Dual Decomposition for Large-Scale Power Balancing

Rasmus Halvgaard * **John B. Jørgensen** *
Lieven Vandenberghe **

* *Technical University of Denmark, Kgs. Lyngby, Denmark (e-mail: {rhal,jbjo}@dtu.dk).*
** *UCLA, Los Angeles, CA 90095 USA (e-mail: lieven.vandenberghe@ucla.edu)*

Abstract: Dual decomposition is applied to power balancing of flexible thermal storage units. The centralized large-scale problem is decomposed into smaller subproblems and solved locally by each unit in the Smart Grid. Convergence is achieved by coordinating the units consumption through a negotiation procedure with the dual variables.

Keywords: Decomposition methods, Decentralized Control, Model Predictive Control, Smart Grid, Smart power applications

1. INTRODUCTION

A large number of flexible thermal storage units, e.g. electrical heating in buildings or cooling in refrigeration systems, will soon part of the Danish power system. These units could potentially provide a large flexible consumption by aggregating or pooling them together. This will enable them to be coordinated and help follow the fluctuating energy production from renewables such as wind power. We formulate an optimization problem of tracking a power reference. To solve this large-scale control problem in real-time, we decompose the original problem into smaller subproblems to be solved locally by each unit. Each unit has its own model and variables and can make decisions based on its own local control strategy. The need for system level flexibility is communicated to the units from an aggregator that broadcasts dual variables to the units and coordinates the negotiation until global convergence is reached. This negotiation procedure is required in every time step and requires fast evaluation of the subproblems that can be cast as linear quadratic optimal control problems. The subgradient method is used to minimize the system level power imbalance. The cost function of this imbalance can be non-differentiable, which is the case in power balancing, due to the nonlinear penalties on imbalances. A simple example with models of thermal storage systems is used to show how an aggregator can apply dual decomposition for power balancing in a smart energy system. Power capacity constraints in the distribution system can also be accounted for by the aggregator.

2. PROBLEM FORMULATION

The centralized large-scale problem to be solved at every time instant t is

$$\begin{aligned} \text{minimize} \quad & g(p(t), q(t)) \\ \text{subject to} \quad & p(t) = \sum_{k=1}^{n} u_k(t) \\ & x_k(t+1) = A_k x_k(t) + B_k u_k(t) \\ & y_k(t) = C_k x_k(t) \\ & y_k^{\min} \leq y_k(t) \leq y_k^{\max} \\ & u_k^{\min} \leq u_k(t) \leq u_k^{\max} . \end{aligned} \tag{1}$$

$q(t), t = 1, ..., N$ represents a desired power consumption profile over a period of length $N.p(t)$ is the actual power demand and is a sum of the power demands $p_k(t), k = 1, ..., n$ for each of the n units. A power capacity limitation can also be included by adding the inequality constraint $p(t) \leq p^{\max}(t)$. $y_k(t)$ is the output of a linear system with input $u_k(t)$. The variables are $p(t)$, $p_k(t)$, $x_k(t)$, and $u_k(t)$.

We define the set F_k as a bounded polyhedron containing the linear state space system and its constraints in (1). To lighten notation further the time argument will be omitted from here on. The unit constraints in F_k can be moved to the objective by expressing them as an indicator function

$$f_k(u_k) = \begin{cases} 0 & \text{if } u_k \in F_k \\ +\infty & \text{otherwise} \end{cases}$$

Finally, the optimization problem to be solved by the receding horizon controller at every time instant is

$$\begin{aligned} \text{minimize} \quad & g(p) + \sum_k f_k(u_k) \\ \text{subject to} \quad & p = \sum_k u_k \end{aligned} \tag{2}$$

3. DUAL DECOMPOSITION

We solve the problem (2) by solving its unconstrained dual problem with the subgradient descent method Vandenberghe (2011); Bertsekas (1999). The dual is obtained via the Lagrangian L

$$L = \frac{1}{2}||p - q||^2 + \sum f_k(u_k) + z^T \left(p - \sum_k u_k \right)$$

where z is the dual variable. The dual function is

$$\inf_p L = \frac{1}{2}||z||^2 + q^T z - ||z||^2 + \sum_k \inf_{u_k} \left(f_k(u_k) - z^T u_k \right)$$

$$= -\frac{1}{2}||z||^2 + q^T z - \sum_k \sup_{u_k} \left(z^T u_k - f_k(u_k) \right)$$

Finally, the dual problem is

$$\text{maximize} \quad -\frac{1}{2}||z||^2 + q^T z - \sum_k S_k(z) \tag{3}$$

with

$$S_k(z) = \sup_{u_k \in F_k} z^T u_k \,.$$

$S_k(z)$ is the support function of F_k. If F_k is a bounded polyhedron, we can evaluate S_k by solving an LP subproblem

$$u_k^+ = \operatorname*{argmin}_{u_k \in F_k} \left(z^T u_k \right) \tag{4}$$

and the optimal u_k gives us a subgradient of S_k at z. Solving (3) with the subgradient projection method gives us the updates

$$z^+ = z + t^+ \left(\sum_k u_k^+ - (z + q) \right) . \tag{5}$$

The step size t^+ must be decreasing at each iteration j, i.e. $t^+ = \frac{t}{j} \to 0$, for $j \to \infty$. If t doesn't decrease the subgradient method will not converge to the minimum.

With the chosen LP subproblems the dual gradient method converges but the primal solution is not easily recoverable from the dual. An extra strictly convex term can be added to the subproblems, e.g. a temperature reference on the output

$$\begin{aligned} &\text{minimize} && \frac{1}{2}||\sum_k u_k - q||^2 + \frac{1}{2}\sum_k ||y_k - r_k||^2 \\ &\text{subject to} && u_k \in F_k \,. \end{aligned}$$

The LP subproblems from (4) are now QPs on the form

$$u_k^+ = \operatorname*{argmin}_{u_k \in F_k} \left(\frac{1}{2}||y_k - r_k||^2 + z^T u_k \right) .$$

This problem formulation is equivalent to the ordinary optimal control problem with an added linear term, that can be solved efficiently by methods based on the Riccati recursion Jørgensen et al. (2004, 2012).

If an upper bound on the power p is added, the dual variable z can be clipped in (5) by keeping $0 \le p \le p^{\max}$.

4. NUMERICAL EXAMPLE

An example with two different first order thermal storage systems is was simulated. The models have unity gain, time constants 5 and 10, and both a temperature reference equal to $r_k = y_k^{\min}$. The results for step size $t = 0.3$ after 100 iterations is shown in Fig. 1. The power tracking profile is seen match most of the time, but it is not possible to control the consumption amplitude of each unit very accurate through the dual variables, since each unit has its own objective leaving the tracking at some compromise.

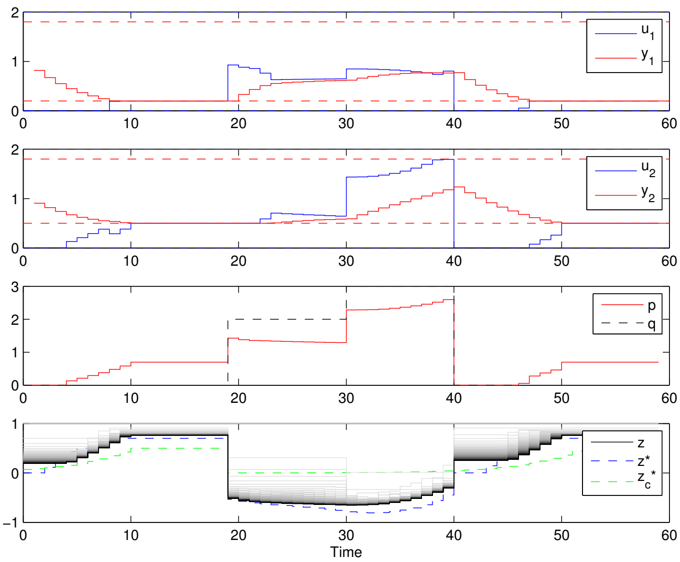

Fig. 1. *Simulation of power balancing with two first order systems. The two input/output pairs (blue/red) with constraints (dotted) are shown above the resulting power tracking profile. The lower plot shows the converged dual variable (black), its iterations (gray), and the optimal dual variable of the original problem (dotted blue). Also the optimal dual variable when using* (4) *as the subproblem is shown (dotted green).*

However, shifting the load in time is quite accurate, since the sharp variations in the dual variables, that can be interpreted as prices, causes the consumption to be placed in this cheap period.

5. CONCLUSION

Controlling the consumption of a large number of flexible thermal storage units in a Smart Grid was achieved by distributing the optimization problem to be solved and coordinating the total consumption through dual variables. The resulting power balancing performance is a compromise between system level balancing needs and the state and objectives of each unit.

REFERENCES

Bertsekas, D.P. (1999). *Nonlinear programming*. Athena Scientific.

Jørgensen, J.B., Frison, G., Gade-Nielsen, N.F., and Damman, B. (2012). Numerical Methods for Solution of the Extended Linear Quadratic Control Problem.

Jørgensen, J.B., Rawlings, J., and Jørgensen, S.B. (2004). Numerical Methods for Large Scale Moving Horizon Estimation and Control. In *7th International Symposium on Dynamics and Control of Process Systems (DYCOPS), Boston, USA*.

Vandenberghe, L. (2011). Lecture notes on Dual Decomposition. URL `www.ee.ucla.edu/~vandenbe/ee236c.html`.