Convex-Concave Procedure for Design of PID Controllers ⋆

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Abstract: Controller design is a rich problem where both performance and robustness must be taken into account. Many of these features can be captured by formulating a constrained optimization problem. Unfortunately, this often leads to nonconvex optimization problems that, in general, can be hard to solve. In this work we present a method for designing PID controllers (Hast et al., 2013) that utilizes the convex-concave procedure (Boyd, 2013). The method admits general process descriptions in terms of frequency response data. The convex-concave procedure is an iterative scheme for solving nonconvex optimization problems where the objective function and the constraints are written as a difference between two convex functions i.e.,

$$f(x) - g(x). \tag{1}$$

A convex approximation of the function is obtained by replacing the concave part, $-g(x)$, with a linearization around the current solution point x_k i.e.,

$$\hat{f}(x) = f(x) - g(x_k) - \nabla g(x_k)^T (x - x_k) \tag{2}$$

The problem is solved using the convex approximations and the obtained solution x_{k+1} is used in the next iteration. The iterative procedure converges to a local minimum and although no guarantees of converging to the global optimum can be given, experience have shown that the method produces good solutions.

We show that classical robustness constraints such as maximum values on the sensitivity and complementary sensitivity functions easily can be formulated in this framework. This idea is also generalized to settings where explicit process uncertainties are known to lie inside a circle with frequency dependent radius. To avoid oscillatory responses we also introduce curvature constraints on the Nyquist plot in a fashion suitable to the convex-concave procedure.

A common criteria for control performance is the integrated error, IE. It has been shown (Åström and Hägglund, 2006) that the integrated error due to a unit step load disturbance applied at the process input is inversely proportional to the integral gain i.e.,

$$IE = \int_0^\infty e(t)dt = \frac{1}{k_i}. \tag{3}$$

In this work we show that by using the convex-concave procedure to solve the optimization problem of maximizing integral gain subject to constraints on robustness and the Nyquist plots curvature gives good PID controllers.

Keywords: Convex optimization, PID control.

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