Adaptive Iterative Learning Control for Discrete-Time Nonlinear Systems without Knowing the Control Gain Signs

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Abstract: An adaptive iterative learning control method is proposed for a class of nonlinear strict-feedback discrete-time systems with random initial conditions and iteration-varying desired trajectories. An n-step ahead predictor approach is employed to estimate the future states in the control design. Discrete Nussbaum gain method is utilized to deal with the lack of *a priori* knowledge of control directions. The proposed control algorithm guarantees the boundedness of all the signals in the controlled system. The tracking error converges to zero asymptotically along the iterative learning axis except for beginning states affected by random initial conditions. The effectiveness of the proposed control scheme is verified through numerical simulation.

Keywords: iterative learning control; unknown control directions; discrete Nussbaum gain; n-step ahead state predictor.

1. INTRODUCTION

During the past decades, iterative learning control (ILC) has been studied extensively and attracts a lot of research interests. It was first proposed by Arimoto et al. (1984) in the application of robot manipulators. Then numerous results have been dedicated to ILC design based on contraction mapping (see e.g. Moore (1993); Chen and Wen (1999); Ahn et al. (2007)). However, there are several critical requirements that limit the applications of ILC, especially to complex nonlinear systems. For instance, the nonparametric uncertainties in nonlinear systems are requested to satisfy the global Lipschitz condition. Meanwhile, the initial value of each trial is required to be the same, which is difficult to realize in practice. In order to relax these prerequisites of traditional ILC, several adaptive ILC methods are developed to deal with parametric uncertainties in nonlinear systems (see e.g. French and Rogers (2000); Xu and Tan (2003); Marino and Tomei (2009); Xu (2011); Yu et al. (2011)). An iterative parameter adaptation law is incorporated into the control design to achieve the pointwise tracking along iterative learning axis.

While most of the existing results focused on ILC of continuous-time systems, ILC of discrete-time systems deserves more efforts since it is more suitable for real implementation. As a discretized version of D-type continuous learning control algorithm, the D-type discrete learning control was proposed in Saab (1995) and applied to robot manipulators. The 2-D system theory was successfully adopted in the analysis of discrete-time ILC utilizing the property that the system progresses in both the time domain and the iteration domain (Kurek and Zaremba (1993); Fang and Chow (2003)). Recently, an adaptive ILC scheme was presented for a class of discrete-time systems in Chi et al. (2008), in which the requirements of identical initial condition and iteration-invariant reference trajectory were removed.

The control direction is of great significance in all ILC control designs since it presents the motion direction of the system under any control. Nonetheless, in some cases, the control direction is difficult to detect or be decided from the physical meaning, which makes the control design much more difficult. The continuous Nussbaum gain method, which was first proposed by Nussbaum (1983), is a popular method to deal with the problem of unknown control direction since it is easy to implement in the control design. It was then adopted in adaptive control design of high-order nonlinear systems (Ye and Jiang (1998); Ge and Wang (2003)) and ILC schemes with unknown control direction (Chen and Jiang (2004); Xu and Yan (2004); Dang and Owens (2006)). Analogous to the continuous Nussbaum gain, the discrete Nussbaum gain method was developed in Lee and Narendra (1986), which met several essential properties of the continuous one. It was then applied in the discrete-time adaptive control design in Ge et al. (2008); Yang et al. (2009) to overcome the difficulties caused by unknown control directions.

In this paper, the discrete Nussbaum gain method is utilized in learning control design to remove the key assumption in all the existing studies on discrete-time ILC that the control direction is known and invariant. A class of high-order nonlinear discrete-time systems with strict-feedback form is considered, whose continuous-time counterpart has been studied in our previous work Yu et al. (2011). In order to solve the problem associated with unknown control directions in the step-by-step deduction, an n-step ahead state predictor approach is applied to estimate future states exploited in the control law. The proposed control scheme is free of controller singularity. The tracking error converges to zero asymptotically along the iterative learning axis under random initial conditions and iteration-varying target trajectories, while all the signals are kept bounded.

2. PROBLEM FORMULATION AND PRELIMINARIES

2.1 Problem formulation

Consider the following strict-feedback discrete-time nonlinear systems

$$
\begin{aligned}
x_1(i, t+1) &= \theta_1^T \xi_1(\bar{x}_1(i,t), t) + b_1 x_2(i,t), \\
x_2(i, t+1) &= \theta_2^T \xi_2(\bar{x}_2(i,t), t) + b_2 x_3(i,t), \\
&\vdots \\
x_n(i, t+1) &= \theta_n^T \xi_n(\bar{x}_n(i,t), t) + b_n u(i,t), \\
y(i, t) &= x_1(i,t),
\end{aligned} \tag{1}
$$

where $\bar{x}_j(i,t) = [x_1(i,t), x_2(i,t), \cdots, x_j(i,t)]^T$ $(1 \le j \le n)$ denote the states at time instant t of ith iteration, $t \in [0, 1, \cdots, T]$, $i = 0, 1, 2, \cdots$; $u(i,t)$ is the system input; θ_j are the unknown bounded parameters; $\xi_j(\bar{x}_j(i,t), t)$ are known vector-valued nonlinear functions with respect to $\bar{x}_j(i,t)$ and t; b_j are the unknown input gains. $x_j(i,0)$, the initial conditions of each iteration, are random and bounded.

To explicate the control design, we make the following assumptions on system (1).

Assumption 1. The nonlinear functions $\xi_j(\bar{x}_j(i,t), t)$ are global Lipschitz, i.e., $\left\| \xi_j(\eta_1) - \xi_j(\eta_2) \right\| \le L_j \|\eta_1 - \eta_2\|$, $\forall 1 \le j \le n$, where L_j are Lipschitz coefficients.

Assumption 2. The input gains b_j are nonsingular, i.e., $b_j \ne 0$, $1 \le j \le n$. The signs of b_j, which are called control directions, are assumed to be unknown.

Rewrite system (1) as

$$
\begin{aligned}
y(i, t+n) &= \theta_1^T \xi_1(\bar{x}_1(i, t+n-1), t+n-1) \\
&\quad + b_1 x_2(i, t+n-1), \\
x_2(i, t+n-1) &= \theta_2^T \xi_2(\bar{x}_2(i, t+n-2), t+n-2) \\
&\quad + b_2 x_3(i, t+n-2), \\
&\vdots \\
x_n(i, t+1) &= \theta_n^T \xi_n(\bar{x}_n(i,t), t) + b_n u(i,t).
\end{aligned} \tag{2}
$$

After straight iteration, it is easy to obtain that

$$
y(i, t+n) = \sum_{k=1}^{n} \Theta_k^T \xi_k(\bar{x}_k(i, t+n-k), t+n-k) + bu(i,t), \tag{3}
$$

where $\Theta_1 = \theta_1, \Theta_k = \theta_k \prod_{j=1}^{k-1} b_j, j = 2, 3, \cdots, n, b = \prod_{j=1}^{n} b_j$. Define $\Theta^T = [\Theta_1^T, \Theta_2^T, \cdots, \Theta_n^T]^T$, $\xi(i, t+n-1) = [\xi_1^T(\bar{x}_1(i, t+n-1), t+n-1), \xi_2^T(\bar{x}_2(i, t+n-2), t+n-2), \cdots, \xi_n^T(\bar{x}_n(i,t), t)]^T$. Then equation (3) could be rewritten as

$$
y(i, t+n) = \Theta^T \xi(i, t+n-1) + bu(i,t). \tag{4}
$$

Define the reference trajectory as $y_d(i,t)$, where $t \in [0, 1, \cdots, T]$, $i = 0, 1, 2, \cdots$. Note that $y_d(i,t)$ can be variant with iterations, which removes the critical assumption of invariant desired trajectory in traditional ILC based on contraction mapping (Xu (1997), Wang (1998)). Then the tracking error is $e(i,t) = y(i,t) - y_d(i,t)$. The control objective is to find a sequence of suitable system inputs $u(i,t)$, $t \in [0, 1, \cdots, T-n]$, $i = 0, 1, 2, \cdots$, such that the output of system (1) converges to the desired trajectory asymptotically along the iteration axis except for

beginning n instants of each iteration, that is $\lim_{i \to \infty} e(i,t) = 0$, $t \in [n, \cdots, T]$.

Remark 1. In view of (1) and (4), the control input of initial instant $u(i,0)$ is involved in the output after n steps $y(i,n)$. The outputs of first n instants $y(i,t)$, $0 \le t \le n-1$, are affected by the random initial conditions, thus are not learnable.

2.2 Preliminaries about discrete Nussbaum gain

The discrete Nussbaum gain was first proposed in Lee and Narendra (1986). Let $\{\chi(k)\}$ be a discrete sequence with $\chi(0) = 0, \chi(k) \ge 0$, for $k = 0, 1, 2, \cdots$, and $|\Delta\chi(k)| = |\chi(k+1) - \chi(k)| \le c$, where c is a constant.

The discrete nonlinear Nussbaum gain is chosen as

$$
N(\chi(k)) = \chi_s(k) s(\chi(k)), \tag{5}
$$

where

$$
\chi_s(k) \overset{\Delta}{=} \sup_{\sigma \le k} \{\chi(\sigma)\}, \tag{6}
$$

and the sign function $s(\chi(k))$ which swings between $+1$ and -1 is defined as follows,

$$
s(\chi(0)) = +1,
$$

At $k = k_1$, if $s(\chi(k_1)) = +1$, then if

$$
\sum_{\sigma=0}^{\sigma=k_1} N(\chi(\sigma)) \Delta\chi(\sigma) > \chi_s^{3/2}(k_1),
$$

set $s(\chi(k_1+1)) = -1$, otherwise set $s(\chi(k_1+1)) = +1$.

But if $s(\chi(k_1)) = -1$, then if

$$
\sum_{\sigma=0}^{\sigma=k_1} N(\chi(\sigma)) \Delta\chi(\sigma) < -\chi_s^{3/2}(k_1),
$$

set $s(\chi(k_1+1)) = +1$, otherwise set $s(\chi(k_1+1)) = -1$.

Obviously it is easy for the digital implementation. Associated with discrete Nussbaum gain, two important properties corresponding to that of continuous Nussbaum gain (Nussbaum (1983)) were derived.

Lemma 1. (Lee and Narendra (1986))(The Oscillating-Unbounded Sum Property) Let

$$
S(\chi(k)) \overset{\Delta}{=} \sum_{\sigma=0}^{\sigma=k} N(\chi(\sigma)) \Delta\chi(\sigma). \tag{7}
$$

If $\chi_s(k)$ increases without bound, then

$$
\sup_{\chi_s(k) \ge a_1} \frac{1}{\chi_s(k)} S(\chi(k)) = +\infty, \tag{8}
$$

$$
\inf_{\chi_s(k) \ge a_1} \frac{1}{\chi_s(k)} S(\chi(k)) = -\infty. \tag{9}
$$

Lemma 2. (Lee and Narendra (1986))(The Bounded Sum Property) If $\chi_s(k) \le \xi_1$, then $|S(\chi(k))| \le \xi_2$ where ξ_1 and ξ_2 are some constants.

Recently, a basic lemma was derived in Ge et al. (2008), which facilitates the application of discrete Nussbaum gain in the adaptive control design.

Lemma 3. (Ge et al. (2008)) Let $V(k)$ be a positive definite function defined $\forall k$, $N(\chi(k))$ be the discrete Nussbaum gain proposed in Lee and Narendra (1986), and θ be a nonzero constant. If the following inequality holds

$$
V(k) \le \sum_{k'=k_1}^{k} (c_1 + \theta N(\chi(k'))) \Delta\chi(k') + c_2 \chi(k) + c_3, \forall k \tag{10}
$$

where c_1, c_2 and c_3 are some constants, k_1 is a positive integer, then $V(k)$, $\chi(k)$, and $\sum_{k'=k_1}^{k}(c_1 + \theta N(\chi(k')))\Delta\chi(k') + c_2\chi(k) + c_3$ must be bounded, $\forall k$.

2.3 Useful definitions and lemmas

Definition 1. (Chen and Narendra (2001)) Let $x(k)$ and $y(k)$ (scalar or vector) be two discrete time signals defined for all $k \in \aleph^+$, where $\aleph^+$ is the set of all nonnegative integers. Let $|\cdot|$ denotes a norm.

- (Large order). We denote $y(k) = O[x(k)]$ if there exist positive constants M_1, M_2, and k_0 such that $|y(k)| \leq M_1 \max_{\tau \leq k}|x(\tau)| + M_2, \forall k \geq k_0$.
- (Small order). We denote $y(k) = o[x(k)]$ if there exists a discrete-time function $\beta(k)$ with the property that $\lim_{k\to\infty}\beta(k) = 0$, and a constant k_0 such that $|y(k)| \leq \beta(k)\max_{\tau \leq k}|x(\tau)|, \forall k \geq k_0$.
- (Equivalence). If $y(k) = O[x(k)]$ and $x(k) = O[y(k)]$, we refer to $x(k)$ and $y(k)$ as being equivalent and denote it as $x(k) \sim y(k)$.

Lemma 4. (Goodwin and Sin (1984)) If the following conditions are satisfied for some given sequences $\{s(t)\}$, $\{\sigma(t)\}$, $\{b_1(t)\}$, and $\{b_2(t)\}$

(1) $\lim_{t\to\infty}\dfrac{s^2(t)}{b_1(t) + b_2(t)\sigma^T(t)\sigma(t)} = 0$, where $\{b_1(t)\}$, $\{b_2(t)\}$, and $\{s(t)\}$ are real scalar sequences and $\{\sigma(t)\}$ is a real $(p \times 1)$ vector sequence.
(2) Uniform boundedness condition: $0 < b_1(t) < K < \infty$, $0 < b_2(t) < K < \infty$, for all $t \geq 1$.
(3) Linear boundedness condition: $\sigma(t) = O[s(t)]$.

it follows that

(1) $\lim_{t\to\infty} s(t) = 0$.
(2) $\|\sigma(t)\|$ is bounded.

3. DISCRETE-TIME ADAPTIVE ILC DESIGN

3.1 N-step ahead state predictor

The error dynamics of the system can be expressed as

$$e(i, t+n) = y(i, t+n) - y_d(i, t+n)$$
$$= \Theta^T\xi(i, t+n-1) + bu(i, t) - y_d(i, t+n), \quad (11)$$

where $t = 0, 1, \cdots, T-n$. Denote $\alpha = b^{-1}\Theta, \beta = b^{-1}$, the system input can be chosen as

$$u(i, t) = -\hat{\alpha}^T(i, t)\xi(i, t+n-1) + \hat{\beta}(i, t)y_d(i, t+n), \quad (12)$$

where $\hat{\alpha}(i, t)$ and $\hat{\beta}(i, t)$ denote the estimates of parameters α and β at time instant t of ith iteration.

The control law (12) is a cancelation controller that aims to exactly cancel the nonlinearities using state feedback and parameter adaptation. Instead of iteratively learning the iteration-invariant desired input $u_d(t)$ in traditional ILC (Xu (1997), Wang (1998)), we mainly focus on the iterative adaptation of unknown parameters $b^{-1}\Theta$ and b^{-1}. Therefore, the requirements of identical initial condition and iteration-invariant reference can be removed. On the other hand, the control law (12) is asymptotically an inverse model algorithm on $[n, T]$ or equivalently a pole allocation algorithm allocating all poles to zero. It will be subject to the robustness issues even for the case

of linear systems (Goodwin and Sin (1984)). Consequently, considering Remark 1, the control objective is to achieve the exact tracking on interval $[n, T]$.

It is noticed the noncausal term $\xi(i, t+n-1)$ appears on the right side of (12). In order to explicate the control process, we shall estimate the states of future n steps at current step to design the appropriate input $u(i, t)$.

Let $\hat{\theta}_j(i, t)$ and $\hat{b}_j(i, t)$ denote the estimates of θ_j and b_j at the j-th step. Denote $\phi_j = [\theta_j^T, b_j]^T$, the estimation errors are $\tilde{\phi}_j = \phi_j - \hat{\phi}_j$.

Define the one-step state predictor as

$$\hat{x}_j(i, t+1|t) = \hat{\phi}_j^T(i, t-n+2)\psi_j(i, t), j = 1, 2, \cdots, n-1 \quad (13)$$

where $\psi_j(i, t) = [\xi_j^T(\bar{x}_j(i, t), t), x_{j+1}(i, t)]^T$.

Define the two-step state predictor as

$$\hat{x}_j(i, t+2|t) = \hat{\phi}_j^T(i, t-n+3)\hat{\psi}_j(i, t+1|t), j = 1, 2, \cdots, n-2 \quad (14)$$

where $\hat{\psi}_j(i, t+1|t) = [\xi_j^T(\bar{x}_j(i, t+1|t), t+1), \hat{x}_{j+1}(i, t+1|t)]^T$.

Define the m-step ($m = 3, \cdots, n-1$) state predictor as

$$\hat{x}_j(i, t+m|t) = \hat{\phi}_j^T(i, t-n+m+1)\hat{\psi}_j(i, t+m-1|t), j = 1, 2, \cdots, n-m \quad (15)$$

where $\hat{\psi}_j(i, t+m-1|t) = [\xi_j^T(\bar{x}_j(i, t+m-1|t), t+m-1), \hat{x}_{j+1}(i, t+m-1|t)]^T$.

The parameter estimates updating law is defined as

$$\hat{\phi}_j(i, t+1) = \hat{\phi}_j(i, t-n+2) - \frac{\tilde{x}_j(i, t+1|t)\psi_j(i, t)}{1 + \psi_j^T(i, t)\psi_j(i, t)}, j = 1, 2, \cdots, n-1 \quad (16)$$

where $\tilde{x}_j(i, t+1|t) = x_j(i, t+1) - \hat{x}_j(i, t+1|t)$.

With respect to (4), define $\hat{\xi}(i, t+n-1|t) = [\xi_1^T(\bar{\hat{x}}_1(i, t+n-1|t), t+n-1), \xi_2^T(\bar{\hat{x}}_2(i, t+n-2|t), t+n-2), \cdots, \xi_n^T(\bar{x}_n(i, t), t)]^T$, we have the following result

Lemma 5. The parameter estimates $\hat{\phi}_j(i, t)$, $j = 1, 2, \cdots, n-1$, are bounded. The estimation errors satisfy

$$\bar{\tilde{x}}_j(i, t+n-j|t) = o[O[y(i, t+n-1)]],$$
$$\tilde{\xi}(i, t+n-1|t) = o[O[y(i, t+n-1)]].$$

Proof: See the proofs of Lemma 6 and 7 in Ge et al. (2008). ∎

3.2 Adaptive iterative learning control design

With the n-step state predictor above, we design the system input as

$$u(i, t) = -\hat{\alpha}^T(i, t)\hat{\xi}(i, t+n-1|t) + \hat{\beta}(i, t)y_d(i, t+n). \quad (17)$$

Substituting (17) into (11), we have

$$e(i, t+n) = -b(\tilde{\alpha}^T(i, t)\xi(i, t+n-1) - \tilde{\beta}(i, t)y_d(i, t+n)$$
$$+ \hat{\alpha}^T(i, t)\tilde{\xi}(i, t+n-1|t)), \quad (18)$$

where $\tilde{\alpha}(i, t) = \hat{\alpha}(i, t) - \alpha, \tilde{\beta}(i, t) = \hat{\beta}(i, t) - \beta$ are the estimation errors.

The parameter estimates updating law along the iteration axis is defined as

$$\varepsilon(i-1,t+n) = \frac{\gamma e(i-1,t+n)}{G(i,t)}$$
$$+\frac{\mathrm{N}(\chi(i,t))\varphi(i,t)\hat{\alpha}^{\mathrm{T}}(i-1,t)\tilde{\xi}(i-1,t+n-1|t)}{G(i,t)},$$

$$\hat{\alpha}(i,t) = \hat{\alpha}(i-1,t) +$$
$$\gamma \frac{\mathrm{N}(\chi(i,t))}{D(i,t)}\xi(i-1,t+n-1)\varepsilon(i-1,t+n),$$

$$\hat{\beta}(i,t) = \hat{\beta}(i-1,t)$$
$$-\gamma\frac{\mathrm{N}(\chi(i,t))}{D(i,t)}y_d(i-1,t+n)\varepsilon(i-1,t+n),$$

$$\Delta\varphi(i,t) = \varphi(i+1,t) - \varphi(i,t)$$
$$= -\mathrm{N}(\chi(i,t))\hat{\alpha}^{\mathrm{T}}(i-1,t)\tilde{\xi}(i-1,t+n-1|t)$$
$$\times\varepsilon(i-1,t+n)/D(i,t),$$

$$\Delta z(i,t) = z(i+1,t) - z(i,t) = \frac{G(i,t)\varepsilon^2(i-1,t+n)}{D(i,t)},$$

$$\chi(i,t) = z(i,t) + \frac{\varphi^2(i,t)}{2},$$

$$G(i,t) = 1 + |\mathrm{N}(\chi(i,t))|,$$

$$D(i,t) = (1+|\varphi(i,t)|)(1+\left|\mathrm{N}^3(\chi(i,t))\right|)$$
$$\times(1 + \|\xi(i-1,t+n-1)\|^2 + y_d^2(i-1,t+n)$$
$$+(\hat{\alpha}^{\mathrm{T}}(i-1,t)\tilde{\xi}(i-1,t+n-1|t))^2$$
$$+\varepsilon^2(i-1,t+n)), \qquad (19)$$

where $\mathrm{N}(\chi(i,t))$ is the discrete Nussbaum function defined in (5) which updates along the iteration axis at time instant t, $t = 0,1,\cdots T-n$. The parameter $\gamma > 0$ is tunable to improve the learning performance.

4. LEARNING CONVERGENCE ANALYSIS

Theorem 1. Consider the discrete-time nonlinear system (1) with random initial conditions and iteration-varying desired trajectories, if Assumptions 1 and 2 are satisfied, applying the proposed adaptive learning control law (17) and parameter estimate updating law (19), all the signals in the system are guaranteed to be bounded. Moreover, the tracking error converges to zero asymptotically along the iterative learning axis except for beginning n instants of each iteration, that is $\lim_{i\to\infty} e(i,t) = 0$, $t = n,\cdots,T$.

Proof: Define a positive definite function as
$$V(i,t) = \tilde{\alpha}^{\mathrm{T}}(i,t)\tilde{\alpha}(i,t) + \tilde{\beta}^2(i,t). \qquad (20)$$
Then the difference of $V(i,t)$ along the iteration axis is derived as

$$\Delta V(i,t) = V(i,t) - V(i-1,t)$$
$$= (\tilde{\alpha}(i,t) - \tilde{\alpha}(i-1,t))^{\mathrm{T}}(\tilde{\alpha}(i,t) - \tilde{\alpha}(i-1,t))$$
$$+2\tilde{\alpha}^{\mathrm{T}}(i-1,t)(\tilde{\alpha}(i,t) - \tilde{\alpha}(i-1,t))$$
$$+(\tilde{\beta}(i,t) - \tilde{\beta}(i-1,t))^2$$
$$+2\tilde{\beta}(i-1,t)(\tilde{\beta}(i,t) - \tilde{\beta}(i-1,t)). \qquad (21)$$

Substituting the parameter estimate updating law (19) into (21) yields

$$\Delta V(i,t) = \gamma^2 \frac{\mathrm{N}^2(\chi(i,t))}{D^2(i,t)}\varepsilon^2(i-1,t+n)$$
$$\times(\|\xi(i-1,t+n-1)\|^2 + y_d^2(i-1,t+n))$$
$$+2\gamma\frac{\mathrm{N}(\chi(i,t))}{D(i,t)}\varepsilon(i-1,t+n)(\tilde{\alpha}^{\mathrm{T}}(i-1,t)$$
$$\times\xi(i-1,t+n-1) - \tilde{\beta}(i-1,t)y_d(i-1,t+n)).$$

Considering the error dynamic (18) and parameter definition (19), it can be obtained that

$$e(i-1,t+n) = (\varepsilon(i-1,t+n)G(i,t) - \mathrm{N}(\chi(i,t))\varphi(i,t)$$
$$\times\hat{\alpha}^{\mathrm{T}}(i-1,t)\tilde{\xi}(i-1,t+n-1|t))/\gamma$$
$$= -b(\tilde{\alpha}^{\mathrm{T}}(i-1,t)\xi(i-1,t+n-1)$$
$$-\tilde{\beta}(i-1,t)y_d(i-1,t+n)$$
$$+\hat{\alpha}^{\mathrm{T}}(i-1,t)\tilde{\xi}(i-1,t+n-1|t)). \qquad (22)$$

Then it can be derived that

$$\Delta V(i,t) = \gamma^2 \frac{\mathrm{N}^2(\chi(i,t))}{D^2(i,t)}\varepsilon^2(i-1,t+n)(\|\xi(i-1,t+n-1)\|^2$$
$$+y_d^2(i-1,t+n)) - \frac{2}{b}\frac{\mathrm{N}(\chi(i,t))}{D(i,t)}\varepsilon^2(i-1,t+n)G(i,t)$$
$$-2\gamma\frac{\mathrm{N}(\chi(i,t))}{D(i,t)}\varepsilon(i-1,t+n)\hat{\alpha}^{\mathrm{T}}(i-1,t)$$
$$\times\tilde{\xi}(i-1,t+n-1|t)$$
$$+\frac{2}{b}\frac{\mathrm{N}(\chi(i,t))}{D(i,t)}\mathrm{N}(\chi(i,t))\varepsilon(i-1,t+n)\varphi(i,t)$$
$$\times\hat{\alpha}^{\mathrm{T}}(i-1,t)\tilde{\xi}(i-1,t+n-1|t)$$
$$\leq \gamma^2\frac{G(i,t)\varepsilon^2(i-1,t+n)}{D(i,t)}$$
$$-\frac{2}{b}\mathrm{N}(\chi(i,t))(\Delta z(i,t) + \varphi(i,t)\Delta\varphi(i,t)) + 2\gamma\Delta\varphi(i,t)$$
$$\leq \gamma^2\Delta z(i,t) + \frac{1}{|b|}\mathrm{N}(\chi(i,t))(\Delta\varphi(i,t))^2 + 2\gamma\Delta\varphi(i,t)$$
$$-\frac{2}{b}\mathrm{N}(\chi(i,t))(\Delta z(i,t) + \varphi(i,t)\Delta\varphi(i,t)$$
$$+\frac{(\Delta\varphi(i,t))^2}{2}). \qquad (23)$$

Since

$$\Delta\chi(i,t) = \Delta z(i,t) + \varphi(i,t)\Delta\varphi(i,t) + \frac{(\Delta\varphi(i,t))^2}{2},$$
$$\mathrm{N}(\chi(i,t))(\Delta\varphi(i,t))^2 \leq \Delta z(i,t), \qquad (24)$$

we have

$$\Delta V(i,t) \leq (\gamma^2 + \frac{1}{|b|})\Delta z(i,t) + 2\gamma\Delta\varphi(i,t) - \frac{2}{b}\mathrm{N}(\chi(i,t))\Delta\chi(i,t).$$
$$(25)$$

Taking the sum of (25) at time instant t along the iterative learning axis, we have

$$V(i,t) \le -\frac{2}{b}\sum_{k=0}^{i} \mathrm{N}(\chi(k,t))\Delta\chi(k,t)$$

$$+(\gamma^2 + \frac{1}{|b|})z(i,t) + 2\gamma\varphi(i,t) + V(-1,t)$$

$$\le -\frac{2}{b}\sum_{k=0}^{i} \mathrm{N}(\chi(k,t))\Delta\chi(k,t)$$

$$+(\gamma^2 + \frac{1}{|b|})\chi(i,t) + 2\gamma^2 \Big/ (\gamma^2 + \frac{1}{|b|}) + V(-1,t). \quad (26)$$

Since $V(-1,t)$ is a constant parameter, applying Lemma 3, we can conclude directly that $V(i,t)$ and $\chi(i,t)$ are bounded. Thus, $\mathrm{N}(\chi(i,t))$, $G(i,t)$, $\hat{\alpha}(i,t)$, $\hat{\beta}(i,t)$ are all bounded, and

$$\lim_{i\to\infty}\Delta z(i,t) = \lim_{i\to\infty}\frac{G(i,t)\varepsilon^2(i-1,t+n)}{D(i,t)} = 0 \quad (27)$$

In order to apply Lemma 4, we should guarantee the linear boundedness condition between $D(i,t)$ and $\varepsilon^2(i-1,t+n)$. From error dynamic (18), it is easy to see that

$$x_j(i,t) = O[y(i,t+j-1)] = O[e(i,t+j-1)],$$
$$u(i,t) = O[y(i,t+n)] = O[e(i,t+n)]. \quad (28)$$

Thus it is derived from the Lipschitz condition in Assumption 1 that

$$\xi(i-1,t+n-1) = O[e(i-1,t+n-1)]. \quad (29)$$

From Lemma 5, we have

$$\tilde{\xi}(i-1,t+n-1|t) = o[O[y(i-1,t+n-1)]] = o[O[e(i-1,t+n-1)]]. \quad (30)$$

It is easy to see that from the parameter definition (19)

$$\varepsilon(i,t) \sim e(i,t). \quad (31)$$

Thus it can be obtained that

$$D(i,t) = O[\varepsilon^2(i-1,t+n)]. \quad (32)$$

Applying Lemma 4, considering the boundedness of $G(i,t)$, we can conclude that $\lim_{i\to\infty} e(i,t) = 0$, $t = n,\cdots,T$. Considering Remark 1, the pointwise tracking is achieved on interval $[n,T]$ except for beginning n instants affected by random initial conditions. Moreover, all the signals in the system are kept bounded. ∎

5. ILLUSTRATIVE EXAMPLE

Consider the following high-order strict-feedback discrete-time nonlinear systems

$$x_1(i,t+1) = a_1 x_1(i,t)\cos^2(x_1(i,t)) + a_2 x_1(i,t)\sin(x_1(i,t))$$
$$+b_1 x_2(i,t),$$
$$x_2(i,t+1) = a_3 \frac{x_1(i,t)\sin(x_2(i,t))}{3 + x_1^2(i,t)} + a_4 \frac{x_2^3(i,t)}{2 + x_2^2(i,t)} + b_2 u(i,t),$$
$$y(i,t) = x_1(i,t),$$

where $a_1 = 0.2$, $a_2 = 0.1$, $a_3 = 0.1$, $a_4 = -0.5$, $b_1 = 1.5$, $b_2 = \mp 0.1$. The initial value is chosen from $(-1,0)\cup(0,1)$ when iteration varies.

The desired trajectory is given as $y_d = m(i)[1.5\sin(\frac{\pi H}{5}t) + 1.5\cos(\frac{\pi H}{20}t)]$, $H = 0.04$, where $m(i)$ is also varying in the interval $(-1,0)\cup(0,1)$ with iteration i. The iteration interval is $t \in [0,1,\cdots,100]$. The tunable parameter γ is chosen to be 4.
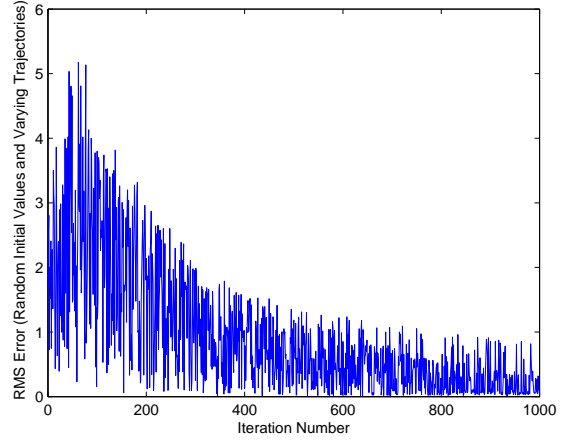

Fig. 1. Root mean square tracking error under random initial values and varying desired trajectories when $b_2 = -0.1$
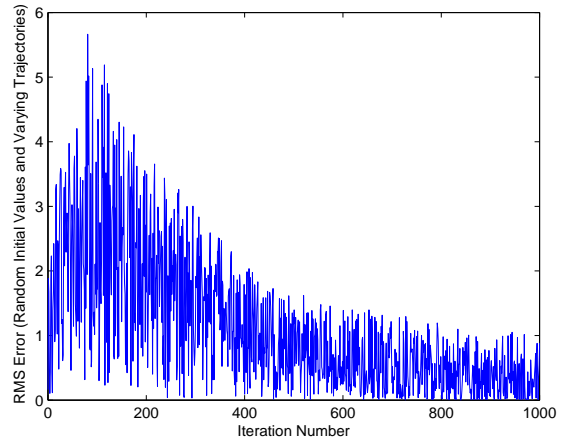

Fig. 2. Root mean square tracking error under random initial values and varying desired trajectories when $b_2 = 0.1$

To demonstrate the effectiveness of the proposed control scheme, we carry out the simulation with b_2 of the same absolute value but opposite signs. Simulation results with random initial values and varying desired trajectories are shown in Fig 1-6. Fig 1 and 2 show the RMS tracking error versus iteration number. It has shown the validity of the control algorithm. The variation of discrete Nussbaum gain along the iteration axis at time instant $t = 1$ is shown in Fig 3 and 4 as an example. We can see that it is first at the wrong direction, then switches to the right direction after several iterations of learning, which coincides with the sign of b_2. In Fig 5 and 6, discrete Nussbaum gain $\mathrm{N}(\chi(t))$ at the 1000th iteration is depicted. It is easy to see that $\mathrm{N}(\chi(t))$ are all be the same sign with b_2 for $t \in [0,\cdots,99]$.

6. CONCLUSIONS

In this paper, the problem of adaptive iterative learning control for a class of strict-feedback discrete-time nonlinear systems with random initial conditions and iteration-varying reference trajectories is tackled. The discrete Nussbaum gain method is exploited to deal with the lack of the prior knowledge of control directions. Under the proposed control scheme and parameter estimates updating law, the tracking error converges to zero pointwisely with all the signals bounded. Simulation results
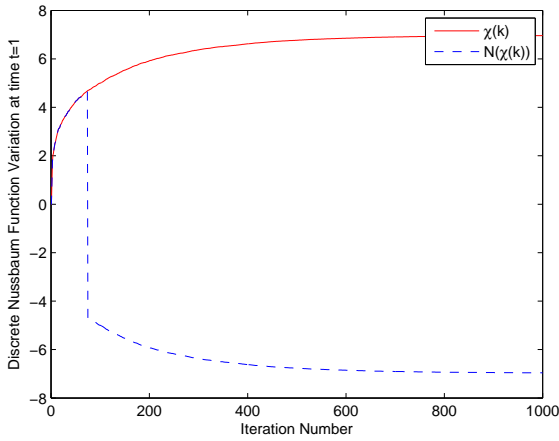
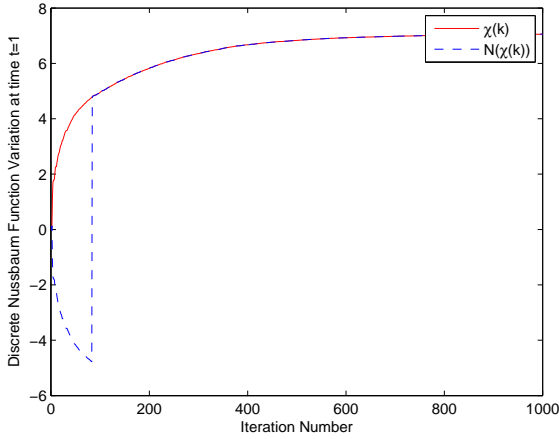Fig. 3. Discrete Nussbaum function along the iteration axis at t=1 when $b_2 = -0.1$

Fig. 4. Discrete Nussbaum function along the iteration axis at t=1 when $b_2 = 0.1$
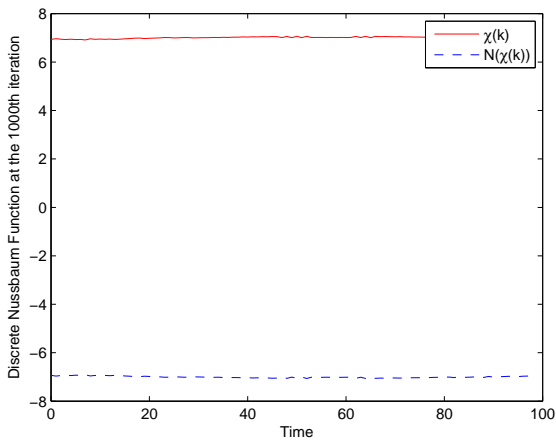

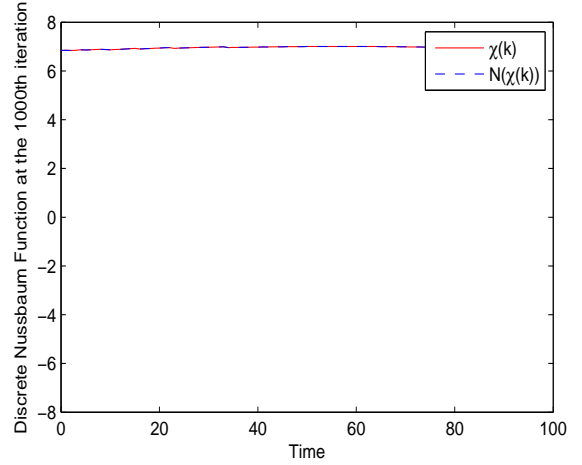Fig. 5. Discrete Nussbaum function of the 1000th iteration when $b_2 = -0.1$

Fig. 6. Discrete Nussbaum function of the 1000th iteration when $b_2 = 0.1$

have demonstrated the effectiveness of the presented control method.

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