Robustness Margins Separating Process Dynamics Uncertainties

Per-Ola Larsson, Tore Hägglund

Dept. of Automatic Control
Lund University, Sweden

Background

Considered controllers for performance comparison

- PI(D)
- Smith Predictor, PI(D)$_\tau$
  - “Bad” reputation, sensitive to modelling errors.

Tuning – Control requirements

- Remove load disturbance errors quickly, IAE.
- Robust

Considerations

- Process properties change simultaneously, e.g., gain, time constant, and time delay.

Specifying robustness in easy ways

Classic measures

- Gain, phase, and dead time margins do not guarantee stability for simultaneous process changes.

Present solution: Robust control

- Lumps all uncertainties together
- Conservative, especially for time delays

- Design should be as simple as possible
- Common design for PI(D) – $\min$ IAE with

$$\|S(s)\|_\infty \leq M_S, \qquad \|T(s)\|_\infty \leq M_T$$

- Works very well for PI(D) design

Specifying robustness in easy ways

Classic measures

- Gain, phase, and dead time margins do not guarantee stability for simultaneous process changes.

Present solution: Robust control

- Lumps all uncertainties together
- Conservative, especially for time delays

- Design should be as simple as possible
- Common design for PI(D) – $\min$ IAE with

$$\|S(s)\|_\infty \leq M_S, \qquad \|T(s)\|_\infty \leq M_T$$

- Works very well for PI(D) design

PI_τ design

- Minimizing IAE with PI_τ controller and constraints

$$\mathrm{PI}_\tau(s) = K\frac{sT_i+1}{sT_i}\,\frac{sT_i}{sT_i+1-e^{-sL_r}}$$

$$\|S(s)\|_\infty \le M_S, \qquad \|T(s)\|_\infty \le M_T$$

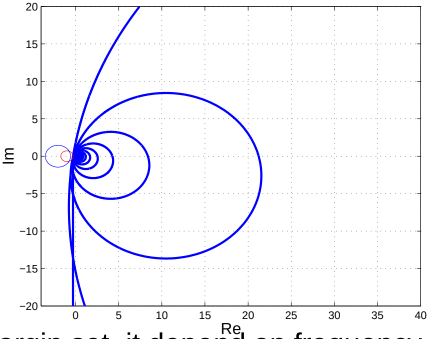
PI$_\tau$ design

- Minimizing IAE with PI$_\tau$ controller and constraints

$$C(s) = K \frac{sT_i + 1}{sT_i} \frac{sT_i}{sT_i + 1 - e^{-sL_r}}$$

$$\|S(s)\|_\infty \le M_S, \qquad \|T(s)\|_\infty \le M_T$$

- Result: Nyquist plot



- Why? No dead time margin set, it depend on frequency.

Robust control – dead times

Example:

- Process $P(s) = \frac{1}{s+1} e^{-s}$ + 20% uncertainty in dead time. Minimize IAE using PI control, appropriate weight on $T(s)$.
- Result: 15% higher IAE than if only dead time margin is used.

Conclusion: Must have frequency dependent weights, but ordinary robust control is (most often) too conservative.

Robust control – dead times

Example:

- Process $P(s) = \dfrac{1}{s+1} e^{-s}$ + 20% uncertainty in dead time. Minimize IAE using PI control, appropriate weight on $T(s)$.
- Result: 15% higher IAE than if only dead time margin is used.

Conclusion: Must have frequency dependent weights, but ordinary robust control is (most often) too conservative.

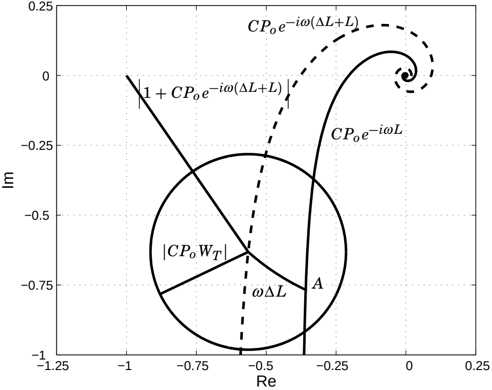
Focusing on dead times – I

- Idea: Separate dead time and other uncertainties
- Why?
  - Dead time uncertainty give rotation of Nyquist curve.
  - Badly approximated by disk.
- Modelling: Multiplicative uncertainty

$$P_\Delta = P_o(1 + W_T\Delta)e^{-s(L+\Delta L)}$$

P_o – nominal process, dead time free.
$W_T\Delta$ – gain, time constants,... ordinary weight, $\|\Delta\|_\infty = 1$.
ΔL – in dead time uncertainty interval $[\Delta L_{\min}, \Delta L_{\max}]$

Focusing on dead times – II



Robust stability cond.: $|CP_o W_T| < |1 + CP_o e^{-i\omega(L+\Delta L)}|, \ \forall \Delta L, \omega$

Focusing on dead times – III

- Condition can be rewritten as

$$\sup_{\omega} |T(i\omega, \Delta L) W_T(\omega)| < 1, \forall \Delta L$$

with extended complementary sensitivity function

$$T(s, \Delta L) = \frac{CPe^{-s\Delta L}}{1 + CPe^{-s\Delta L}}$$

- Graphical interpretation in Nyquist plot: Circles with centers and radii

$$\frac{1}{W_T^2(\omega) - 1}(\cos \omega \Delta L, \sin \omega \Delta L), \quad \frac{W_T(\omega)}{|1 - W_T^2(\omega)|}$$

Example — PI$_\tau$ control of FOTD

- Control of

$$P(s) = \frac{1}{s+1}\, e^{-s}$$

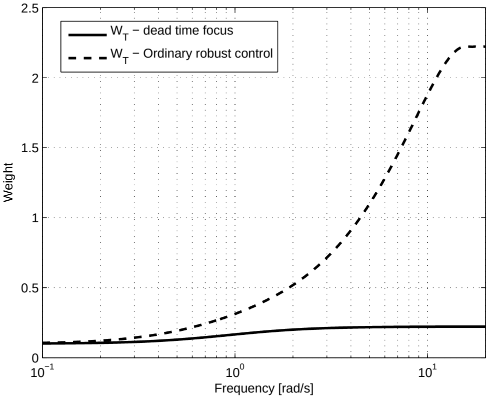
with the PI$_\tau$-controller

$$\mathrm{PI}_\tau(s) = 2.6\, \frac{0.75s+1}{0.75s} \frac{0.75s}{0.75s+1-e^{-1.25s}}$$

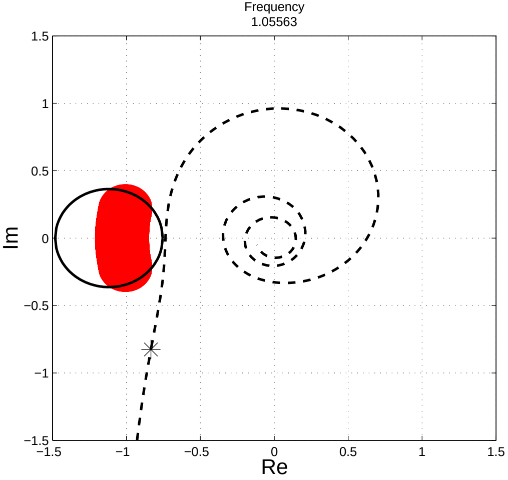
- 10% uncertainty in gain and time constant
  20% (symmetric) uncertainty in dead time.
- Robustly stable?

Example — Weights W_T

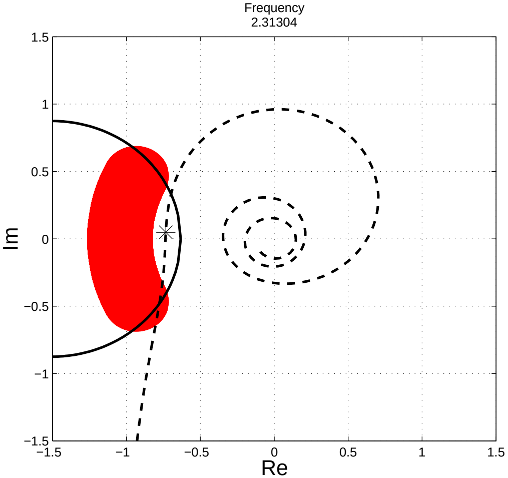
Weights on (extended) complementary sensitivity function



Example — Graphical interpretation

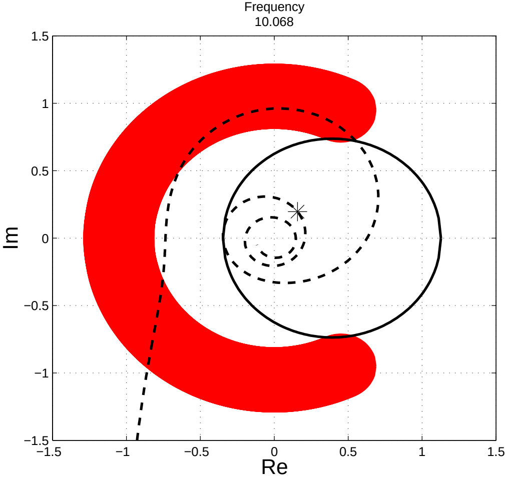


Example — Graphical interpretation

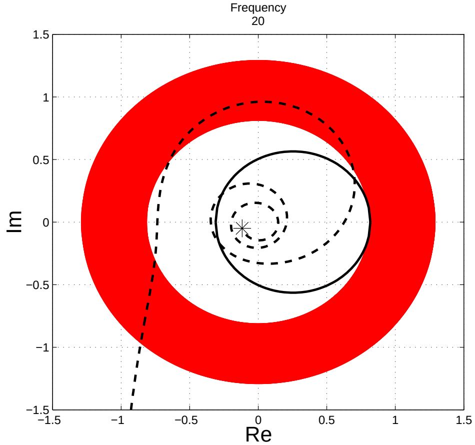


- No guarantees from ordinary robust control

Example — Graphical interpretation



Example — Graphical interpretation



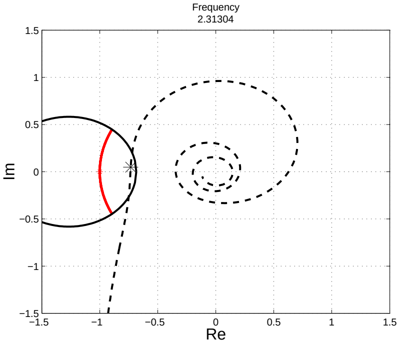
- Conclusion: Robustly stable by separating uncertainties.
- Actually, IAE is minimized with active constraints.

Relations to other margins

- If no dead time uncertainty, we have ordinary robust control

$$|T(i\omega, \Delta L) W_T(\omega)| = |T(i\omega) W_T(\omega)| < 1$$

- If only dead time uncertainty
  - radii are 0
  - recover ordinary delay margin



Focusing on dead times – VI

- Inverse multip. uncertainty $P_\Delta = P_o(1 + W_S\Delta)^{-1}e^{-s(L+\Delta L)}$ gives the condition

$$\sup_\omega |S(i\omega, \Delta L)W_S(\omega)| < 1$$

- Graphical interpretation in Nyquist plot: Circles with centers and radii

$$-(\cos\omega\Delta L, \sin\omega\Delta L), \quad W_S(\omega)$$

- Robust performance, i.e., $\sup_\omega |S_\Delta(i\omega)W_p(\omega)| < 1$, gives

$$\sup_\omega \left(|S(i\omega, \Delta L)W_p(\omega)| + |T(i\omega, \Delta L)W_T(\omega)|\right) < 1$$

  or equivalently

$$\sup_\omega |S(i\omega, \Delta L)\tilde{W}_p(\omega)| < 1$$

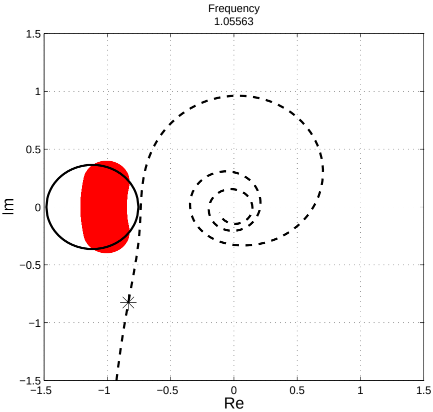
$$\tilde{W}_p(\omega) = W_p(\omega) + |CP_oW_T(\omega)|$$

Computational effort

- Algorithms developed to compute margins, e.g.,
  - decide if robustly stable (shown in example)
  - given weight $W_X(\omega)$, compute $[\Delta L_{\min},\ \Delta L_{\max}]$
  - given uncertainty interval $[\Delta L_{\min},\ \Delta L_{\max}]$, compute $W_X(\omega)$
- Based on graphical interpretation
- Fast

Always better?

- Depends on process and controller
- Phase of $e^{-s\Delta L}$ not taken into account
- Solution: Combine allowed areas. Better or equal performance.



Summary

- Explores dead time characteristics
- $T(s) \rightarrow T(s, \Delta L)$, $S(s) \rightarrow S(s, \Delta L)$ + robust control
- In between robust control and classic measures
- Gives good insight on inherent problems of time delays
- Algorithms available
- Combine allowed areas for better or equal performance