Disturbance Handling in Economic Model Predictive Control

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1. INTRODUCTION

Economic model predictive control (EMPC) is a recent development of MPC where the usual quadratic objective is replaced by an economic objective reflecting the real cost of operation (Ellis et al., 2014). The target in EMPC is hence to directly minimize the cost of operation whereas the usual target in MPC is to keep the process close to a predetermined steady-state or trajectory (sometimes determined by a supervisory economic optimization layer). A benefit of this approach is that the controller itself is economically aware, which may lead to better economic performance during transient modes of operation (Ellis et al., 2014). In this work we consider the use of EMPC to utilize the extra degrees of freedom that exist in fat systems and also address the problem of handling unmeasured disturbances.

A fat system has more inputs than outputs and hence provides the controller with extra degrees of freedom. These degrees of freedom may be appropriately utilized in EMPC where economic optimization is used to determine which combination of inputs is currently the most beneficial. In standard MPC, the extra degrees of freedom may instead cause problems with uniqueness, and systems are therefore frequently *squared* where some potential control inputs are locked at so called design values, thereby reducing the available actuation power to deal with e.g., large disturbances.

A complicating factor in EMPC is that the EMPC objective is likely to be monotone in some of the control inputs (decision variables). For example, costs of energy and raw materials typically scale linearly with usage. Therefore EMPC may be expected to often operate with active constraints, which makes it sensitive to disturbances and noise since even small disturbances may move the plant into an infeasible region. To ensure feasibility, it will in general be necessary to back away from the constraints. In robust MPC, a set-based disturbance model is introduced and the constraints are required to be fulfilled for all possible disturbance realizations in the set. This corresponds to a worst case approach and may as such be very conservative (Bemporad and Morari, 1999; Mayne, 2014). An arguably more general but also more complex framework is provided by stochastic MPC in which a stochastic disturbance model is employed. In stochastic MPC the constraints are frequently interpreted as probabilistic and only required to be fulfilled with a specified probability (Mayne, 2014). However, both the robust and the stochastic approach are computationally expensive and may be difficult to scale for larger applications.

Here we consider an adaptive approach where instead of introducing a disturbance model we adapt the constraints in the EMPC optimization problem based on feedback from the actual closed loop cost. The desired back off is achieved by adding a bias to the true constraints when solving the EMPC optimization problem. To locate the economically best constraint bias, we close an outer loop where the method of simultaneous perturbation stochastic approximation (SPSA) is used to minimize the achieved closed loop stage cost with respect to the constraint bias.

We hence consider two sets of constraints: the true constraints relevant for the problem, and an adapted set of constraints used in the optimization problem solved at each iteration. Assume that the relevant constraints are soft such that they may be added to the problem as a large term in the stage cost whenever they are violated. We may then express the optimization problem to be solved at each sampling instant as:

$$\min_{\mathbf{u}} \sum_{k=0}^{N_p} \ell(x_k, u_k, c)$$

Subject to:

$$x^+ = f(x, u)$$
$$u_{min} \leq u_k \leq u_{max}, \quad k = 0, \dots, N_p$$

The stage cost $\ell : \mathbf{R}^{n_x} \times \mathbf{R}^{n_u} \times \mathbf{R}^{n_c} \to \mathbf{R}$ includes the soft constraints with terms like $C \max(0, x_{min} + c - x)$, where C is a large multiplier and c is the bias added to the constraint. A cost is incurred only if the biased constraint is violated, i.e., $x < x_{min} + c$, otherwise the cost is zero.

We define the closed loop stage cost as the real cost of operating the system, i.e., the stage cost evaluated for the measured states x_m with the original constraints:

$$J(c) = \ell(x_m(c), u(c), 0).$$

The cost function J will be stochastic in nature due to the disturbances present in the process, however, it will also depend on the constraint bias c which affects the closed loop trajectory of the states and inputs through the optimization problem above. We now want solve the problem

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$$\min_{\mathbf{c}} \sum_{i=0}^{\infty} J(c_i),$$

by adapting c. Here we perform the adaptation with SPSA (Spall, 1998).

2. EXAMPLE: REGULATION OF HOT WATER TANK

Consider the heater-tank system illustrated in Fig. 1. The aim is to keep the level h and temperature T of the tank within bounds (see Tab. 1) at minimum cost. The control inputs are the inflow F_{in}, and the effect of two different heaters, P_e and P_p^{sp}. Heater P_p is both more powerful and cheaper to operate, but contains slow first order dynamics whereas P_e has direct effect. The outflow F_{out} and the inlet

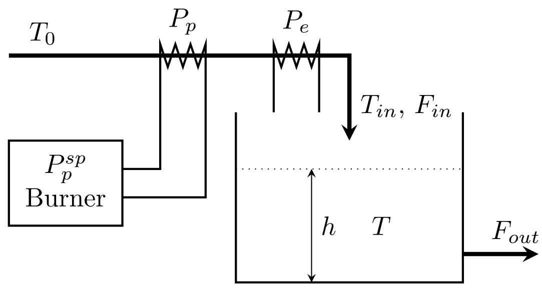

Figure 1. Tank with heated inflow.

temperature T_0 acts as stochastic disturbances. The exact nature of these disturbances is assumed unknown, i.e., no disturbance model is available.

Table 1. Bounds and costs for CVs and MVs.

	T	h	P_p	P_e	F_{in}
LB	19.5 °C	0.95 m	0 kW	0 kW	0.6 l/min
UB	20.5 °C	1.05 m	60 kW	20 kW	24 l/min
Cost	-	-	0.5kr/kW	1.5kr/kW	20 kr/m^3

The heater-tank system may be modeled as

$$A\frac{dh}{dt} = F_{in} - F_{out}$$
$$\frac{dT}{dt} = \frac{F_{in}(T_{in} - T)}{Ah}, \quad T_{in} = T_0 + (P_e + P_p)/(CF_{in})$$
$$\frac{dP_p}{dt} = \frac{-P_p + P_p^{sp}}{\tau_p}$$

where C is the specific heat of water and A is the cross-section of the tank. The EMPC prediction model is a linearized and discretized version of the above. The stage cost is defined as

$$l(x, u, c) = W_x x + W_u u + C_{x_{min}} \max(0, x_{min} + c - x) + C_{x_{max}} \max(0, x - x_{max})$$

where $x = [h\ T\ P_p]^T$, $u = [F_{in}\ P_p^{sp}\ P_e]^T$, $W_u = [20\ 0\ 1.5]$, $W_x = [0\ 0\ 0.5]$, and $C_{x_{min}} = C_{x_{min}} = [10^4\ 10^3\ 10^3]$.

Figure 2 relates the achieved closed loop cost to the bias added to the lower bound of the level and temperature. Each colored dot represents the average cost during a simulation with a constant bias added to the EMPC problem. The nominal problem, corresponding to the origin where no bias is added, is clearly associated with large closed loop cost which is due to frequent constraint violations. It is hence clearly necessary to back off from the nominal constraints when disturbances are present. How much to back off is difficult to see from the figure. However, a closer examination shows that operating in the upper right corner is more expensive than operating near the center of the plot. The solid red line in the figure shows the path of the constraints during a simulation where the constraints are adapted using SPSA. As can be seen in the figure, SPSA moves the constraints to a region of low cost.

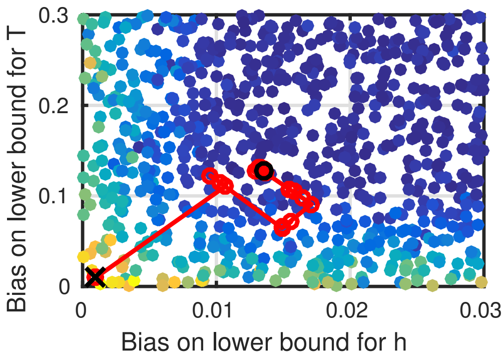

Figure 2. The dots correspond to a Monte Carlo simulation with different constant constraint biases over a fixed simulation horizon. The dot colors correspond to the normalized average closed loop cost. The red solid line shows the constraints during a simulation where SPSA is used to adapt the constraints online. The black cross shows the initial constraints and the black circle shows the constraints at the end of the simulation.

3. CONCLUSIONS

EMPC will frequently operate with constraints active and is thus sensitive to disturbances. We here show how constraint adaptation can be used to improve closed loop performance for such cases. The adaptation introduces negligible overhead compared to standard MPC and is hence computationally attractive as compared to robust and stochastic EMPC. Since the adaptation is based on online measurements, it does not require *a priori* knowledge of the disturbances. However, such information may be incorporated in terms of a more accurate initial guess, for example as calculated by off-line simulations. Future work includes investigating different methods of adaptation, analysis of stability and convergence, and also tuning considerations.

REFERENCES

Bemporad, A. and Morari, M. (1999). Robust model predictive control: A survey. In *Robustness in identification and control*, 207–226. Springer.

Ellis, M., Durand, H., and Christofides, P.D. (2014). A tutorial review of economic model predictive control methods. *Journal of Process Control*, 24(8), 1156–1178.

Mayne, D.Q. (2014). Model predictive control: Recent developments and future promise. *Automatica*, 50(12), 2967–2986.

Spall, J.C. (1998). Implementation of the simultaneous perturbation algorithm for stochastic optimization. *Aerospace and Electronic Systems, IEEE Transactions on*, 34(3), 817–823.