Optimal Neumann boundary control for a freezing process with phase change

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Abstract: The system studied in this paper is a model of heat exchange phenomena, known as the heat equation. This is a parabolic partial differential equation (PDE). The application is the freezing of a fish block in a vertical plate freezer where liquid ammonia (NH_3) at minimal 235 K is used as the cooling medium. A pump forces the ammonia through the plate freezer, where it partly vaporizes due to the heat taken out from the fish block. The amount of heat added to the ammonia is removed in a compression/condensation/throttling - process with the consequence that the whole process is a cycle process

The freezing of fish in a vertical plate freezer is a thermodymical process where certain thermodynamical phenomena hold. Fish consists to a large amount of water and when water undergoes phase change while crossing the freezing point (liquid to solid state) one can observe that for a certain period of time the temperature will remain constant at the freezing point. Although not observable by measurement due to the constant temperature, there is still energy removed from the fishblock, the so called latent heat of fusion. Physically, latent heat of fusion is a hidden amount of energy that is needed to break the grid structure of the solid phase when melting ice. This phenomenon is not directly modeled in the standard heat equation and thus it is modeled by adapting the thermodynamical properties *specific heat capacity* c_p and *thermal conductivity* λ of the fish around the freezing point.

The approach of solving the optimal control problem (OCP) with Neumann boundary conditions (defining heat flow at the boundaries) is a two-dimensional spatial discretization of the fish block leading to a set of ordinary differential equations (ODEs). For every discretization-step in y-direction a new input function u_i is defined which acts exclusively in x-direction. At $y = 0$ (top of the fishblock) there is heat exchange with the surrounding air due to convection, whereas at $y = H$ (bottom of the fishblock) there is perfect isolation assumed. Both, convection with air and perfect isolation happen along the x-direction.

To solve the OCP the software package ACADO for MATLAB developed by Moritz Diehl and coworkers has been used. The bounds on the OCP are the discretized system equations and the maximal and minimal temperatures that the fish block can tend to: Due to basic thermodynamical laws it cannot become warmer than the surrounding temperature and not colder than the temperature of the ammonia cooling it down. Furtherermore, bounds on the input function defining maximal and a minimal heat flow as well as terminal state constraints are introduced. It has to be mentioned that the input functions are chosen as the quotient of *heat flow* $\dot{q}$ and *thermal conductivity of ammonia* $\lambda_{\mathrm{NH_3}}$, where the heat flow $\dot{q}$ can be exchanged by the product of the *massflow of ammonia* $\dot{m}_{\mathrm{NH_3}}$ and the *enthalpy difference of the ammonia* $\Delta h_{\mathrm{NH_3}}$. This is valid because the same amount of heat that leaves the fish gets absorbed by the ammonia.