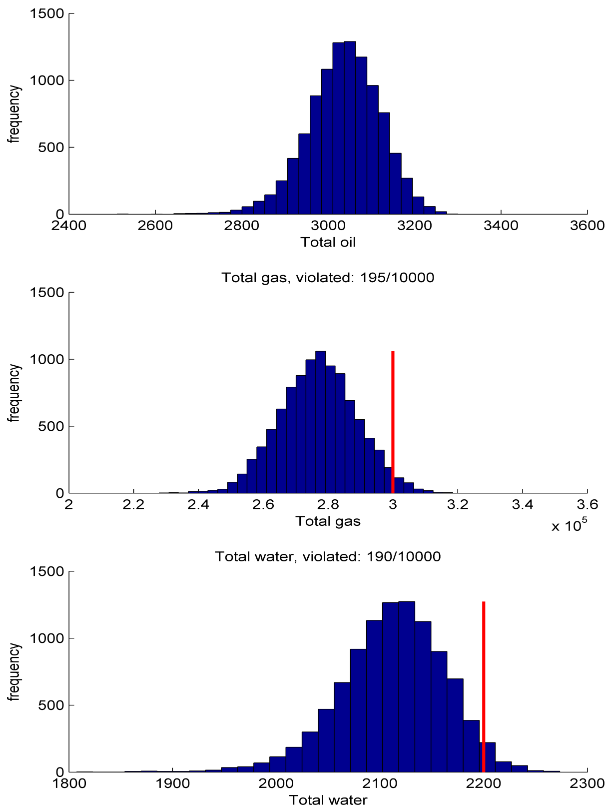

Fig. 4. Stochastic solution, $\alpha^g = \alpha^w = 0.05$

valid for the linearly interpolated curves. Because of the concave shape of the WPC, the linear interpolation is an underestimate of the MARLIM model. This means more realizations will break the constraints for the true MARLIM simulator.

We have presented how a production optimization problem containing uncertainty can be formulated, and especially how constraints can be handled using CVaR. Although this is not new within the stochastic programming community, it is to the authors knowledge, the first time it is applied to the production optimization problem. The solution obtained is a more conservative solution than the expected value solution, but less conservative than the robust formulation. More importantly, CVaR provides a means to control how conservative the solution should be by adjusting α.

REFERENCES

Aitokhuehi, I. and Durlofsky, L.J. (2005). Optimizing the performance of smart wells in complex reservoirs using continuously updated geological models. *Journal of Petroleum Science and Engineering*, 48(3-4), 254–264.

Alhuthali, A.H., Datta-Gupta, A., Yuen, B., and Fontanilla, J.P. (2010). Optimizing smart well controls under geologic uncertainty. *Journal of Petroleum Science and Engineering*, 73(1-2), 107–121.

Artzner, P., Delbaen, F., Eber, J.M., and Heath, D. (1999). Coherent measures of risk. *Mathematical Finance*, 9(3), 203–228.

Bieker, H., Slupphaug, O., and Johansen, T. (2007a). Real-time production optimization of oil and gas production systems: A technology survey. *SPE Production & Operations*, 22(4).

Bieker, H.P., Slupphaug, O., and Johansen, T.A. (2006). Optimal well-testing strategy for production optimization: A monte carlo simulation approach. In *SPE Eastern Regional Meeting*. Society of Petroleum Engineers, Canton, Ohio, USA.

Bieker, H.P., Slupphaug, O., and Johansen, T.A. (2007b). Well management under uncertain gas/ or water/oil ratios. In *Proceedings of Digital Energy Conference and Exhibition*. Society of Petroleum Engineers, Houston, Texas, U.S.A.

Capolei, A., Suwartadi, E., Foss, B., and Jørgensen, J.B. (2013). Waterflooding optimization in uncertain geological scenarios. *Computational Geosciences*, 991–1013.

Chen, C., Li, G., and Reynolds, A.C. (2011). Robust constrained optimization of short and long-term npv for closed-loop reservoir management. In *SPE Reservoir Simulation Symposium*. Society of Petroleum Engineers, The Woodlands, Texas, USA.

Chen, Y., Oliver, D.S., and Zhang, D. (2009). Efficient ensemble-based closed-loop production optimization. *SPE Journal*, 14(04), 634–645.

Dilib, F. and Jackson, M. (2013). Closed-loop feedback control for production optimization of intelligent wells under uncertainty. *SPE Production & Operations*, 28(4), 345–357.

Elgsæter, S., Slupphaug, O., and Johansen, T.A. (2008). Production optimization; system identification and uncertainty estimation. In *Intelligent Energy Conference and Exhibition*, February. Society of Petroleum Engineers, Amsterdam, The Netherlands.

Elgsæter, S.M., Slupphaug, O., and Johansen, T.A. (2010). A structured approach to optimizing offshore oil and gas production with uncertain models. *Computers & Chemical Engineering*, 34(2), 163–176.

Foss, B.A. and Jensen, J.P. (2011). Performance analysis for closed-loop reservoir management. *SPE Journal*, 16(01), 183–190.

Li, L., Jafarpour, B., and Mohammad-Khaninezhad, M.R. (2012). A simultaneous perturbation stochastic approximation algorithm for coupled well placement and control optimization under geologic uncertainty. *Computational Geosciences*, 17(1), 167–188.

Löfberg, J. (2004). Yalmip. In *Proceedings of the CACSD Conference*. Taipei, Taiwan.

Rockafellar, R.T. and Uryasev, S. (2000). Optimization of conditional value-at-risk. *Journal of risk*, 2, 21–41.

Shapiro, A., Dentcheva, D., and Ruszczyński, A. (2014). *Lectures on Stochastic Programming: Modeling and Theory*. MOS-SIAM Series on Optimization, second edition.

van Essen, G., Zandvliet, M., Van den Hof, P., Bosgra, O., and Jansen, J.D. (2009). Robust waterflooding optimization of multiple geological scenarios. *SPE Journal*, 14(01), 202–210.

Wang, H., Echeverría-Ciaurri, D., Durlofsky, L., and Cominelli, A. (2012). Optimal well placement under uncertainty using a retrospective optimization framework. *SPE Journal*, 17(01), 112–121.

