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Vector Based Kinematic Closed-Loop Attitude Control-System for Directional Drilling ⋆

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Abstract: An attitude control system is presented here, where the attitude of a drilling tool is represented by a unit vector and hence non-linearities of Euler angles are avoided. It is shown in simulation that the control system has a stable orbit about the desired attitude in the presence of drop and turn rate biases, and with a spatial delay due to the sensors. The attitude controller is embedded into a way-point tracking control system where it is demonstrated that the controller tracks the position and attitude of each way-point and minimizes the strain energy along the path.

Keywords: Attitude control, Path tracking, Nonlinear control systems, Directional Drilling

1. INTRODUCTION

In the petroleum industry, the accuracy of the placement of a borehole is important in order to maximize the recovery of hydrocarbons from any reservoir. The process of strategically placing a borehole involves steering a drilling tool in a desired direction along a path defined by a multidisciplinary team of: reservoir engineers, drilling engineers, geosteerers and geologists amongst others. Most wells drilled nowadays are horizontal wells, which consist of a vertical part, a curved part known as a build section, and a horizontal section which is steered with respect to geological features in order to maximize oil recovery from a reservoir (Williams, 2010; Jiang et al., 1999; Li et al., 2009). Oil reservoirs are typically located by finding 'traps' from seismic surveys, which are impermeable rocks under which hydrocarbons have accumulated over many millions of years. Gas, oil and water tend to be found together, with gas being lighter than oil, and oil being lighter than water. The horizontal section of a well is usually placed between the oil-water interface and the oil-gas interface. It is hence important that a drilling tool can stabilize the direction in which it is penetrating in (the attitude) to the desired attitude for better well placement and better control of the steerability of the drilling tool.

The technology which enables the steering of the drill allows for turn radii as low as 120 metres (15°/100 ft), enabling complex three dimensional wells to be drilled. Directional drilling can be achieved by either Rotary Steerable Systems (RSS) (Baker, 2000; Yonezawa et al., 2002) and conventional slide directional drilling approaches (Baker, 2000; Kuwana et al., 1994). For the case of RSS directional drilling tools the Bottom Hole Assembly (BHA) lies inside the borehole and is connected to the surface by a series of steel tubular pipes collectively referred to as the drill string.

In practice the Measurement-While-Drilling (MWD) sensors used to determine the attitude are some distance (sometimes several tens of feet) behind the steering unit for which the attitude measurement is being made. This introduces a significant measurement delay in the attitude feedback measurement which any outer attitude control loop should be robust enough to deal with in terms of stability and performance. Additionally there can be a significant dynamic response between the applied tool-face from the actuator and the response tool-face of the steering unit.

In this paper a feedback control law for stabilizing the attitude of a general directional steering drilling system is outlined. Although the controller was originally proposed in Panchal et al. (2012), here we demonstrate by simulation the performance of the proposed controller for following paths, in particular paths generated using an optimal geometric Hermite curve technique (Panchal et al., 2011). Section 2 details the model used to represent the transient behavior of the tools attitude based on an angle axis representation. Section 3 describes a feedback control law used to stabilize the tool towards its desired attitude, where the stability is proved by Lyapunov's Direct Method. Section 4, describes the coordinate transformations required to move from the global reference frame of the earth to the local reference frame of the tool through the use of tri-axis accelerometer and tri-axis magnetometor signals. Section 5 presents a summary of a way-point tracking trajectory following controller in (Panchal et al., 2011),

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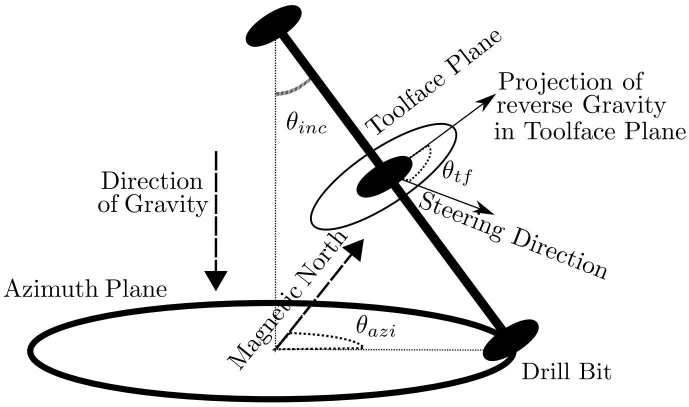

Fig. 1. Conventional attitude and steering parameters for a Bottom Hole Assembly(BHA)

which determines a minimum strain energy path from the tool to a desired position, where the tangents to the path at the start and end points, are coincident with the attitude of the tool and the desired attitude of the tool at the way-point. Finally in Section 6, transient simulations of the attitude control law and the attitude control law embedded in the minimum strain energy way-point tracking control law is presented with engineering constraints modeled.

The overall control architecture is shown in Figure 2 and has the feature that the input demand signal and the output signals are inclination and azimuth angles (Fig. 1), consistent with standard practice of how drillers visualize the direction which the tool is propagating with respect to the gravitational and magnetic fields. When the attitude controller is embedded into a trajectory following loop, the coordinate transformations of moving from the Euler angle representation to the vector based representation are redundant, and it is sufficient to directly interface vectors between the trajectory tracking outer loop and the attitude tracking inner loop.

2. KINEMATIC MODEL OF THE DRILL

2.1 Attitude Response Model

The Bottom Hole Assembly (BHA) is modeled kinematically using an angle-axis representation to describe the response of the tools attitude in time. Where the tool attitude vector is rotated only, and this rotation rate is small and the translational motion is neglected. Furthermore, the motion of the BHA is constrained by the well and hence momentum terms are redundant. Here the kinematic system representing the time varying response of the tools attitude (Wen and Kreutz-Delgado, 1991) can be represented as

$$\dot{\mathbf{x}} = K\omega \times \mathbf{x} \tag{1}$$

where $\mathbf{x} \in B$ is a unit vector representing the tools attitude and $\omega \in B$ is the angular velocity vector parameter, $K \in \mathbb{R}^+$ is referred to here as the *build rate*, and the direction of ω represents the axis which the attitude $\mathbf{x}(t)$ rotates about to achieve the attitude $\mathbf{x}(t+\delta t)$. The set $B \subset \mathbb{R}^3$ is defined to be:

$$B := \{\mathbf{x} \in \mathbb{R}^3 | \|\mathbf{x}\|_2 = 1\}. \tag{2}$$

The set B can be interpreted to be the unit ball. The direction of $\dot{\mathbf{x}}$ represents the direction perpendicular to $\mathbf{x}$ where the bit propagates, and it is assumed that the response in the axis is much greater than the response in the tools attitude $\|\omega(t)-\omega(t+\delta t)\|_2 \gg \|\mathbf{x}(t)-\mathbf{x}(t+\delta t)\|_2$.

Proposition 1. Given an initial attitude, $\mathbf{x}(0) = x_0 \in B$, and a control $\omega \in B$, such that $\omega \cdot \mathbf{x}_0 = 0$ then the resulting trajectory, $\mathbf{x}(t) \in B$ lies on the surface of the unit sphere.

Proof 1. The resultant attitude response from (1) is equivalent to Rodrigues' rotation formula

$$\mathbf{x}(t) = \mathbf{x}_0 \cos Kt + \omega \times \mathbf{x}_0 \sin(Kt) + \omega\omega \cdot \mathbf{x}_0(1-\cos(Kt)). \tag{3}$$

Since $\omega, \mathbf{x}_0 \in B$, it follows that $\omega \times \mathbf{x}_0 \in B$, and given that $\omega \cdot \mathbf{x}_0 = 0$, it follows from evaluating the 2-norm of $\mathbf{x}(t)$ gives $\|\mathbf{x}(t)\|_2 = 1$. □

The plane perpendicular to $\mathbf{x}$ spanned by ω and $\dot{\mathbf{x}}$ is known as the tool-face plane.

2.2 Translational Response Model

The measured position vector of the drill is defined to be $\mathbf{p}_m$ along with the measured attitude $\mathbf{x}_m$, the way-point position $\mathbf{p}_{wp}$ and the way-point attitude $\mathbf{x}_{wp}$. The dynamic relationship for the evolution of the measured position of the drill is given by

$$\frac{d\mathbf{p}_m}{dt}(t) = V_{rop}\mathbf{x}(t), \tag{4}$$

where V_{rop} is the scalar rate of penetration of the tool, and $\mathbf{x}(t)$ is the aforementioned attitude of the tool. This attitude is an input signal for the path tracker and it is assumed that the bandwidth of the attitude controller is higher than that of the path tracker, where a desired space curve can be produced from a sequence of demand attitude signals where the attitude controller is closed loop stable.

3. ATTITUDE CONTROL LAW AND STABILITY

In this section an attitude control law from (Panchal et al., 2012) is stated and its stability is proved by Lyapunov direct method using a lemma that is derived directly from the Lyapunov Theorem of Local Stability (Slotine and Li, 1991). The following definition (Slotine and Li, 1991) is first required.

Definition 1. An equilibrium point $\tilde{\mathbf{x}}$ is a state of the system (1) such that when $\mathbf{x}(t)$ is equal to $\tilde{\mathbf{x}}$, it remains so for all subsequent times. That is $\dot{\mathbf{x}} = 0$.

Lemma 1. For a dynamical system to be locally stable, there exists a scalar valued function $V(\mathbf{x})$ with continuous first partial derivatives in a neighbourhood B about the equilibrium point such that $V(\mathbf{x})$ is positive definite and $\dot{V}(\mathbf{x})$ is negative semi-definite. In addition, if the derivative $\dot{V}(\mathbf{x})$ is negative definite in B, then the stability is asymptotic.

3.1 Constant build-rate controller

The choice of ω to drive the tools attitude $\mathbf{x}$ towards a desired attitude $\mathbf{x}_d$ is chosen such that ω is perpendicular to both these vectors and is orientated such that $\dot{\mathbf{x}} \cdot \mathbf{x}_d \geq 0$. This gives us the following proposition:

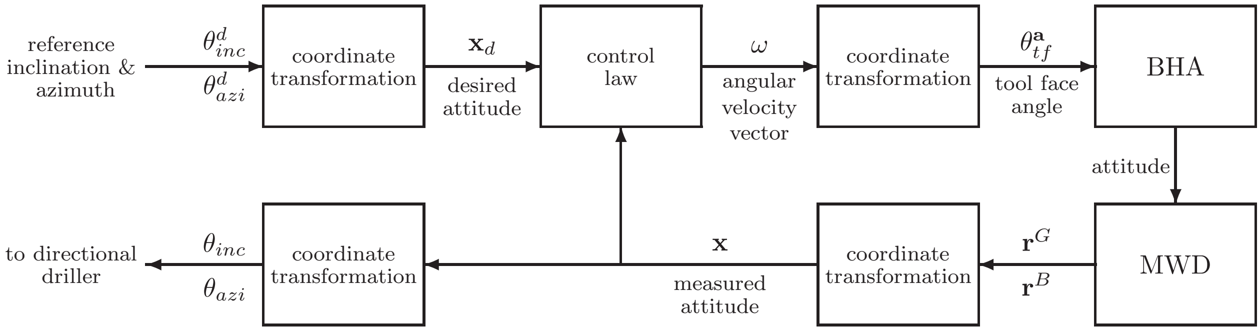

Fig. 2. System structure

Proposition 2. The dynamical system given by (1) with the feedback control law

$$\omega = \begin{cases} \dfrac{\mathbf{x} \times \mathbf{x}_d}{\|\mathbf{x} \times \mathbf{x}_d\|} & \text{for } \mathbf{x} \neq \mathbf{x}_d \\ 0 & \text{for } \mathbf{x} = \mathbf{x}_d, \end{cases} \tag{5}$$

is locally asymptotically stable at the equilibrium point $\mathbf{x} = \mathbf{x}_d$ for $\mathbf{x} \in B$ where K is the constant build rate.

Proof 2. From (5) by definition $\mathbf{x} = \mathbf{x}_d$ is an equilibrium point. We use the Lyapunov function $V(\mathbf{x}) = \frac{1}{2}\left[1 - (\mathbf{x} \cdot \mathbf{x}_d)^2\right]$, which is positive definite and where the first derivative is given by

$$\dot{V}(\mathbf{x}) = \begin{cases} \dfrac{-K[1 - (\mathbf{x}.\mathbf{x}_d)^2]}{\|\mathbf{x} \times \mathbf{x}_d\|} & \text{for } \mathbf{x} \neq \mathbf{x}_d \\ 0 & \text{for } \mathbf{x} = \mathbf{x}_d, \end{cases} \tag{6}$$

which is negative definite and hence by Lemma 1 the system (1) with control law (5) is locally asymptotically stable. □

Definition 2. A geodesic on a unit sphere B between two points $\mathbf{x}_1, \mathbf{x}_2 \in B$ is the shortest arc of great circle given by the intersection of the sphere and a plane spanned by $\mathbf{x}_1, \mathbf{x}_2$ passing through the origin of the sphere, where the great circle lies on this plane (Pressley, 2001, p. 218).

Corollary 1. The control given by (5) solves the minimum time optimal control problem defined as

$$\min_{\omega} \int_0^{t_f} dt \\ \text{subject to (1)},\ \mathbf{x}(0) = x_0,\ \|\mathbf{x}_0\| = 1,\ \mathbf{x}(t_f) = \mathbf{x}_d. \tag{7}$$

Proof 3. Let $\hat{\mathbf{x}}(t)$ be the trajectory that is the solution to the initial value problem for system (1) with the control given by (5) and $\mathbf{x}(0) = \mathbf{x}_0 \in B$. The tangent to the curve $\hat{\mathbf{x}}(t)$ is given by

$$\omega \times \mathbf{x} = K \frac{\mathbf{x}_d - \mathbf{x}(\mathbf{x} \cdot \mathbf{x}_d)}{\|\mathbf{x} \times \mathbf{x}_d\|}. \tag{8}$$

Since the tangent vector is spanned by the vectors $\mathbf{x}_d$ and $\mathbf{x}$ which pass through the origin on the sphere, the state trajectory $\hat{\mathbf{x}}(t)$ is a geodesic by Definition 2. Clearly, the path of minimum length is the path of the minimal time trajectory if the speed along the path is always maximal. From (5), the build rate is maximum (except when the target is reached), hence the trajectory is minimum time. □

The control law given by (5) assumes that build rate K can be made zero when $\mathbf{x} = \mathbf{x}_d$. Since a drilling tool is always steering at its maximum build rate, this assumption will not work in practice. For drilling tools straight line holes of constant attitude $\mathbf{x}$ are achieved through spinning ω about the $\mathbf{x}$-axis. Since we assume here for the analysis of the feedback control law that the rate in which ω can be changed is very fast, the $\omega = 0$ in (5) can be interpreted as an infinitely fast switching of the polarity of the $\omega-$axis.

4. CONTROLLER IMPLEMENTATION

Several coordinate transformations are required in order to implement the controller within the structure shown in Figure 2. The attitude of the BHA has to be determined from measurements of the accelerometers and magnetometers, the desired attitude, x_d, has to be calculated as a unit vector from desired Euler angles, and finally the tool face angle to enact the calculated control, ω, has to be computed. In this section, the calculations for these coordinate transformations are presented.

4.1 Determining the Attitude in the earth Frame from Accelerometers and Magnetometers

The earth frame is the inertial frame which is fixed, and corresponds locally to the geology in which a drilling operation would take place. It is assumed that the variation in the earth's magnetic and gravitational field over the region of an oil well is small. In this frame there are two normalized reference vectors: the magnetic field $\mathbf{r}^B$, and gravitational field $\mathbf{r}^G$. These are given relative to a basis in the earth frame which is easily measured and known. On the drill the accelerometers $\mathbf{b}^G$ and magnetometers $\mathbf{b}^B$ provide another basis where the magnetic and gravitational field is related to the earth frame by the transformation

$$\mathbf{x} = R_y(\theta_{inc}) R_x(\theta_{azi}) \mathbf{r}^G \tag{9}$$

where

$$\theta_{azi} = \arctan\left(\frac{-\mathbf{b}^B_z}{\mathbf{b}^B_y}\right), \qquad \theta_{inc} = \arccos \frac{\mathbf{b}^G_x}{\|\mathbf{b}^G\|}, \tag{10}$$

and

$$R_x(\cdot) := \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\cdot) & \sin(\cdot) \\ 0 & -\sin(\cdot) & \cos(\cdot) \end{bmatrix}, \tag{11}$$

$$R_y(\cdot) := \begin{bmatrix} \cos(\cdot) & 0 & -\sin(\cdot) \\ 0 & 1 & 0 \\ \sin(\cdot) & 0 & \cos(\cdot) \end{bmatrix}. \tag{12}$$

4.2 Control Signal

Directional drills will generally have an internal tool face control system. The tool face angle (see Figure 1) is the clockwise difference in angle between the projection of $\mathbf{a}$ in the tool face plane and the steering direction in this plane. The tool face angles are determined from the control, ω, by

$$\theta_{tf}^{\mathbf{a}} = \begin{cases} \frac{3\pi}{2} - \arccos\left(\frac{\omega \cdot (\mathbf{a}\times\mathbf{x})}{\|\omega\|\|\mathbf{a}\times\mathbf{x}\|}\right) \text{ for } \omega\cdot(\mathbf{a}\times\mathbf{x}) > 0 \\ \frac{\pi}{2} - \arccos\left(\frac{\omega \cdot (\mathbf{a}\times\mathbf{x})}{\|\omega\|\|\mathbf{a}\times\mathbf{x}\|}\right) \text{ for } \omega\cdot(\mathbf{a}\times\mathbf{x}) < 0 \\ \pi \text{ for } \omega\cdot(\mathbf{a}\times\mathbf{x}) = 0 \text{ and } \mathbf{a}\cdot\omega - (\mathbf{a}\cdot\mathbf{x})(\omega\cdot\mathbf{x}) > 0 \\ 0 \text{ for } \omega\cdot(\mathbf{a}\times\mathbf{x}) = 0 \text{ and } \mathbf{a}\cdot\omega - (\mathbf{a}\cdot\mathbf{x})(\omega\cdot\mathbf{x}) < 0 \end{cases} \tag{13}$$

where $\mathbf{a}$ is either $-\mathbf{r}^G$ for the case of Gravity Tool Face (GTF) or $\mathbf{r}^B$ for Magnetic Tool Face (MTF).

The required tool-face angle to implement the control laws proposed in Section 3 can be calculated from the components of the gravitational vector $\mathbf{r}^G$, the magnetic field vector $\mathbf{r}^B$ and the demand attitude $\mathbf{x}^d$ in the toolface plane as follows:

$$\theta_{tf}^{\mathbf{G}} = \theta_G - \theta_r, \tag{14}$$

$$\theta_{tf}^{\mathbf{B}} = \theta_B - \theta_r, \tag{15}$$

where (14) and (15) represent $\theta_{tf}^{\mathbf{a}}$ in GTF and MTF respectively, with

$$\theta_r = \operatorname{atan2}\left(\mathbf{b}_z^d, \mathbf{b}_y^d\right), \qquad \theta_G = \operatorname{atan2}\left(\mathbf{b}_z^G, \mathbf{b}_y^G\right),$$
$$\theta_B = \operatorname{atan2}\left(\mathbf{b}_z^B, \mathbf{b}_y^B\right), \qquad \mathbf{b}^d = R_y(\theta_y)R_x(\theta_x)\mathbf{x}^d,$$

and where $R_x(\cdot)$ and $R_y(\cdot)$ are defined by (11) and (12) respectively.

5. PATH FOLLOWING USING OPTIMIZED GEOMETRIC HERMITE CURVES

The path tracking problem considered here is to meet a sequence of way-points where for a given way-point, and attitude is also specified and it is desired for the tool to meet this too as presented in (Panchal et al., 2011). Since for any drilled path would do work on the drill string, it is also desired to minimize the geometric strain energy for the path to the way-point. Using Euler-Bernoulli beam theory, assume an axisymmetric beam with a constant cross-sectional area and uniform mass distribution, as a general model for the drill-pipe and section of casing.

The relationship between the local curvature κ to the bending moment M is given by (Farouki, 2008, p325)$\kappa = \frac{M}{EI}$, where I is the second area moment and E is the modulus of elasticity of a section of casing or drill-pipe and are both assumed constant.

When deflected into a space curve $\xi(t)$, the work done on an element $\xi(t+\delta t) - \xi(t)$ (Farouki, 2008, p325) is given by $\frac{1}{2}Md\theta$, where $t \in [0,1]$ parameterizes the curve $\xi(t)$ from the start position to the end position, and M is the moment acting on the element dt deflecting it by an angle $d\theta$ where $\theta = \frac{d\xi}{dt}(t)$. The curvature κ at $\xi(t)$ is given by $\kappa = \frac{d\theta}{dt} = \frac{d^2\xi}{dt^2}(t)$.

The total strain energy over the length of the curve which we wish to minimize is given by

$$\Phi(\xi(t)) = \frac{1}{2}EI\int_t \left[\frac{d^2\xi}{dt^2}(t)\right]^2 dt = \frac{1}{2}EI\int_t \kappa^2(t)dt. \tag{16}$$

The path tracking problem is thus

$$\min_{\mathbf{p(t)}} \Phi(\mathbf{p}(t)) = \frac{1}{2}EI\int_t \kappa^2(t)dt \tag{17}$$

subject to the constraints

$$\mathbf{p}(0) = \mathbf{p}_m, \qquad \mathbf{x}(0) = \mathbf{x}_m,$$
$$\mathbf{p}(1) = \mathbf{p}_{wp}, \qquad \mathbf{x}(1) = \mathbf{x}_{wp},$$
$$\frac{d\mathbf{p}_m}{dt}(t) = V_{rop}\mathbf{x}(t).$$

5.1 Solution Method

From the tools measured position $\mathbf{p}_m$ and attitude $\mathbf{x}_m$; the way-point position $\mathbf{p}_{wp}$ and attitude $\mathbf{x}_{wp}$; a cubic hermite interplant spline $\mathbf{p}(t)$ is chosen

$$\mathbf{p}(t) = b_0\mathbf{p}_m + b_1(\mathbf{p}_m + \frac{1}{3}a_0\mathbf{x}_m) + b_2(\mathbf{p}_{wp} - \frac{1}{3}a_1\mathbf{x}_{wp}) + b_3\mathbf{p}_{wp}. \tag{18}$$

The cubic Bernstein polynomial coefficients are given by

$$b_i = \binom{3}{i} t^i(1-t)^{(3-i)}. \tag{19}$$

The values for constants a_0 and a_1 that provide the OGH curve are found from the following theorem (Yong and Cheng, 2003).

Theorem 1. Given two endpoints $\mathbf{p}_m$ and $\mathbf{p}_{wp}$, and two endpoint tangent vectors $\mathbf{x}_m$ and $\mathbf{x}_{wp}$, an OGH curve $f(t)$, $t \in [0,1]$ is obtained at $a_0 = a_0^*$ and $a_1 = a_1^*$ where

$$a_0^* = \frac{6(\mathbf{p}_{wp} - \mathbf{p}_m)\cdot\mathbf{x}_m\mathbf{x}_{wp}^2 - 3(\mathbf{p}_{wp} - \mathbf{p}_m)\cdot\mathbf{x}_{wp}\mathbf{x}_m\cdot\mathbf{x}_{wp}}{4\mathbf{p}_m^2\mathbf{p}_{wp}^2 - (\mathbf{p}_m\cdot\mathbf{p}_{wp})^2} \tag{20}$$

and

$$a_1^* = \frac{3(\mathbf{p}_{wp} - \mathbf{p}_m)\cdot\mathbf{x}_m\mathbf{x}_{wp}\cdot\mathbf{x}_m - 6(\mathbf{p}_{wp} - \mathbf{p}_m)\cdot\mathbf{x}_{wp}\mathbf{x}_m^2}{(\mathbf{p}_m\cdot\mathbf{p}_{wp})^2 - 4\mathbf{p}_m^2\mathbf{p}_{wp}^2}. \tag{21}$$

The curve $\mathbf{p}(t)$ now represents a feasible trajectory from the tool to the way-point optimized for geometric strain energy. The attitude of $\mathbf{p}(t)$ along the curve one drilling cycle ahead is taken and used as a demand signal for the attitude controller. A drilling cycle represents the sampling time for the trajectory controller. The drilling cycle distance is easily determined from the drilling cycle time and V_{rop}, and since the path $\mathbf{p}(t)$ is parameterized for $t \in [0,1]$, one needs to know the arclength L of the OGH curve to determine the value for t used as the control input. The arclength is found by (Chi et al., 2008)

$$L = \frac{6(\mathbf{p}_1 - \mathbf{p}_0)^2}{5} - \frac{\alpha_0^*(\mathbf{p}_1 - \mathbf{p}_0)\cdot\mathbf{x}_0}{5} - \frac{\alpha_1^*(\mathbf{p}_1 - \mathbf{p}_0)\cdot\mathbf{x}_1}{5} - \frac{\alpha_0^*\alpha_1^*\mathbf{x}_0\cdot\mathbf{x}_1}{15} + \frac{2(\alpha_0^*)^2\mathbf{x}_0^2}{15} + \frac{2(\alpha_1^*)^2\mathbf{x}_1^2}{15} \tag{22}$$

hence the attitude demand vector is given by

$$\mathbf{x}_d = \frac{d\mathbf{P}}{dt}\left(t = \ell/L\right). \tag{23}$$

For each subsequent drilling cycle, the path tracking algorithm is iterated.

6. SIMULATION RESULTS

In this section, a transient simulation of the attitude controller is performed for three cases where given the tool is at an initial attitude, a final attitude is specified where it is to be shown that the tool stabilizes in the neighborhood of this attitude. In the following simulation, the tool face response is modeled as a second order servo actuator, and also there is a modeled lag in the measurements for an assumed constant V_{rop} corresponding to sensors being spaced behind the bit. The parameters for these three cases is shown in Table 1, where the drilling tool is initially horizontal and with a 180°azimuth for all three cases. Cases 3 and 1 differ by their target inclination, and case 2 differs to case 1 by having its sensors further behind the BHA to investigate the effect this has on the closed loop attitude holding capability. Secondly the attitude controller is embedded in the way-point following controller where initially a path is generated by steering the tool open-loop with a series of tool-face commands, and from this 16- way-point positions and associated attitudes are generated.

6.1 Attitude Controller Simulation

The inclination and azimuth responses are shown in Figure 3 and Figure 4 respectively. It can be seen that in all three cases that the tool reached its target attitude in about 600ft. In the reaching phase the drop rate disturbance has the effect of dipping the inclination of the tool by about 3 °for all three cases. Furthermore the effect of having the sensors further behind the BHA for case 2 is of an oscillation of inclination and azimuth about the target attitude. This effect is better illustrated in Figure 5 where the state trajectory of the inclination error and azimuth error is shown. Here for both cases there is a limit cycle about **0** where the effect of a larger spatial lag on the sensors for case 2 gives rise to a larger amplitude limit cycle.

Table 1. Simulation Parameters

Case	1	2	3
Nominal ROP $ft(hr)^{-1}$	100	100	100
Drilling Cycle s	50	50	50
Drop Rate Disturbance $°(100ft)^{-1}$	1	1	1
Turn Rate Disturbance $°(100ft)^{-1}$	0.5	0.5	0.5
Spatial Measurement Lag ft	10	40	10
Max Curvature Response of tool $°(100ft)^{-1}$	15	15	15
Initial azimuth °	180	180	180
Initial inclination °	90	90	90
Target azimuth °	270	270	270
Target inclination °	90	90	70

6.2 Trajectory Following Simulation Results

The resultant trajectory from the way-point following controller is shown in the visualization Figure 6, where the triangle markers show the reference way-points. Here when the drilling tool is within 20ft of its target way-point, then the next way-point is subsequently assigned to be the target. It can be seen here that the resultant trajectory passes through the way-points, and furthermore Figure 8 shows the norm distance to the nearest way-point during the simulation. It can be seen that the distance strictly decreases. Since the way-point tracking controller solves for a path to minimize the strain energy of the path solved on-line to the target, this value is plotted in Figure 7 where tracking is demonstrated since each time the strain energy

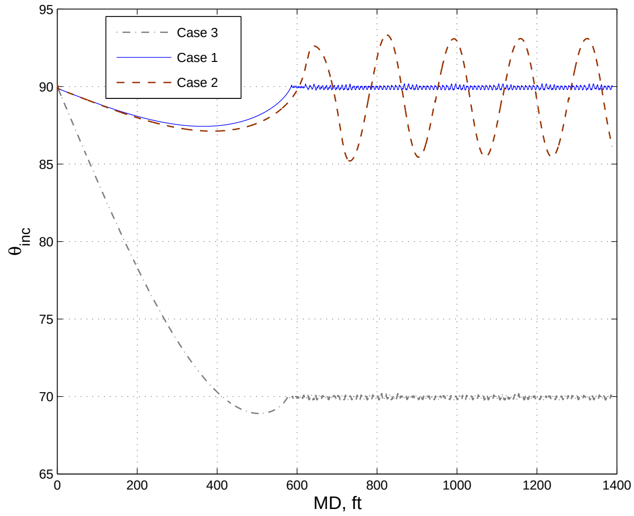

Fig. 3. Inclination response verses measured depth for 3 test cases

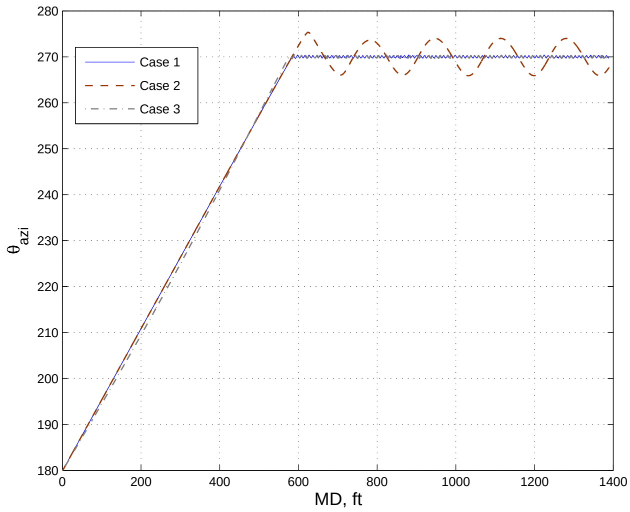

Fig. 4. Azimuth response verses measured depth for 3 test cases

of the path to the target is determined it decreases. This is also the case for the attitude error as shown in Figure 9 where the attitude error is defined to be the norm error between the unit vector representing the target attitude and the current measured tool attitude.

7. CONCLUSION

In this paper, a vector based attitude control law for directional drilling tools is presented along with a stability analysis. This control law is also shown to be a minimum time control law. Implementation of the attitude controller with respect to calculating the attitude from accelerometers and magnetometers is presented. The attitude controller is used as an inner loop for way-point tracking control system. In time domain simulations, the attitude controller is demonstrated to reach and hold an attitude with drop and turn rate disturbances and tool-face actuator dynamics modeled, where the controller limit cycles around the desired attitude. This limit cycle is shown to increase in amplitude for larger spatial delays for direction and inclination sensors. By means of simulations

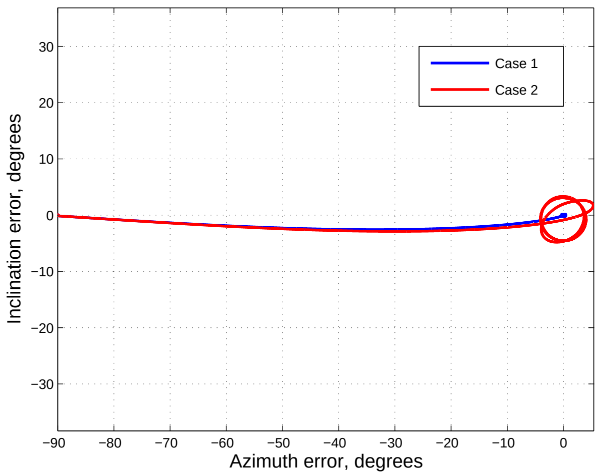

Fig. 5. Azimuth response verses measured depth for 2 test cases showing limit cycle behaviour

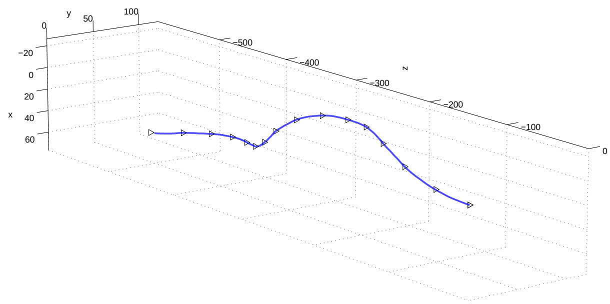

Fig. 6. Way-point tracking visualisation

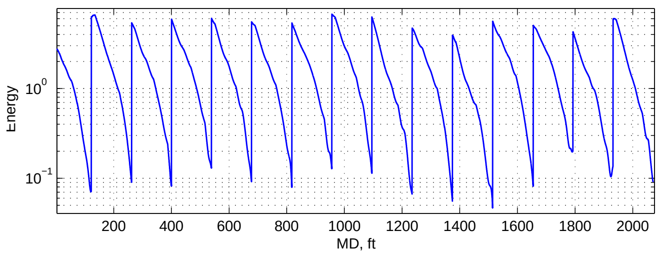

Fig. 7. Cost-to-go strain energy of correction path from tool to way-point verses measured depth

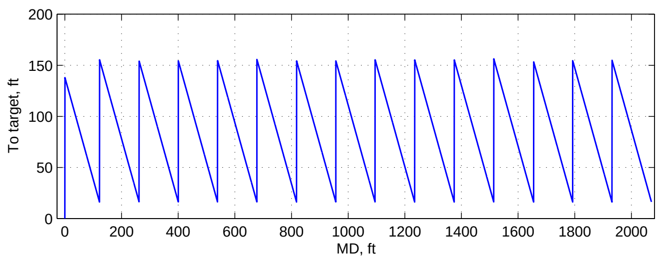

Fig. 8. Distance to way-point verses measured depth

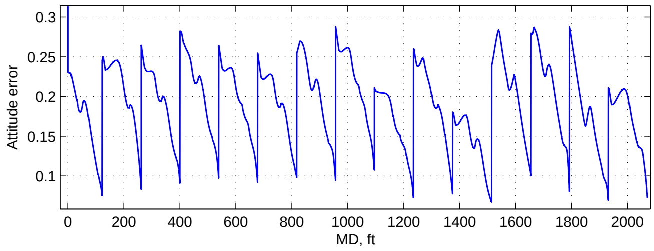

Fig. 9. attitude error verses measured depth

we show that the positive definite measures of the position and attitude errors are decreasing along the trajectories of the closed loop system. Furthermore the geometric strain energy of the correction path from the tool to the way-point is shown to decrease showing practical stability.

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