Early Termination of Dantzig-Wolfe Algorithm for Economic MPC

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Abstract: In this paper we apply the Economic Model Predictive Control (MPC) for balancing the power supply and demand in the future power systems in the most economic way. The control problem is formulated as a linear program, having a block-angular structure solved by the implementation of the Dantzig-Wolfe decomposition. For real-time applications we introduce an early termination technique. Simulations demonstrate that the algorithm developed operates efficiently a power system, reducing significantly computational time.

Keywords: Economic Model Predictive Control, Linear Programming, Distributed Optimization, Power Systems

1. INTRODUCTION

During the past decades climate has dramatically changed. Scientists define the recent global warming as unprecedented and emphasize the need of accellerated and urgent actions (Vidal, 2013). CO_2 emissions and other pollutants are collecting in the atmosphere like a thickening blanket, trapping the sun's heat and causing the planet to warm up. The combustion of fossil fuels to generate electricity is one of the largest source of CO_2 emissions (United States Enviromental Protection Agency, 2013). Hence, a transistion from fossil to non-fossil fuels plays a key role in our future, leading to a new electricity system. Future electric grids will consist of independent energy sources and customers; these are characteristics of Smart Grids (European Technology Platform SmartGrids, 2012; The Danish Energy Agreement of March 2012 , Ministry of Climate, Energy and Building, 2012). Renewable energy sources (RES) take part in the Smart Grids with their intermittent energy production. This innovative scenario requires control actions so as to ensure the total energy production satisfies customers' demands.

We propose an optimization-based controller to balance power production and consumption in an economically efficient way. As a case study we consider a large scale system in which multiple power generators that are dynamically decoupled, operate in a coordinated way to serve as a single power portfolio. We address two issues related to the power management in a large scale scenario. The first issue concerns minimizing the cost of producing enough power to meet the market demand. The second issue concerns providing supply security. Our control strategy is an Economic MPC applied to a power portfolio in a large scale scenario. The optimization problem of the proposed controller shows a block-angular constraints matrix; because of this, we solve the control problem by using Dantzig-Wolfe decomposition. However, real-time applications require fast computation of the optimal control sequence: because of this, an early termination strategy is applied on the Dantzig-Wolfe decomposition algorithm. Such early termination provides a suboptimal solution of MPC and reduces significantly computational times.

Recent applications for energy systems have included the Economic MPC: refrigeration systems (Hovgaard et al., 2010, 2011, 2012a,b), heat pumps for residential buildings (Halvgaard et al., 2012c), solar-heated water tanks (Halvgaard et al., 2012a), and batteries in electrical vehicles (Halvgaard et al., 2012b). Due to computational complexity and the communication bandwidth limitation, distributed control structures have been developed for large-scale systems (Scattolini, 2009). The interest in distributed MPC has led to the use of decomposition techniques applied to large-scale linear programs, (Lasdon, 1970; Chvatal, 1983; Nazareth, 1987; Dantzig and Thapa, 2003; Conejo et al., 2006). The Dantzig-Wolfe decomposition algorithm for large linear programs was first introduced in 1960 (Dantzig and Wolfe, 1960, 1961). However, recently, the Dantzig-Wolfe algorithm has been used in a number application connected to the MPC: in an oil field by (Gunnerud and Foss, 2010; Gunnerud et al., 2010), control of building temperature (Morsan et al., 2011) and power balancing (Edlund et al., 2011). Suboptimal MPC controllers are stabilizing and guarantee feasibility and stability of the controller (Pannocchia et al., 2010). However, often real-time suboptimal MPC is a combination of offline and online optimization (Scokaert et al., 1999; Zeilinger et al., 2008). Other strategies involve online active set and bounds on the CPU time (Ferreau et al., 2008) and early termination approach for interior point methods (Wang and Boyd, 2010).

The outline of the paper is as follows: Section 2 introduces power systems. Section 3 formulates a linear Economic MPC for linear power systems. Section 4 describes the Dantzig-Wolfe decomposition algorithm. The early termination strategy is explained in Section 5. Section 6.1 proposes a model for the power generators included in the portfolio; Section 6.2 reports simulation results and, finally, the conclusion and suggestions for future work are presented in Section 7.

2. POWER SYSTEMS

Power system consists of a number of independent power units, such as power producers and consumers. Figure 1 depicts a generic power system, where power units are connected only with *operation* center. The total power supply includes the production from each of these independent power producers. Such power systems are also called Distributed Energy Sources (DES). Moreover, power units are independent and dynamically decoupled systems; such decoupled models are ubiquitous in power systems. Accordingly, the energy units considered in this paper can be described as a linear discrete time state space model

$$x_{k+1} = Ax_k + Bu_k, \tag{1a}$$
$$y_k = Cx_k, \tag{1b}$$
$$z_k = C_z x_k. \tag{1c}$$

x_k denotes the states, u_k the manipulated variables (MVs), y_k denotes the measurement used for feedback, and z_k is output variables.

The manipulated variable, u_k, is subject to bounds and rate-of-movements constraints

$$u_{min} \leq u_k \leq u_{max} \tag{2a}$$
$$\Delta u_{min} \leq \Delta u_k \leq \Delta u_{max} \tag{2b}$$

These are hard constraints and not mean-value constraints.

The system output z_k denotes the power produced by the generator and it must satisfy the customers' demand, r. Often the electricity demand is forecast in advance and defined by an interval as $[r_{\min,k},\ r_{\max,k}]$; we assume to have such demand interval from external forecasts. However, due to the manifold power units involved, it might be impossible to have the total power production z_k within the demand interval; because of this, the constraints on the power produced include slack variables s_k. The slack variables, s_k, may represent selling or buying power from the short-term market, violation of temperature limits, or violation of state-of-charge limits. Every time s_k is non-zero, a penalty cost, e.g. the cost of buying or selling power on the short-term market must be paid.

$$r_{\min,k} - s_k \leq z_k \leq r_{\max,k} + s_k \tag{3a}$$
$$s_k \geq 0 \tag{3b}$$

The cost of producing power over a period of time, is ϕ_k. This economic cost, ϕ_k, consists of the cost of operating a power generator, c_k, and the penalties, ρ_k, related to the use of slack variables, s_k

$$\phi_k = \sum_{j=0}^{N-1} c_k' u_k + \sum_{j=0}^{N-1} \rho_k' s_k. \tag{4}$$

3. ECONOMIC MPC FOR OPERATIONS

Figure 1 illustrates a power system where the *operations* center has the task to coordinate and control power untis.

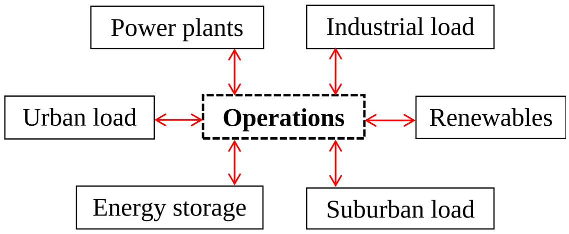

Fig. 1. A generic future power system. Power producers and consumers are independent units, and *Operations* coordinates and controls these power units to guarantee power supply in response to the customers' demand.

Operating such power system means making real-time decisions as planning the power production in response to the customers' demand. This section introduces Economic Model Predictive Control (MPC) to operate a power system as the one in Figure 1 balancing power supply nad demand in the most economic way.

Consider a power system, as described in Section 2, which consists of P power producers. These power generators collectively produce the total portfolio power production $\hat{\tilde{z}}_{k+j+1|k}$ subject to the following connecting constraints

$$\hat{z}_{k+j+1|k} = \sum_{i=1}^{P} \tilde{C}_i \hat{x}_{i,k+j+1|k}, \tag{5a}$$
$$\hat{z}_{k+j+1|k} + s_{k+j+1|k} \geq \hat{r}_{\min,k+j+1|k}, \tag{5b}$$
$$\hat{z}_{k+j+1|k} - s_{k+j+1|k} \leq \hat{r}_{\max,k+j+1|k}, \tag{5c}$$
$$s_{k+j+1|k} \geq 0. \tag{5d}$$

Constraints (5b)-(5d) are equivalent to the constraints (3) but referring to the total power produced by the power system.

The Economic MPC is formulated as a linear program because of the linear dynamics of the power units (1), linear cost functions (4), and linear constraints (2)-(3) and (5). In addition, a Kalman filter predicts $\hat{x}_{k+1+j|k}$. Accordingly, the Linear Economic MPC to operate a power system of P power units, is formulated as

$$\min \quad \phi_k = \sum_{i=1}^{P} \phi_{i,k} + \sum_{j=0}^{N-1} \hat{\rho}'_{k+j+1|k} s_{k+j+1|k} \tag{6}$$

subject to the local constraints $\forall i \in \mathcal{P}$ and $\forall j \in \mathcal{N}$

$$\hat{x}_{i,k+j+1|k} = A_i \hat{x}_{i,k+j|k} + B_i u_{i,k+j|k} \tag{7a}$$
$$\hat{z}_{i,k+j+1|k} = C_{z,i} \hat{x}_{i,k+j+1|k} \tag{7b}$$
$$u_{\min,i} \leq u_{i,k+j|k} \leq u_{\max,i} \tag{7c}$$
$$\Delta u_{\min,i} \leq \Delta u_{i,k+j|k} \leq \Delta u_{\max,i} \tag{7d}$$
$$\hat{z}_{i,k+j+1|k} + s_{i,k+j+1|k} \geq \hat{r}_{\min,i,k+j+1|k} \tag{7e}$$
$$\hat{z}_{i,k+j+1|k} - s_{i,k+j+1|k} \leq \hat{r}_{\max,i,k+j+1|k} \tag{7f}$$
$$s_{i,k+j+1|k} \geq 0 \tag{7g}$$

and subject to the connecting constraints $\forall j \in \mathcal{N}$ in (5).

The optimization control problem (5)-(7) has a block-angular structure that is suitable for the implementation of Dantzig-Wolfe decomposition to solve efficiently the control linear program.

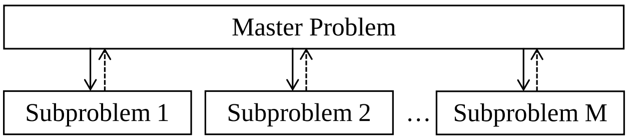

Fig. 2. Dantzig-Wolfe structure. Each subproblem comunicates exclusively with the master problem that must coordinate such units.

4. DANTZIG-WOLFE DECOMPOSITION TECHNIQUE

The Dantzig-Wolfe decomposition algorithm is a decomposition technique to solve efficiently linear programs having a block-angular structure, as (5)-(7), (Dantzig and Wolfe, 1960, 1961). The Economic MPC expressed as a linear program in (5)-(7), can be formulated as

$$\min_{q_{i,k}{}_{i=1}^{M}} \quad \varphi = \sum_{i=1}^{M} e_i' q_{i,k} \tag{8a}$$

$$s.t. \begin{bmatrix} F_1 & F_2 & \dots & F_M \\ G_1 & & & \\ & G_2 & & \\ & & \ddots & \\ & & & G_M \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_M \end{bmatrix} \geq \begin{bmatrix} g \\ h_1 \\ h_2 \\ \vdots \\ h_M \end{bmatrix} \tag{8b}$$

where $i \in \mathcal{M} = \{1, ..., P, P+1\}$ as the slack variables $s_{k+j+1|k}$ in (5) and (6) are considered as an extra unit. Therefore,

$$e_j' = \begin{bmatrix} p_{j,0}' & \dots & p_{j,N}' & \hat{\rho}_N' \end{bmatrix}, q_j' = \begin{bmatrix} \bar{u}_{j,0}' & \dots & \bar{u}_{j,N}' & s_N' \end{bmatrix}$$

where the variables p_k' and $\bar{u}_k'$ are from the objective function (4)

$$\phi_k = \sum_{j=0}^{N-1} c_k' u_k + \sum_{j=0}^{N-1} \rho_k' s_k = \sum_{j=0}^{N-1} p_k' \bar{u}_k$$

The M diagonal blocks in the linear program (8) denotes M subproblems having their own set of constraints. Moreover, a master problem coordinates such subproblems as Figure 2 shows. From here on we assume that the feasible region of each subproblems is closed and bounded.

In a view of describing the Dantzig-Wolfe decomposition technique, it is necessary to introduce the *convex combination* theorem (Dantzig and Thapa, 2003).

Theorem 1. (Convex Combination). Consider $\mathcal{Q} = \{q \mid Gq \geq h\}$ be nonempty, bounded and closed set, i.e. a polytope. v^j denotes the extreme point of $\mathcal{Q}$ with $j \in \{1, 2, ..., V\}$.

Then any point q in the polytope $\mathcal{Q}$ can be written as a convex combination of its extreme points

$$q = \sum_{j=1}^{V} \lambda_j v^j \tag{9a}$$

$$s.t \quad \lambda_j \geq 0, \qquad j = 1, 2, ..., V \tag{9b}$$

$$\sum_{j=1}^{V} \lambda_j = 1 \tag{9c}$$

Proof. See (Dantzig and Thapa, 2003).

Substituting (9) into (8) yields to the following linear program

$$\min_{\lambda} \quad \varphi = \sum_{i=1}^{M} \sum_{j=1}^{V_i} f_{ij} \lambda_{ij} \tag{10a}$$

$$s.t \quad \sum_{i=1}^{M} \sum_{j=1}^{V_i} p_{ij} \lambda_{ij} \geq g \tag{10b}$$

$$\sum_{j=1}^{V_i} \lambda_{ij} = 1, \qquad i = 1, 2, ..., M \tag{10c}$$

$$\lambda_{ij} \geq 0, \qquad i = 1, 2, ..., M; j = 1, 2, ..., V_i \tag{10d}$$

where the coefficients are

$$f_{ij} = e_i' v_i^j, \quad p_{ij} = F_i v_i^j \tag{11}$$

The linear program (10), known as *Master Problem (MP)*, is equivalent to the block-angular linear problem (8). It is worth noting that (10) has fewer constraints than the original problem (8). However the MP considers the extreme points of each subproblem, thus the number of variables is larger than in the original problem (8). The Dantzig-Wolfe decomposition algorithm overcomes this problem by including a reduced number of extreme points, and adding new vertices when needed. As a result, the *Reduced Master Problem (RMP)* is defined as

$$\min_{\lambda} \quad \varphi = \sum_{i=1}^{M} \sum_{j=1}^{l} f_{ij} \lambda_{ij} \tag{12a}$$

$$s.t \quad \sum_{i=1}^{M} \sum_{j=1}^{l} p_{ij} \lambda_{ij} \geq g \tag{12b}$$

$$\sum_{j=1}^{l} \lambda_{ij} = 1, \qquad i = 1, 2, ..., M \tag{12c}$$

$$\lambda_{ij} \geq 0, \qquad i = 1, 2, ..., M; j = 1, 2, ..., l \tag{12d}$$

where $l \leq V_i$ for all $i \in \{1, 2, ..., M\}$. Solving the RMP provides the Lagrangian multipliers π associated with the inequality constraint (12b), the Lagrangian multipliers ρ, associated with equalities (12c), and the Lagrange multipliers κ for the positivity constraints (12d). These are playing a key role in the Dantzig-Wolfe algorithm as they represent the information sent from the Master Problem to each subproblem. The Lagrangian associated to the Master Problem (10) yields to the following necessary and sufficient optimality conditions, for $i = 1, 2, \dots, M$ and $j = 1, 2, \dots, V_i$

$$\nabla_{\lambda_{ij}} \mathcal{L} = f_{ij} - p_{ij}' \pi - \rho_i - \kappa_{ij} = 0 \tag{13a}$$

$$\sum_{i=1}^{M} \sum_{j=1}^{V_i} p_{ij} \lambda_{ij} - g \geq 0 \quad \perp \quad \pi \geq 0 \tag{13b}$$

$$\sum_{j=1}^{V_i} \lambda_{ij} - 1 = 0 \tag{13c}$$

$$\lambda_{ij} \geq 0 \quad \perp \quad \kappa_{ij} \geq 0 \tag{13d}$$

We notice that the conditions (13a) and (13d) imply

$$\kappa_{ij} = f_{ij} - p_{ij}' \pi - \rho_i = \left[e_i - F_i' \pi \right]' v_i^j - \rho_i \geq 0 \tag{14}$$

such that the KKT-conditions for (10) may be stated as for $i = 1, 2, \dots, M$ and $j = 1, 2, \dots, V_i$

$$\sum_{i=1}^{M}\sum_{j=1}^{V_i} p_{ij}\lambda_{ij} - g \geq 0 \quad \perp \quad \pi \geq 0 \tag{15a}$$

$$\sum_{j=1}^{V_i} \lambda_{ij} - 1 = 0 \tag{15b}$$

$$\lambda_{ij} \geq 0 \quad \perp \quad \kappa_{ij} = [e_i - F_i'\pi]'\, v_i^j - \rho_i \geq 0 \tag{15c}$$

An optimal solution must satisfy the KKT conditions (15). We denote λ_{ij}^{RMP} a solution of RMP, such that a feasible solution to Master Problem (10) is

$$\lambda_{ij} = \lambda_{ij}^{RMP} \qquad i = 1, 2, \ldots, M;\ j = 1, 2, \ldots, l \tag{16a}$$

$$\lambda_{ij} = 0 \qquad i = 1, 2, \ldots, M; j = l+1, l+2, \ldots, V_i \tag{16b}$$

This solution satisfies (15a) and (15b). To be optimal it also needs to satisfy (15c). These conditions are already satisfied for $i = 1, 2, \ldots, M$ and $j = 1, 2, \ldots, l$. We need to verify whether they are satisfied for all $i = 1, 2, \ldots, M$ and $j = l+1, l+2, \ldots, V_i$. This is complicated by the fact that we only know the extreme points, v_i^j for $i = 1, 2, \ldots, M$ and $j = 1, 2, \ldots, l$. An efficient initialization technique is introduced in (Standardi et al., 2012). Condition (15c) is satisfied for all $i = 1, 2, \ldots, M$ and $j = 1, 2, \ldots, V_i$ if $\min_i \quad \psi_i - \rho_i \geq 0$ where

$$\psi_i = \min_{v_i^j} [e_i - F_i'\pi]' v_i^j \tag{17}$$

v_i^j is an extreme point of the polytope $\mathcal{Q}_i = \{q_i \mid G_i q_i \geq h_i\}$. Therefore, using the Simplex Algorithm we compute the solution of (17) as a solution of the following linear program

$$\psi_i = \min_{q_i} \varphi = [e_i - F_i'\pi]' q_i \tag{18a}$$

$$s.t \qquad G_i q_i \geq h_i \tag{18b}$$

These linear programs are called *subproblems* and can be solved via either parallel or sequential computation; this possible parallel computation of the subproblems represents one of the advantages of the Dantzig-Wolfe decomposition algorithm. Let (ψ_i, q_i) be the optimal value-minimizer pair for the linear problem (18); then if

$$\psi_i - \rho_i \geq 0 \qquad \forall i \in \{1, 2, ..., M\} \tag{19}$$

is satisfied, then the solution computed from the RMP is optimal. Therefore the solution of the original control problem (8) is given by

$$q_i^* = \sum_{j=1}^{l} v_i^j \lambda_{ij} \qquad i \in \{1, 2, ..., M\} \tag{20}$$

Otherwise, if (19) is not satisfied, then the number of extreme points considered, l , is not enough and a new vertex v_i^{l+1} needs to be included.

The Dantzig-Wolfe algorithm needs an initial feasible solution. As this decomposition agorithm solves an Economic MPC, the previous solution is available and utilized as initial value at the next sampling time. To initialize the slack variables in the control problem (5)-(7) the output constraints (3) are utilized.

5. EARLY TERMINATION

The Dantzig-Wolfe decomposition solves the control problem reducing computational times (Standardi et al., 2012).

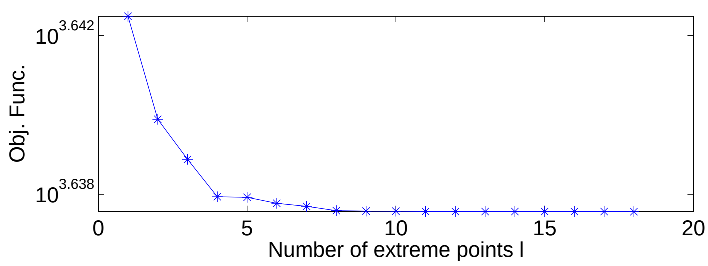

Fig. 3. Master Problem (10) objective function Vs. number of extreme points of the polytope (9).

However, many real-time applications include limits on the computational time that restrict the applicability of the MPC; real-time constraints and high-speed application may prevent the computation of the optimal controller as well. Early termination strategy and suboptimal MPC mantain feasibility and stability, as demonstrated in (Zeilinger et al., 2008; Scokaert et al., 1999; Pannocchia et al., 2010; Wang and Boyd, 2010). Section 4 illustrates that l extreme points of the feasible polytope are necessary to compute the optimal solution q_i^* (20); the Dantzig-Wolfe algorithm includes one vertex of the polytope at each iteration until the stopping criteria (19) is not satisfied. However, a smaller number of vertices can compute a solution that is not optimal but feasible though. The computation of such suboptimal solution reduces the number of iterations in the Dantzig-Wolfe algorithm, hence, reduces the computational time.

6. APPLICATION TO A POWER SYSTEM

In this section we apply the Economic MPC controller to a power system consisting of power plants, and the Dantzig-Wolfe decomposition computes the optimal control trajectory. In addition, we implement the early termination strategy in order to reduce computational times.

6.1 Boiler Load Generators

Section 2 introduces power units as independent and dynamically decoupled systems; these power units are coupled only through the objective to follow the customers' demand. This work includes boiler load units as power unit and the models are (Edlund et al., 2009)

$$Z_i(s) = G_i(s)U_i(s) \qquad G_i(s) = \frac{1}{(\tau_i s + 1)} \tag{21}$$

where $z_i(t)$ is the produced power at unit i, while $u_i(t)$ is the corresponding reference signal.

6.2 Simulations Results

We apply the algorithm developed in this paper on a power system consisting of five power plants as described in Section 6.1. Open-loop simulation provides Figure 3 that illustrates the reason of early termination effectiveness. Section 4 describes that the Dantzig-Wolfe algorithm computes the optimal solution considering a certain number of extreme points of the feasible polytope (9). With reference to the number of extreme points necessary to compute the optimal solution, Section 5 introduces the

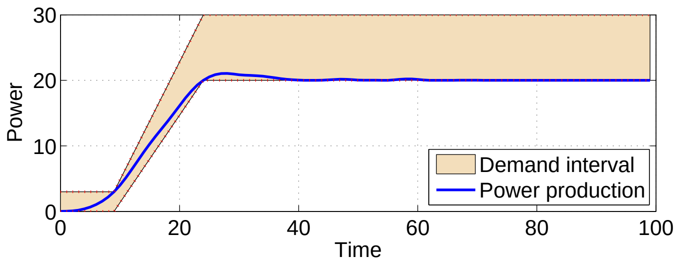

Fig. 4. Closed-loop simulations results. The total power production is within the customers' demand interval.

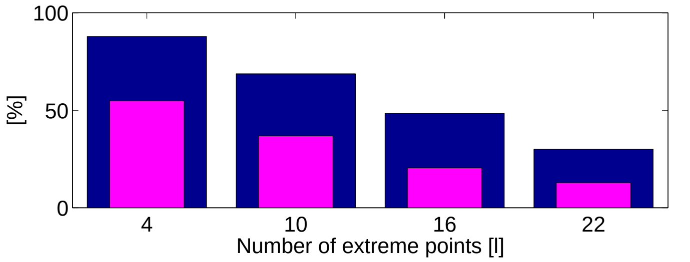

Fig. 5. CPU time vs. Extra Costs. The early termination strategy decreases CPU time, blu bars. However, such strategy leads to extra costs shown in the magenta bars. Consequently, a smaller number of extreme points of the feasible polytope yields to a decrease of the CPU time and extra costs to pay. For instance, if we set 16 as upper bound on the number of extreme points, then there is a decrease of 50% on the CPU time and 20% of extra costs to pay.

early termination strategy. Accordingly, Figure 3 shows that the master problem objective function φ (10) reaches its optimal value before the stopping criteria (19) of the Dantzig-Wolfe algorithm is satisfied.

The Economic MPC strategy controls a power system, and the Dantzig-Wolfe decomposition solves efficiently the control linear problem. The controller performances are in Figure 4, where the power system output is kept within the interval demand for the entire closed-loop simulation. Figure 5 reports the early termination effects. In closed-loop simulation the Dantzig-Wolfe algorithm computes the optimal control trajectory at each sampling time; in average, the decomposition algorithm takes 25 extreme points of the feasible polytope. The early termination utilizes fewer extreme points by setting bounds on these vertices. Such strategy reduces the computational time appreaciably even higher that 50%. Whereas, the early termiantion leads to extra costs upwards of 10%.

7. CONCLUSION

Future power systems need new control algorithms to balance power supply and demand efficiently. The Economic MPC can operate power systems efficiently. The work of this paper differs from the recent applications of Economic MPC to energy systems because we compute the optimal control trajectory implementing a decomposition technique, known as Dantzig-Wolfe. Moreover, the early termination approach provides valuable results reducing substantially computational times. The controller developed coordinates the power production of a power system consisting of several power generators, i.e. boiler load units. Future work should focus on the early termination in order to minimize the associated extra costs.

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