Stabilizing gain design for PFC (Predictive Functional Control) with estimated disturbance feed-forward

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Abstract: PFC (Predictive Functional Control) is a simple but very effective SISO (Single-Input Single-Output) predictive controller. A disturbance estimator is proposed based on PFC algorithm in case of non-measurable disturbances. The algorithm considers both external disturbance signal and internal, structural disturbances (process/model mismatch). Estimated disturbance feed-forward control can lead to instability because of the additional closed-loop of the estimator. Stabilizing technique by filtering the estimated disturbance is investigated for aperiodic processes including dead time.

Keywords: Predictive functional control, disturbance estimator, feed-forward control, stability

1. INTRODUCTION

PFC (Predictive Functional Control) is a simple predictive control algorithm (Richalet, et al., 1978) which does not require any matrix inversion or numerical minimization of a cost function. PFC algorithms can be easily realized for all types of SISO (Single-Input, Single-Output) processes. Furthermore the manipulated variable and/or its increment, as well other variables can be constrained. PFC can be extended by feed-forward control in the case of measurable disturbances.

The idea of IMC (Internal Model Control) is to estimate the disturbance (either an external one or a model mismatch between process and model used in the controller) and to use this signal in the control algorithm. Dr. Richalet recommended a different scheme to estimate the disturbance with PFC and to use the estimated signal for feed-forward control. The advantage of the method is that it considers both external not-measurable disturbances and internal, structural model mismatch. For this reason the expression "global disturbance estimator". In the case of external disturbances the method works in a similar way as if the disturbance would have been measured and feed-forward controlled.

Feed-forward control with measured disturbance is an open-loop control. However a feed-forward control with estimated disturbance has an additional closed loop and influences the stability. Therefore some modification must be done to enhance the stability. The simplest technique is using an attenuation factor multiplied by the estimated disturbance (Richalet and O'Donavan, 2009).

The paper is structured as follows. In Section 2 the PFC algorithm is shortly shown. In Section 3 the disturbance estimation using PFC algorithm is presented. In Section 4 the estimated disturbance feed-forward control is shown. Section 5 presents a sufficient stability criterion for the feed-forward gain for both using and not using a first-order stabilizing filter. A case study of a mixer with heating jacket illustrates the theoretical results in Section 6.

2. PREDICTIVE FUNCTIONAL CONTROL

The principle of PFC is that the controlled variable y achieves the reference trajectory at the target point (or points) using one change (or minimal number of changes) in the manipulated variable u. The desired change in the controlled variable y during the prediction horizon n_p (from the actual time k) is calculated from the desired change of the reference trajectory and the predicted change of the model output y_m. The manipulated variable u can be calculated easily from the change of the reference trajectory and the predicted change of the model output during the prediction horizon, see Fig.1.

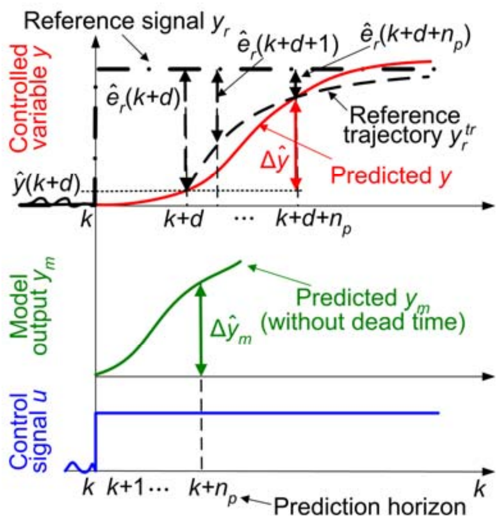


Fig.1: PFC principle

The desired changes in the controlled variable y during n_p prediction step can be defined supposing that y reaches the reference trajectory at the target point (n_p step ahead):

$$\hat{y}(k+n_p \mid k) - y(k) = e(k) - \hat{e}(k+n_p \mid k) \tag{1}$$

where $e(k) = y_r - y(k)$ and y_r is the assumed constant reference signal.

The reference trajectory can be chosen an exponential function for simplicity. Then the control error is decreasing monotonously:

$$\begin{aligned} &\hat{e}(k+1 \mid k) = \lambda_r e(k) \\ &\hat{e}(k+2 \mid k) = \lambda_r \hat{e}(k+1 \mid k) = \lambda_r^2 e(k) \\ &\dots \\ &\hat{e}(k+n_p \mid k) = \lambda_r^{n_p} e(k) \end{aligned} \tag{2}$$

where λ_r is the reduction ratio of the trajectory's error.

The reference trajectory provides the settling time $t_{95\%}=T_c$ for the closed loop control system if $\lambda_r = \exp(-3\Delta t/T_c)$, where Δt is the sampling time.

From (1) and (2), the desired change in y is defined as follows:

$$\Delta\hat{y}(k+n_p \mid k) = \hat{y}(k+n_p \mid k) - y(k) = (1-\lambda_r^{n_p})e(k) \tag{3}$$

The changes of y can be predicted using a proportional, first-order model equation without dead time (chosen for simplicity) in discrete-time as

$$y_m(k) = -a_m\, y_m(k-1) + K_m(1+a_m)u(k-1) \tag{4}$$

where y_m is the model output, u is the model input, a_m is the discrete-time model parameter and K_m is the static gain of the model.

Supposing that the actual input signal u is kept constant during the prediction horizon, the predicted model output becomes after n_p steps:

$$\begin{aligned} \hat{y}_m(k+1 \mid k) &= -a_m\, y_m(k) + K_m(1+a_m)u(k) \\ \hat{y}_m(k+2 \mid k) &= -a_m\, y_m(k+1 \mid k) + K_m(1+a_m)u(k) \\ &= (-a_m)^2 y_m(k) + (-a_m+1)K_m(1+a_m)u(k) \\ &= (-a_m)^2 y_m(k) + K_m[1-(-a_m)^2]u(k) \\ &\dots \\ \hat{y}_m(k+n_p \mid k) &= (-a_m)^{n_p} y_m(k) + K_m[1-(-a_m)^{n_p}]u(k) \end{aligned}$$

Then, the predicted change in y_m becomes:

$$\begin{aligned} \Delta\hat{y}_m(k+n_p \mid k) &= \hat{y}_m(k+n_p \mid k) - y_m(k) \\ &= [1-(-a_m)^{n_p}][K_m u(k) - y_m(k)] \end{aligned} \tag{5}$$

Simple comparison between the predicted change of the reference trajectory in (3) and the predicted change of y_m in (5) results in the manipulated variable:

$$u(k) = k_0[y_r - y(k)] + k_1 y_m(k) \tag{6a}$$

where:

$$k_0 = \frac{1-\lambda_r^{n_p}}{K_m[1-(-a_m)^{n_p}]}, \qquad k_1 = \frac{1}{K_m} \tag{6b}$$

If the process has dead time $d = d_m$ then $y(k)$ in (6a) has to be replaced by $\hat{y}(k+d_m \mid k)$

$$\hat{y}(k+d_m \mid k) = y(k) + [y_m(k) - y_m(k-d_m)] \tag{7}$$

In case of higher-order aperiodic processes the transfer function can be partitioned in parallel connection of first-order processes

$$\hat{y}_{m,i}(k) = -a_{m,i}\hat{y}_{m,i}(k-1) + K_{m,i}(1+a_{m,i})u(k-1) \tag{8}$$

with the corresponding parameters $K_{m,i}$ and $a_{m,i}$ of the i-th sub-process. (If the process has multiple poles then different but very similar poles have to be assigned to each multiple pole.)

The basic algorithm can be easily extended for this case, as well (Khadir and Ringwood, 2008):

$$u(k) = k_0[y_r - y(k)] + \sum_{i=1}^{n} k_i y_{m,i}(k) \tag{9a}$$

where n is the order of the process,

$$k_0 = \frac{1-\lambda_r^{n_p}}{\sum_{j=1}^{n} K_{m,j}[1-(-a_{m,j})^{n_p}]}, \quad k_i = \frac{1-(-a_{m,i})^{n_p}}{\sum_{j=1}^{n} K_{m,j}[1-(-a_{m,j})^{n_p}]} \tag{9b}$$

3. PFC WITH GLOBAL DISTURBANCE ESTIMATOR

The following global disturbance observer was developed by Dr. Richalet based on PFC, see the scheme in Fig. 2. The main advantage of this scheme is that it uses same tools as the already installed PFC, which can be easily implemented.

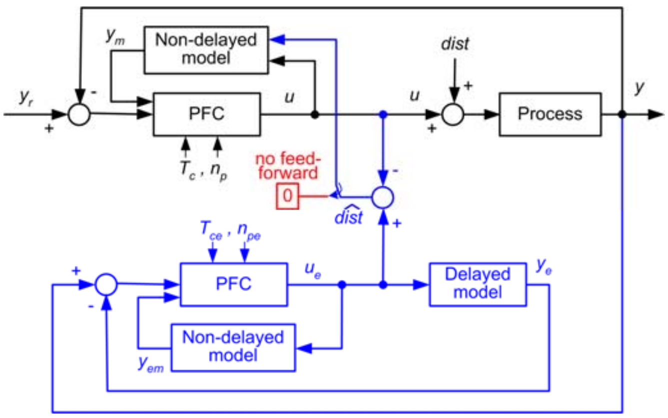


Fig. 2: Richalet's scheme of PFC with disturbance estimator

A simulated process model is controlled by a fast PFC in the estimator. Both the "real" controller and the estimator controller use the same process model without dead time. The controlled variable y is applied as the reference signal of the estimator PFC.

As the estimator control loop is not disturbed the difference between both manipulated/control signals are equal to the external disturbance acting on the process input if the process model and the controllers are perfect. Consequently this difference is the estimated disturbance acting to the process’s input. A detailed description of the disturbance estimator is given in (Richalet and O’Donavan, (2009).

The estimator controller’s tuning parameters are: T_{ce} and n_{pe}. The principle of the estimator in this scheme is to define the control signal $u_e(k)$ which force the estimator process output y_e to reach the controlled variable y during the prediction horizon n_{pe} after the dead time d_m.

The desired changes in y_e during n_{pe} steps are:

$$\begin{aligned}\Delta\hat{y}_e(k+d_m+n_{pe}|k) &= (1-\lambda_{re}^{n_{pe}})[y(k)-\hat{y}_e(k+d_m|k)] \\ &= (1-\lambda_{re}^{n_{pe}})[y(k)-y_{em}(k)]\end{aligned} \tag{10}$$

where $\lambda_{re} = \exp(-3\Delta t/T_{ce})$ is the reduction ratio of the bias between $y(k)$ and $y_{em}(k)$. Here y_{em} is the non-delayed model output of the estimator loop $y_{em}(k - d_m) = y_e(k)$.

The desired estimator closed loop settling time $t_{e95\%}=T_{ce}$ has to be chosen very small to provide a fast estimation of the disturbance.

The predicted change of the estimator model output y_{em} is:

$$\begin{aligned}\Delta\hat{y}_{em}(k+n_{pe}|k) &= u_e(k)\sum_{j=1}^{n} K_{m,j}[1-(-a_{m,j})^{n_{pe}}] \\ &\quad -\sum_{j=1}^{n}[1-(-a_{m,j})^{n_{pe}}]y_{em,j}(k)\end{aligned} \tag{11}$$

Comparing (10) and (11) results in

$$u_e(k) = k_{0e}[y(k)-y_{em}(k)] + \sum_{i=1}^{n} k_{ie} y_{em,i}(k) \tag{12}$$

where n is the order of the model and

$$k_{0e} = \frac{1-\lambda_{re}^{n_{pe}}}{\sum_{j=1}^{n} K_{m,j}[1-(-a_{m,j})^{n_{pe}}]}, \quad k_{ie} = \frac{1-(-a_{m,i})^{n_{pe}}}{\sum_{j=1}^{n} K_{m,j}[1-(-a_{m,j})^{n_{pe}}]} \tag{13}$$

The estimated disturbance is

$$\hat{dist}(k) = u_e(k) - u(k-d_m-n_{pe}) \tag{14}$$

In the following simulations the following processes and PFC tuning parameters are supposed, see Table 1.

Table 1: Process and controller parameters

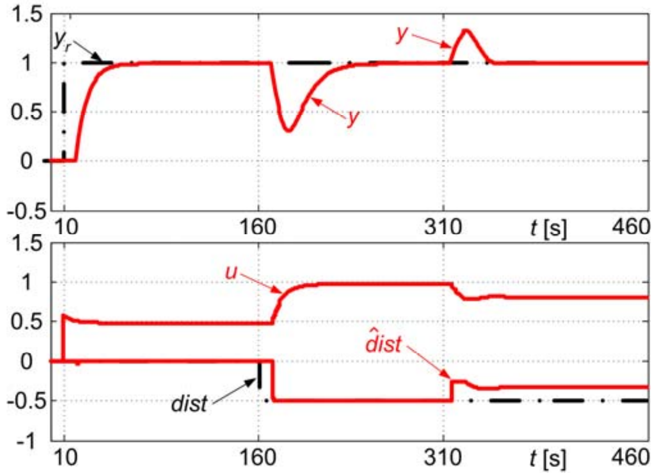
Process/Model	T_c	n_p
Delayed first-order: $\dfrac{2e^{-10s}}{1+10s}$	25s	1
Delayed third-order: $\dfrac{2e^{-10s}}{(1+3.33s)^3}$	25s	10

Delayed first-order model of these processes is assumed in the PFC algorithm with model gain K_m = 2, time constant T_m = 10 s, and dead time T_{dm} = 10 s. A first-order approximating model was used with the third-order process, in order to show the robust behaviour of the algorithm. Using a third-order model with a third-order process - as usual - would lead to a better control behaviour.

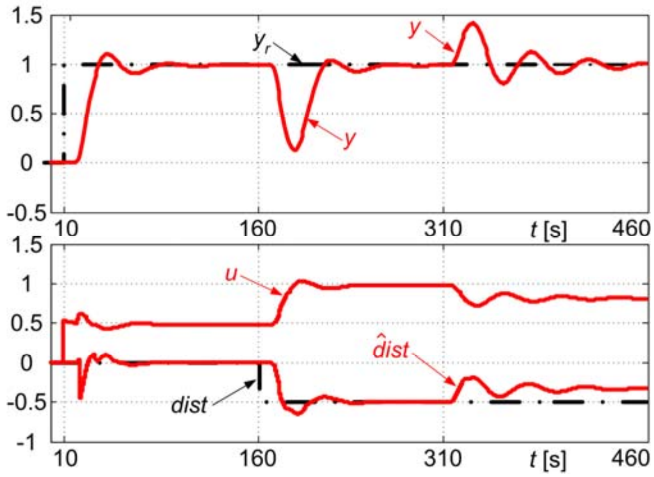
Both external disturbance signal and process gain variation are assumed in the simulations. The control scenario is:

- sampling time Δt=1s and simulation time = 460s,
- at t=10s stepwise increase of y_r from 0 to 1,
- at t=160s stepwise external disturbance (0 → -0.5).
- at t=310s process gain increase by 50%.

Fig. 3 shows the estimation of the disturbance for the given processes (without feed-forward control). The estimator parameters are: T_{ce} = 1s and n_{pe} = 1 for first-order process, T_{ce} = 5s and n_{pe} = 3 for third-order process.



a) Delayed first-order process and first-order model



b) Delayed third-order process and first-order model

Fig. 3: PFC with disturbance estimation without feed-forward control

The plots in Fig. 3/a and 3/b show that the real external disturbances are estimated well, of course only after the dead time. The structural disturbance coming from the processes gain change is estimated as an external disturbance signal at the process input. The estimated disturbance has the same effect as the process parameters change. The plot in Fig. 3/b shows estimated structure disturbance at about t=20 s, mainly because of the mismatch between the third-order process and its approximating first-order model.

4. PFC WITH ESTIMATED DISTURBANCE FEED FORWARD CONTROL

To compensate the disturbance effects on the process, PFC with estimated disturbance feed-forward algorithm can be used, see Fig. 2.

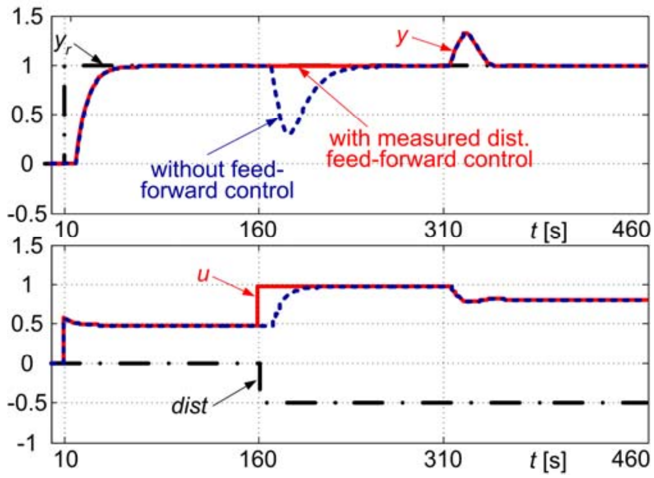
PFC control equation with disturbance feed-forward is:

$$u(k) = k_0[y_r - y(k) - y_m(k) + y_m(k-d_m)] + \sum_{i=1}^{n} k_i y_{m,i}(k) - \overline{dist}(k) \tag{15}$$

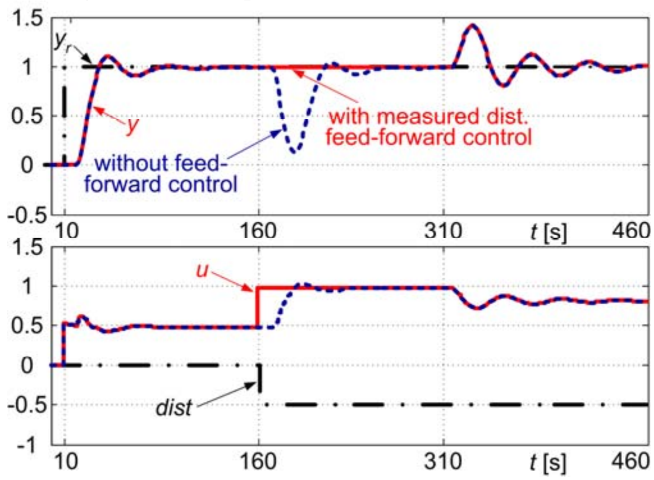
where $\overline{dist}(k)$ represents either measured or estimated disturbance signal:

$$y_m(k) = -\sum_{i=1}^{n} a_{m,i} y_{m,i}(k-1) + [u(k-1) + \overline{dist}(k-1)] \sum_{i=1}^{n} K_{m,i}(1 + a_{m,i}) \tag{16}$$

Fig. 4 shows the simulation of PFC with feed-forward algorithm based on the measured disturbance.



a) Delayed first-order process and first-order model

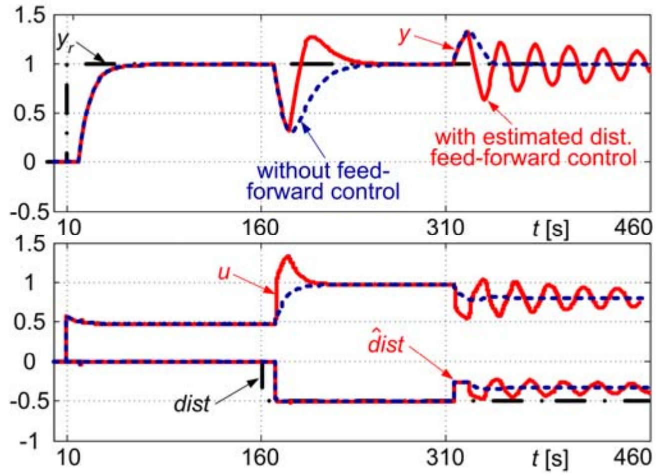


b) Delayed third-order process and first-order model

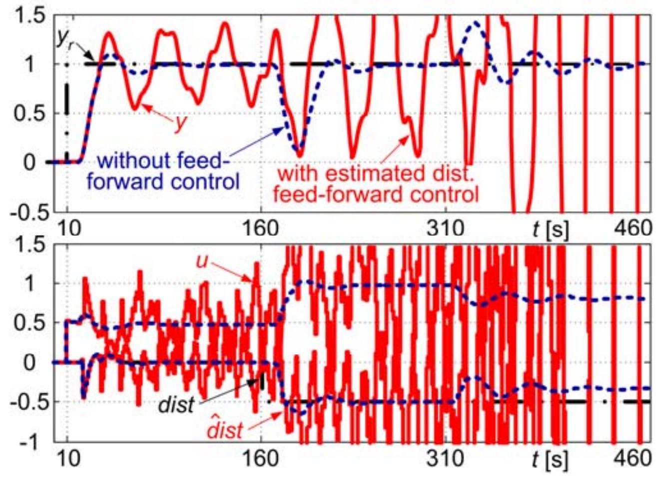
Fig. 4: PFC with measured disturbance feed-forward control

The plot shows that the external disturbance signal is compensated completely. The (usually non-measurable) structure disturbances are compensated as slowly as the external disturbances by the control without disturbance feed-forward algorithm.

Fig. 5 shows the simulation of PFC with estimated disturbance feed-forward algorithm.



a) Delayed first-order process and first-order model



b) Delayed third-order process and first-order model

Fig. 5: PFC with estimated disturbance feed-forward control

Plot 5/a shows that the estimated external and structural disturbances are compensated in case of the first-order process, however with high oscillation when the structural disturbance occurred. Plot 5/b shows instability of the process control resulting from the mismatch between the third-order process and its first-order approximating model. It is emphasized that if a correct third-order model was used with the third-order process then the control and manipulated signals would not oscillate so much. In the next section we shall show how the control behaviour can be improved even in the case of such a model mismatch.

Stabilizing the process control with estimated disturbance feed-forward algorithm can be fulfilled by applying the estimated disturbance through a filter as recommended by Richalet and O'Donavan (2009).

5. STABILIZING PFC WITH ESTIMATED DISTURBANCE FEED-FORWARD CONTROL

To enhance the stability of the closed-loop system control with estimated disturbance feed-forward algorithm, the estimated disturbance is fed forward through a filter.

Zero-order filter with gain K_f is assumed first in the following analysis. The control signal equation using K_f is:

$$u(k) = k_0[y_r - y(k) - y_m(k) + y_m(k-d_m)] + \sum_{i=1}^{n} k_i y_{m,i}(k) - K_f \cdot \hat{dist}(k-1) \tag{17}$$

Assuming nominal dead time $d_p = d_m$, and process gain variation as a structure disturbance $K_p = (1+\delta_K)K_m$, the process pulse-transfer function has the form: $q^{-d_m}G_p(q^{-1})$.

Pulse-transfer function of the control signal with estimated disturbance fed-forward (see the Appendix 1) is:

$$u(k) = \frac{k_0 y_r - k_0 q^{-d_m} G_p(q^{-1}) dist(k)}{1 + k_0 q^{-d_m} G_p(q^{-1}) + \sum_{i=1}^{n}[k_0(1-q^{-d_m}) - k_i]G_{m,i}(q^{-1})} - \frac{K_f\{1 + \sum_{i=1}^{n}[k_0(1-q^{-d_m}) - k_i]G_{m,i}(q^{-1})\}\hat{dist}(k-1)}{1 + k_0 q^{-d_m} G_p(q^{-1}) + \sum_{i=1}^{n}[k_0(1-q^{-d_m}) - k_i]G_{m,i}(q^{-1})} \tag{18}$$

Pulse-transfer function of the estimator control signal is:

$$u_e(k) = \frac{k_{0e}}{1 + \sum_{i=1}^{n}(k_{0e} - k_{ie})G_{m,i}(q^{-1})} y(k) \tag{19}$$

Pulse-transfer function of the estimated disturbance is:

$$\hat{dist}(k) = \frac{k_{0e} q^{-d_m} G_p(q^{-1})}{1 + \sum_{i=1}^{n}(k_{0e} - k_{ie})G_{m,i}(q^{-1})} dist(k) - \left[q^{-n_{pe}} - \frac{k_{0e} G_p(q^{-1})}{1 + \sum_{i=1}^{n}(k_{0e} - k_{ie})G_{m,i}(q^{-1})} \right] q^{-d_m} u(k) \tag{20}$$

The characteristic equation of the closed-loop control system can be introduced from (20) and (18) as:

$$1 = \frac{K_f\{1 + \sum_{i=1}^{n}[k_0(1-q^{-d_m}) - k_i]G_{m,i}(q^{-1})\} q^{-d_m}}{1 + k_0 q^{-d_m} G_p(q^{-1}) + \sum_{i=1}^{n}[k_0(1-q^{-d_m}) - k_i]G_{m,i}(q^{-1})} \cdot \frac{q^{-n_{pe}}[1 + \sum_{i=1}^{n}(k_{0e} - k_{ie})G_{m,i}(q^{-1})] - k_{0e}G_p(q^{-1})}{1 + \sum_{i=1}^{n}(k_{0e} - k_{ie})G_{m,i}(q^{-1})} \tag{21}$$

The sufficient stability condition of the closed-loop system can be introduced in the frequency domain for $\left|q^{-1}\right| = \left|e^{-j\Omega}\right| = 1 \quad \forall \Omega \in [0, 2\pi)$ as:

$$K_f < \left| \frac{1 + k_0 q^{-d_m} G_p(q^{-1}) + \sum_{i=1}^{n}[k_0(1-q^{-d_m}) - k_i]G_{m,i}(q^{-1})}{1 + \sum_{i=1}^{n}[k_0(1-q^{-d_m}) - k_i]G_{m,i}(q^{-1})} \right| \cdot \left| \frac{1 + \sum_{i=1}^{n}(k_{0e} - k_{ie})G_{m,i}(q^{-1})}{q^{-n_{pe}}[1 + \sum_{i=1}^{n}(k_{0e} - k_{ie})G_{m,i}(q^{-1})] - k_{0e}G_p(q^{-1})} \right| \tag{22}$$

The frequency response of the process can be defined parametrically according to the identified process model, or defined non-parametrically according to the impulse response y_I (or the step response y_s) taking into account the variations of the process gain as follows:

$$G_p(e^{-j\Omega}) = (1+\delta)\sum_{k=0}^{k_s} y_I(k) e^{-jk\Omega} \tag{23}$$

whereas the impulse response tends to zero at instant k_s.

Assuming a filter $G_f(q^{-1})$ with static gain K_f to damp the oscillations of the estimated disturbance signal, the sufficient stability condition of the closed-loop system can be introduced in the frequency domain for $\left|q^{-1}\right| = 1$ as:

$$K_f < \left| \frac{1 + k_0 q^{-d_m} G_p(q^{-1}) + \sum_{i=1}^{n}[k_0(1-q^{-d_m}) - k_i]G_{m,i}(q^{-1})}{G_f(q^{-1})\{1 + \sum_{i=1}^{n}[k_0(1-q^{-d_m}) - k_i]G_{m,i}(q^{-1})\}} \right| \cdot \left| \frac{1 + \sum_{i=1}^{n}(k_{0e} - k_{ie})G_{m,i}(q^{-1})}{q^{-n_{pe}}[1 + \sum_{i=1}^{n}(k_{0e} - k_{ie})G_{m,i}(q^{-1})] - k_{0e}G_p(q^{-1})} \right| \tag{24}$$

The stabilizing gain in (24) can be higher than the gain in (22) whereas for an aperiodic filter (first-order filter usually) the absolute value $\left|G_f(q^{-1})\right| \le 1 \quad \forall \left|q^{-1}\right| = 1$. This is expected because the filter is stabilizing the control more than without filter.

The resulted stabilizing gain conditions (22) and (24) are applied for the simulated processes. The designed gains are presented in Table 2.

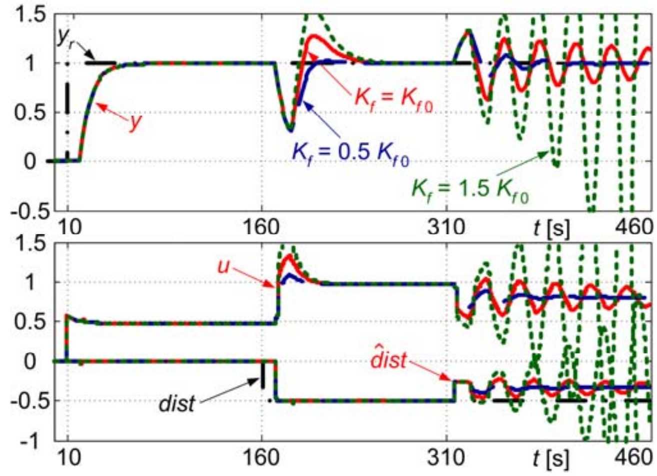
Table 2: Designed stabilizing filter's gain

Delayed process	Filter	
	Zero-order K_{f0}	First-order K_{f1}
First-order	0.9566	1.0776
Third-order	0.2449	0.2667

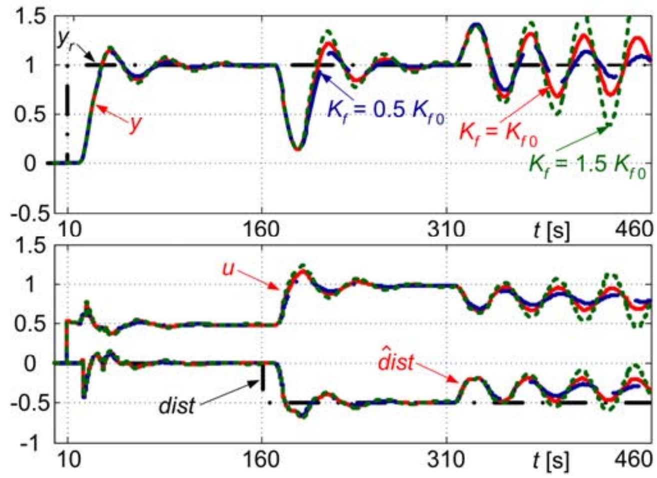
Fig. 6 shows PFC with estimated disturbance fed-forward using a gain K_{f0} (without filter, $T_f = 0\,\text{s}$) (see Table 2). The plots in Fig. 6a and Fig. 6b show that the designed stabilizing gain K_{f0} results in oscillating signals with very small damping, which shows that K_{f0} is very close to the boundary stabilizing gain and it is smaller than the critical value. Applying a smaller filter's gain damps the oscillations, and applying a higher filter's gain moves the process control toward instability.

Fig. 6c shows PFC with estimated disturbance feed-forward of the delayed third-order process assuming nominal third-order model with dead time. The gain $K_f = 1.5K_{f0} \cong 0.366$, which leads to instability in Fig. 6b, is applied in Fig. 6c. Also higher values are applied to show that with nominal model the stabilizing gain can be increased whereas it is stable for $K_f = 0.75$ and unstable for $K_f = 1$.

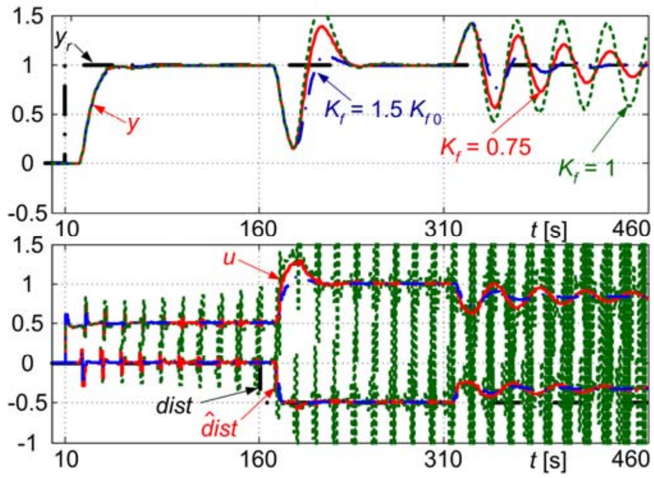
Fig. 7 shows the simulation of PFC with estimated disturbance feed-forward using first-order stabilizing filter with gain K_{f1} (see Table 2) and time constant $T_f = 3\,\text{s}$. The plot shows that the designed stabilizing gain K_{f1} results in oscillating signals with very small damping, which demonstrates that K_{f1} is smaller than and very close to the



a) Delayed first-order process and first-order model

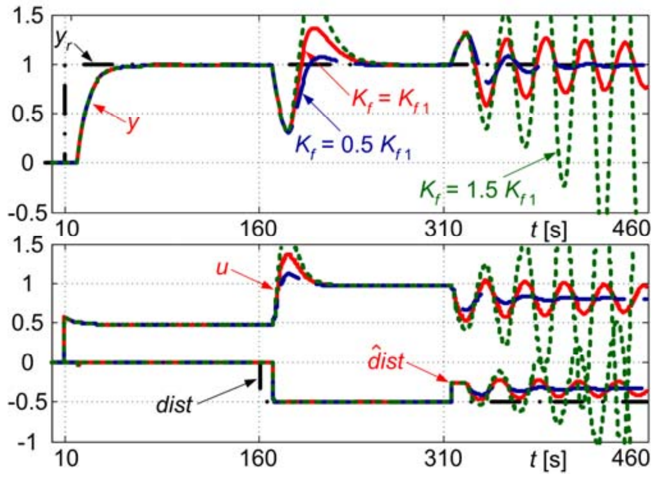


b) Delayed third-order process and first-order model

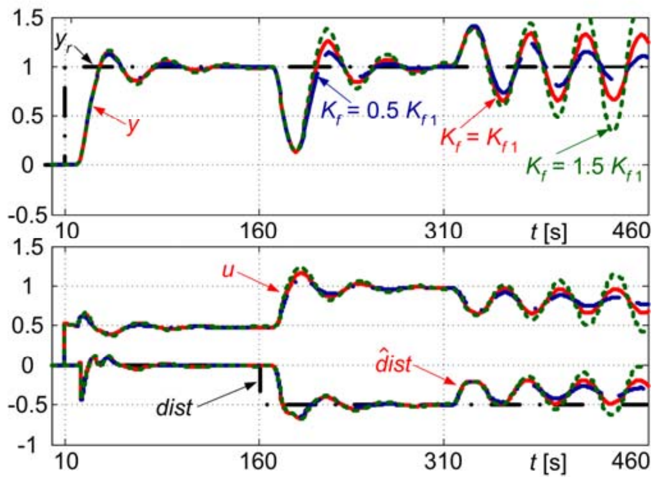


c) Delayed third-order process and third-order model

Fig. 6: PFC with estimated disturbance feed-forward control using stabilizing gain



a) Delayed first-order process and first-order model



b) Delayed third-order process and first-order model

Fig. 7: PFC with estimated disturbance feed-forward control using first-order stabilizing filter

limit value of the stabilizing gain. Applying a smaller filter's gain damps the oscillations, and applying a higher filter's gain moves the process control toward instability.

6. CASE STUDY

Mixing and tempering of liquids is a fundamental operation in the chemical industry. Therefore, this process has been chosen as a case study to illustrate the new design method. A fluid is fed into the mixer. This liquid is stirred and tempered. The temperature is controlled via a heating jacket which is supplied with water. The schema of the mixer is shown in Fig. 8.

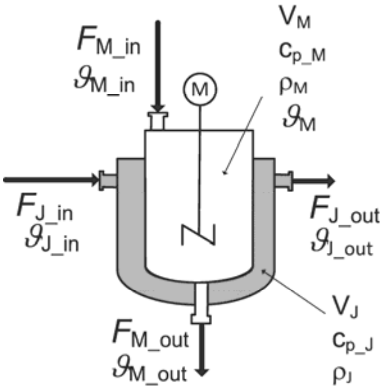


Fig. 8: Schema of the mixer

The following symbols and parameters are used:

ϑ_{M_in}: inlet temperature of fluid in mixer
ϑ_{M_out}: outlet temperature of fluid in mixer
F_{M_in} and F_{M_out}: inlet and outlet flow of fluid in mixer
$F_M = 2\ \mathrm{m^3/h} = 0.00056\ \mathrm{m^3/s}$; flow of fluid in mixer
V_M: volume of mixer
$c_{p\_M} = 3730\ \mathrm{J\cdot kg/K}$: heat capacity of fluid in mixer
$\rho_M = 841\ \mathrm{kg/m^3}$: density of fluid in mixer
ϑ_M: temperature of fluid in mixer
ϑ_{J_in}: water inlet temperature into jacket
ϑ_{J_out}: water outlet temperature from jacket
F_{J_out} and F_{J_out}: water inlet and outlet flow in jacket
$F_J = 10\ \mathrm{m^3/h} = 0.0028\ \mathrm{m^3/s}$: water flow in jacket
V_J=1 m^3: volume of jacket
$c_{p\_J} = 4180\ \mathrm{J\cdot kg/K}$: heat capacity of water
$\rho_J = 1000\ \mathrm{kg/m^3}$: density of water
A=2m^2: reactor heat transfer surface.
$U = 150\ \mathrm{W/(m^2K)}$: thermal transmittance
$UA = U\cdot A = 150\ \mathrm{W/(m^2K)}\cdot 2\,\mathrm{m^2} = 300\ \mathrm{W/K}$

The jacket is described by (25).

$$V_J\cdot\rho_J\cdot c_{p\_J}\frac{\mathrm{d}\vartheta_j(t)}{\mathrm{d}t} = UA\cdot[\vartheta_M(t)-\vartheta_J(t)] + \rho_J\cdot c_{p\_J}\cdot F_J\cdot[\vartheta_{J\_in}(t)-\vartheta_J(t)] \tag{25}$$

From (25) we obtain the following differential equation

$$T_J\frac{\mathrm{d}\vartheta_j(t)}{\mathrm{d}t} + \vartheta_J(t) = K_{J,M}\cdot\vartheta_M(t) + K_{J,J\_in}\cdot\vartheta_{J\_in}(t)$$

This equation reflects the following parameters of the jacket:

$T_J = \dfrac{V_J\cdot\rho_J\cdot c_{p\_J}}{(UA+\rho_J\cdot c_{p\_J}\cdot F_J)} = 351\mathrm{s} = 5{,}85\,\mathrm{min}$: time constant

$K_{J,M} = \dfrac{UA}{(UA+\rho_J\cdot c_{p\_J}\cdot F_J)} = 0{,}0252$: gain of input ϑ_M

$K_{J,J\_in} = \dfrac{\rho_J\cdot c_{p\_J}\cdot F_J}{(UA+\rho_J\cdot c_{p\_J}\cdot F_J)} = 0{,}9748$: gain of input ϑ_{J_in}

The mixer is described by (26).

$$V_M\cdot\rho_M\cdot c_{p\_M}\frac{\mathrm{d}\vartheta_M(t)}{\mathrm{d}t} = UA\cdot[\vartheta_J(t)-\vartheta_M(t)] + \rho_M\cdot c_{p\_M}\cdot F_M\cdot[\vartheta_{M\_in}(t)-\vartheta_M(t)] \tag{26}$$

By transforming of (26) we get the following differential equation:

$$T_M\frac{\mathrm{d}\vartheta_M(t)}{\mathrm{d}t} + \vartheta_M(t) = K_{M,J}\cdot\vartheta_J(t) + K_{M,M\_in}\cdot\vartheta_{M\_in}(t)$$

This equation reflects the following parameters of the mixer:

$T_M = \dfrac{V_M\cdot\rho_M\cdot c_{p\_M}}{(UA+\rho_M\cdot c_{p\_M}\cdot F_M)} = 3071\mathrm{s} = 51\,\mathrm{min}$: time constant

$K_{M,J} = \dfrac{UA}{(UA+\rho_M\cdot c_{p\_M}\cdot F_M)} = 0{,}1469$: gain of input ϑ_J

$K_{M,M\_in} = \dfrac{\rho_M\cdot c_{p\_M}\cdot F_M}{(UA+\rho_M\cdot c_{p\_M}\cdot F_M)} = 0{,}8531$: gain of input ϑ_{M_in}

The process model is shown in Fig. 9 with the controller.

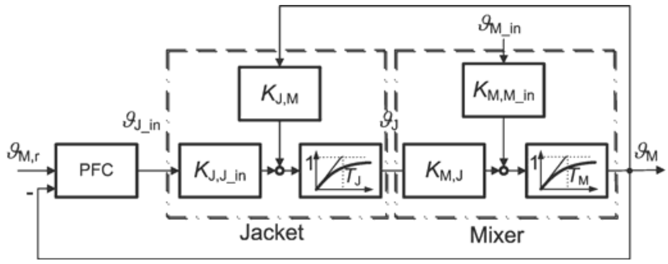


Fig. 9: Control schema of the mixer model

The process was simulated by sampling time of $\Delta t = 1\,\mathrm{min}$. First-order model with static gain $K_m = 0.1437$ and time constant $T_m = 160/3\,\mathrm{min}$ was assumed in PFC, the tuning parameters were: $T_c = 160\,\mathrm{min}$ and $n_p = 10$. Estimated disturbance feed-forward control was applied with PFC estimator parameters: $T_{ce} = 30\,\mathrm{min}$ and $n_{pe} = 10$.

Set point level $\vartheta_{M,r}$: 40°C→41°C was applied at t=50 min and after 500 min a disturbance signal step ϑ_{M_in}: 40°C→41°C °C was used in the inlet fluid temperature to the mixer.

Fig. 10 shows simulations of the temperature control without and with estimated disturbance feed-forward control using a stabilizing gain (that means no first-order filter) Fig. 10a shows the simulations using a designed critical stabilizing gain $K_f = K_{f0} = 0.976$; this plot shows good disturbance rejection with decreasing oscillations. Smaller gain $K_f = 0.75K_{f0}$ is simulated in Fig. 9b; the plot shows good disturbance rejection with smaller oscillating signals. Higher gain $K_f = 1.025 K_{f0}$ is simulated in Fig. 10c, the plot shows very strongly oscillating signals.

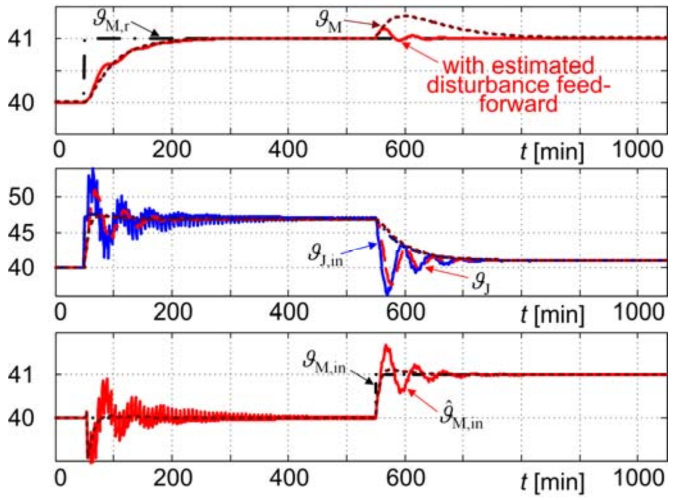


Fig. 10a: Stabilizing gain $K_f = K_{f0}$

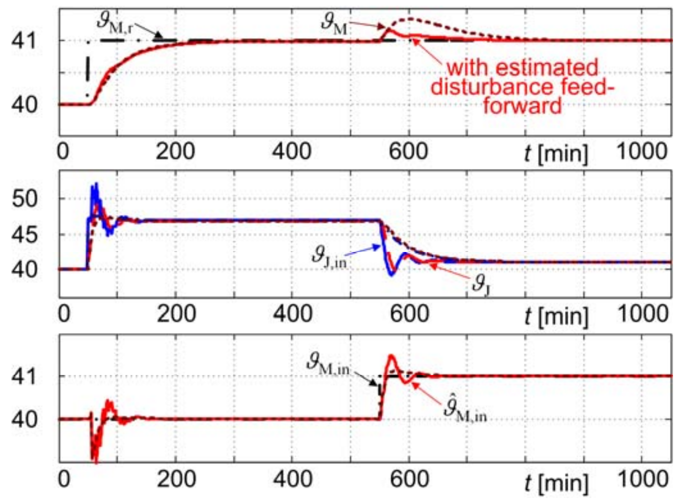


Fig. 10b: Stabilizing gain $K_f = 0.75\ K_{f0}$

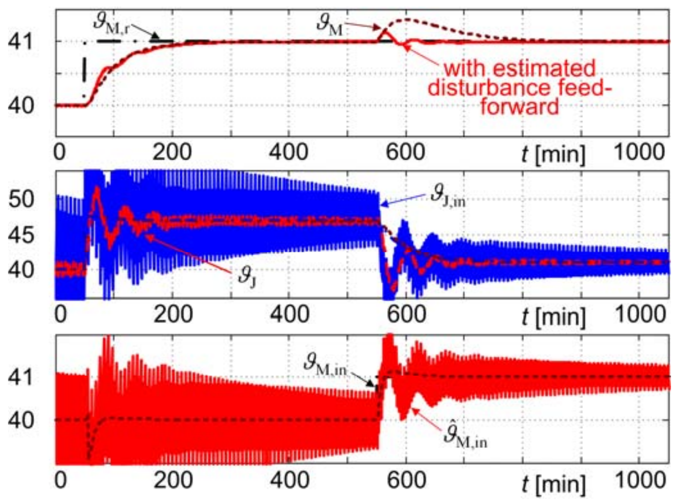


Fig. 10c: Stabilizing gain $K_f = 1.025\ K_{f0}$

Fig. 11 shows simulations of the mixer temperature control without and with estimated disturbance feed-forward control using a first-order stabilizing filter with $T_f = 10$ s.

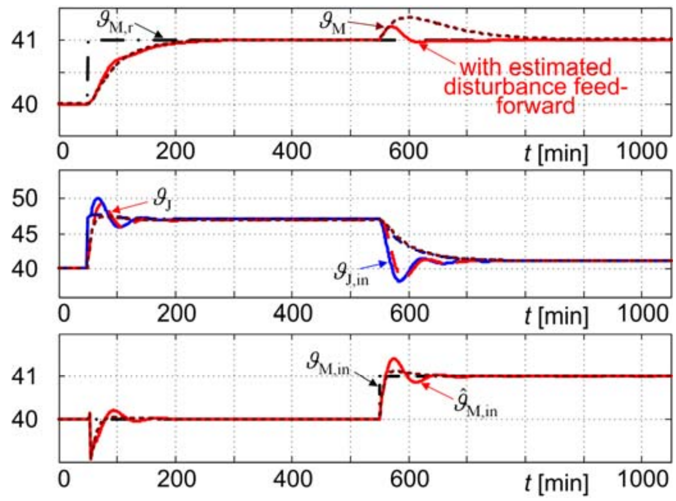


Fig. 11a: Stabilizing first-order filter with $K_f = K_{f1}$

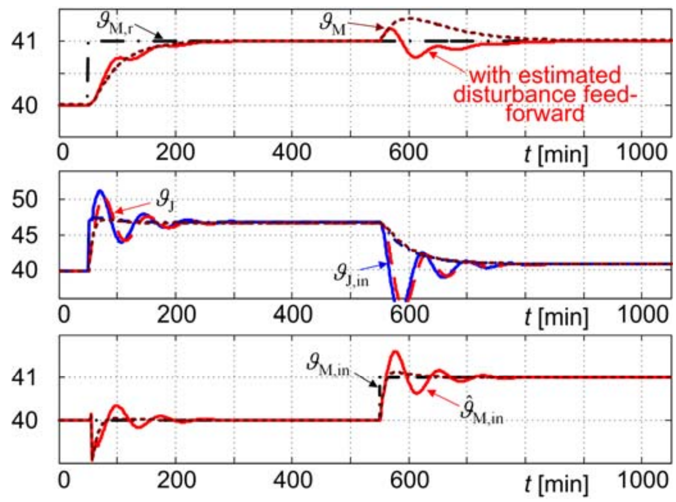


Fig. 11b: Stabilizing first-order filter with $K_f = 1.5\ K_{f1}$

Fig. 11a shows the simulations using a designed critical stabilizing gain $K_f = K_{f1} = 0.988$, the plot shows good disturbance rejection with smooth signals (because of the filter effect). Higher gain $K_f = 1.5 K_{f1}$ is simulated in Fig. 11b; the plot shows slower disturbance rejection with stronger oscillating signals.

7. CONCLUSION

PFC is a simple predictive algorithm which can be implemented easily mainly for SISO processes. The disturbance estimator using PFC algorithm was recommended by Dr. Richalet to avoid an additional implementation of another disturbance estimator algorithm. The advantage of this disturbance observer is that the recommended algorithm uses the same tools like the already implemented PFC.

The algorithm was simulated for delayed first- and higher-order processes using first-order model with dead time. Good estimation of the external and structural (gain variation) disturbances were shown even in the case that the third-order process was approximated by a first-order model.

PFC can be extended by disturbance feed-forward algorithm easily. Therefore, the estimated disturbance was used in the feed-forward control. Applying PFC with estimated disturbance feed-forward algorithm the control can become unstable, mainly if the model matching is not exact. A stabilizing technique was applied by feeding the estimated disturbance through a stabilizing filter. The stabilizing filter gain was designed based on derived sufficient stability condition in the frequency domain. The theoretical results were confirmed by simulations.

The new design method was illustrated for estimating and compensating the temperature disturbances in a mixer with heating jacket. PFC temperature control is often used for chemical reactors (see e.g. Bouhenchir, 2006). The proposed method can be used also for compensating temperature disturbances occurring with chemical reactions.

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APPENDIX

Pulse-transfer function of the manipulated variable with estimated disturbance fed-forward through a gain K_f is:

$$u(k)=k_0[y_r-y(k)-y_m(k)+y_m(k-d_m)] + \sum_{i=1}^{n} k_i y_{m,i}(k) - K_f \hat{dist}(k-1)$$

$$\begin{aligned} u(k)=\;& k_0 y_r - k_0 q^{-d_m} G_p(q^{-1})[u(k)+dist(k)] \\ & - k_0(1-q^{-d_m})G_m(q^{-1})[u(k)+K_f\hat{dist}(k-1)] \\ & + \sum_{i=1}^{n} k_i G_{m,i}(q^{-1})[u(k)+K_f\hat{dist}(k-1)] - K_f\hat{dist}(k-1) \end{aligned}$$

$$u(k)=\frac{k_0 y_r - k_0 q^{-d_m} G_p(q^{-1}) dist(k)}{1+k_0 q^{-d_m} G_p(q^{-1}) + \sum_{i=1}^{n}[k_0(1-q^{-d_m})-k_i]G_{m,i}(q^{-1})}$$

$$\frac{K_f\{1+\sum_{i=1}^{n}[k_0(1-q^{-d_m})-k_i]G_{m,i}(q^{-1})\}\hat{dist}(k-1)}{1+k_0 q^{-d_m} G_p(q^{-1}) + \sum_{i=1}^{n}[k_0(1-q^{-d_m})-k_i]G_{m,i}(q^{-1})} \tag{A1}$$

Pulse-transfer function of the estimator control signal is:

$$u_e(k)=k_{0e}[y(k)-y_{em}(k)] + \sum_{i=1}^{n} k_{ie} y_{em,i}(k)$$

$$u_e(k)=k_{0e} y(k) - \sum_{i=1}^{n}(k_{0e}-k_{ie})G_{m,i}(q^{-1})u_e(k)$$

$$u_e(k)=\frac{k_{0e}}{1+\sum_{i=1}^{n}(k_{0e}-k_{ie})G_{m,i}(q^{-1})}y(k) \tag{A2}$$

Pulse-transfer function of the estimated disturbance is:

$$\hat{dist}(k)=u_e(k)-u(k-d_m-n_{pe})$$

$$\hat{dist}(k)=\frac{k_{0e}}{1+\sum_{i=1}^{n}(k_{0e}-k_{ie})G_{m,i}(q^{-1})}y(k) - q^{-(d_m+n_{pe})}u(k)$$

$$\hat{dist}(k)=\frac{k_{0e} q^{-d_m} G_p(q^{-1})[u(k)+dist(k)]}{1+\sum_{i=1}^{n}(k_{0e}-k_{ie})G_{m,i}(q^{-1})} - q^{-(d_m+n_{pe})}u(k)$$

$$\begin{aligned} \hat{dist}(k) =\;& \frac{k_{0e} q^{-d_m} G_p(q^{-1})}{1+\sum_{i=1}^{n}(k_{0e}-k_{ie})G_{m,i}(q^{-1})} dist(k) \\ & - \left[q^{-n_{pe}} - \frac{k_{0e} G_p(q^{-1})}{1+\sum_{i=1}^{n}(k_{0e}-k_{ie})G_{m,i}(q^{-1})} \right] q^{-d_m} u(k) \end{aligned} \tag{A3}$$

Substituting (A3) into (A1) results in:

$$\begin{aligned} u(k)=\;& \frac{k_0 y_r - k_0 q^{-d_m} G_p(q^{-1}) dist(k)}{1+k_0 q^{-d_m} G_p(q^{-1}) + \sum_{i=1}^{n}[k_0(1-q^{-d_m})-k_i]G_{m,i}(q^{-1})} \\ & - \frac{K_f\{1+\sum_{i=1}^{n}[k_0(1-q^{-d_m})-k_i]G_{m,i}(q^{-1})\}}{1+k_0 q^{-d_m} G_p(q^{-1}) + \sum_{i=1}^{n}[k_0(1-q^{-d_m})-k_i]G_{m,i}(q^{-1})} \\ & \cdot \frac{k_{0e} q^{-d_m} G_p(q^{-1})}{1+\sum_{i=1}^{n}(k_{0e}-k_{ie})G_{m,i}(q^{-1})} dist(k) \end{aligned}$$

$$
+\frac{K_f\{1+\sum_{i=1}^{n}[k_0(1-q^{-d_m})-k_i]G_{m,i}(q^{-1})\}}{1+k_0q^{-d_m}G_p(q^{-1})+\sum_{i=1}^{n}[k_0(1-q^{-d_m})-k_i]G_{m,i}(q^{-1})}
\cdot\left[q^{-n_{pe}}-\frac{k_{0e}G_p(q^{-1})}{1+\sum_{i=1}^{n}(k_{0e}-k_{ie})G_{m,i}(q^{-1})}\right]q^{-d_m}u(k)
$$

Characteristic equation of the closed-loop control system is

$$
1-\frac{K_f\{1+\sum_{i=1}^{n}[k_0(1-q^{-d_m})-k_i]G_{m,i}(q^{-1})\}}{1+k_0q^{-d_m}G_p(q^{-1})+\sum_{i=1}^{n}[k_0(1-q^{-d_m})-k_i]G_{m,i}(q^{-1})}
\cdot\left[q^{-n_{pe}}-\frac{k_{0e}G_p(q^{-1})}{1+\sum_{i=1}^{n}(k_{0e}-k_{ie})G_{m,i}(q^{-1})}\right]q^{-d_m}=0
$$

$$
\Rightarrow 1=\frac{K_f\{1+\sum_{i=1}^{n}[k_0(1-q^{-d_m})-k_i]G_{m,i}(q^{-1})\}}{1+k_0q^{-d_m}G_p(q^{-1})+\sum_{i=1}^{n}[k_0(1-q^{-d_m})-k_i]G_{m,i}(q^{-1})}
\cdot\left[q^{-n_{pe}}-\frac{k_{0e}G_p(q^{-1})}{1+\sum_{i=1}^{n}(k_{0e}-k_{ie})G_{m,i}(q^{-1})}\right]q^{-d_m}
$$

$$
\Rightarrow K_f=\frac{1+k_0q^{-d_m}G_p(q^{-1})+\sum_{i=1}^{n}[k_0(1-q^{-d_m})-k_i]G_{m,i}(q^{-1})}{q^{-d_m}\{1+\sum_{i=1}^{n}[k_0(1-q^{-d_m})-k_i]G_{m,i}(q^{-1})\}}
\cdot\frac{1+\sum_{i=1}^{n}(k_{0e}-k_{ie})G_{m,i}(q^{-1})}{q^{-n_{pe}}[1+\sum_{i=1}^{n}(k_{0e}-k_{ie})G_{m,i}(q^{-1})]-k_{0e}G_p(q^{-1})}
\tag{A4}
$$

The sufficient condition of the stability in the frequency domain ($\left|q^{-1}\right|=1$) is:

$$
K_f<\left|\frac{1+k_0q^{-d_m}G_p(q^{-1})+\sum_{i=1}^{n}[k_0(1-q^{-d_m})-k_i]G_{m,i}(q^{-1})}{1+\sum_{i=1}^{n}[k_0(1-q^{-d_m})-k_i]G_{m,i}(q^{-1})}\right|
\cdot\left|\frac{1+\sum_{i=1}^{n}(k_{0e}-k_{ie})G_{m,i}(q^{-1})}{q^{-n_{pe}}[1+\sum_{i=1}^{n}(k_{0e}-k_{ie})G_{m,i}(q^{-1})]-k_{0e}G_p(q^{-1})}\right|
\tag{A5}
$$

Assuming a stabilizing filter with gain K_f, the sufficient condition of the stability in the frequency domain is:

$$
K_f<\left|\frac{1+k_0q^{-d_m}G_p(q^{-1})+\sum_{i=1}^{n}[k_0(1-q^{-d_m})-k_i]G_{m,i}(q^{-1})}{G_f(q^{-1})\{1+\sum_{i=1}^{n}[k_0(1-q^{-d_m})-k_i]G_{m,i}(q^{-1})\}}\right|
\cdot\left|\frac{1+\sum_{i=1}^{n}(k_{0e}-k_{ie})G_{m,i}(q^{-1})}{q^{-n_{pe}}[1+\sum_{i=1}^{n}(k_{0e}-k_{ie})G_{m,i}(q^{-1})]-k_{0e}G_p(q^{-1})}\right|
\tag{A6}
$$

For an aperiodic filter (with unity gain), the absolute value of the pulse-transfer function in the frequency domain is:

$$
\left|G_f(q^{-1})\right|\le 1 \tag{A7}
$$

First-order filter (with unity gain) as an example has an absolute value of the pulse-transfer function as:

$$
\frac{1+a_f}{1-a_f}\le\left|\frac{(1+a_f)q^{-1}}{1+a_fq^{-1}}\right|\le 1 \qquad \forall\left|q^{-1}\right|=1
$$

From (A7) we can see that the upper limit of the stabilizing gain in (A6) is higher or equals the upper limit of the stabilizing gain in (A5)

$$
\left|\frac{1+k_0q^{-d_m}G_p(q^{-1})+\sum_{i=1}^{n}[k_0(1-q^{-d_m})-k_i]G_{m,i}(q^{-1})}{G_f(q^{-1})\{1+\sum_{i=1}^{n}[k_0(1-q^{-d_m})-k_i]G_{m,i}(q^{-1})\}}\right|
\cdot\left|\frac{1+\sum_{i=1}^{n}(k_{0e}-k_{ie})G_{m,i}(q^{-1})}{q^{-n_{pe}}[1+\sum_{i=1}^{n}(k_{0e}-k_{ie})G_{m,i}(q^{-1})]-k_{0e}G_p(q^{-1})}\right|
$$

$$
\ge\left|\frac{1+k_0q^{-d_m}G_p(q^{-1})+\sum_{i=1}^{n}[k_0(1-q^{-d_m})-k_i]G_{m,i}(q^{-1})}{1+\sum_{i=1}^{n}[k_0(1-q^{-d_m})-k_i]G_{m,i}(q^{-1})}\right|
\cdot\left|\frac{1+\sum_{i=1}^{n}(k_{0e}-k_{ie})G_{m,i}(q^{-1})}{q^{-n_{pe}}[1+\sum_{i=1}^{n}(k_{0e}-k_{ie})G_{m,i}(q^{-1})]-k_{0e}G_p(q^{-1})}\right|
$$

$$
\Rightarrow\left|\frac{1}{G_f(q^{-1})}\right|\ge 1
$$