Design of multivariable LQ-optimal PID controllers based on convex optimization

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NTNU

Outline

- Paradigms for implementation of the optimal solution
- What variables should we control?
- Nullspace method
- Full information: $u = -Kx$ is optimal!
- Extensions to output feedback
- Closed-loop optimization
- Examples:
  - Underdamped second-order plant
  - Distillation column

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Paradigm 1

On-line optimizing control where measurements are primarily used to update the model. With arrival of new measurements, the optimization problem is resolved for the inputs.

Paradigm 2

Pre-computed solutions based on off-line optimization. Typically, the measurements are used to update the inputs using feedback control schemes.

Implementation of optimal operation

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On-line optimizing control where measurements are primarily used to update the model. With arrival of new measurements, the optimization problem is resolved for the inputs.

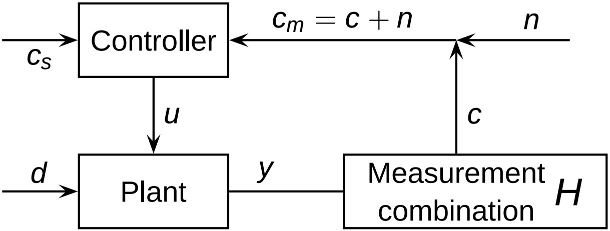
Example: Classical (implicit) MPC.

Paradigm 2

Pre-computed solutions based on off-line optimization. Typically, the measurements are used to update the inputs using feedback control schemes.

Examples: Explicit MPC and MIMO-PID (with “acceptable” loss)

What variables should we control?



Self-optimizing control

Choice of H such that acceptable operation is achieved with constant setpoints (c_s constant).

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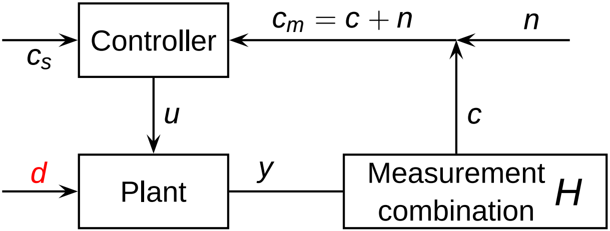

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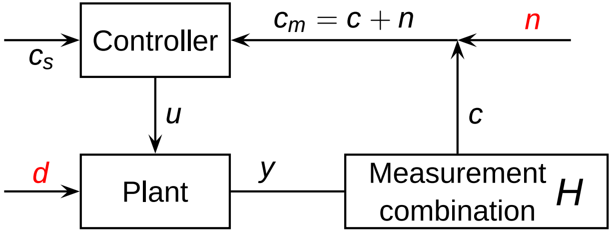

Self-optimizing control

Choice of H such that acceptable operation is achieved with constant setpoints (c_s constant).

- Optimal c_s is invariant with respect to disturbances d
- Insensitive to measurement errors n

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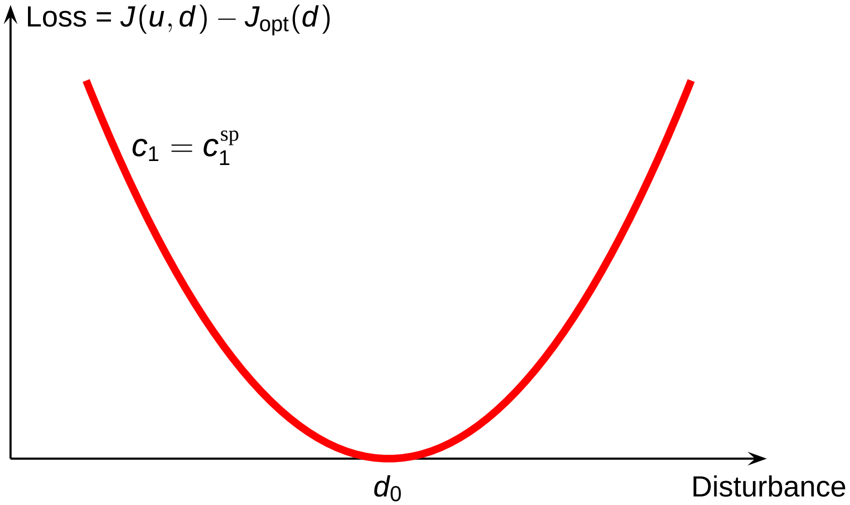

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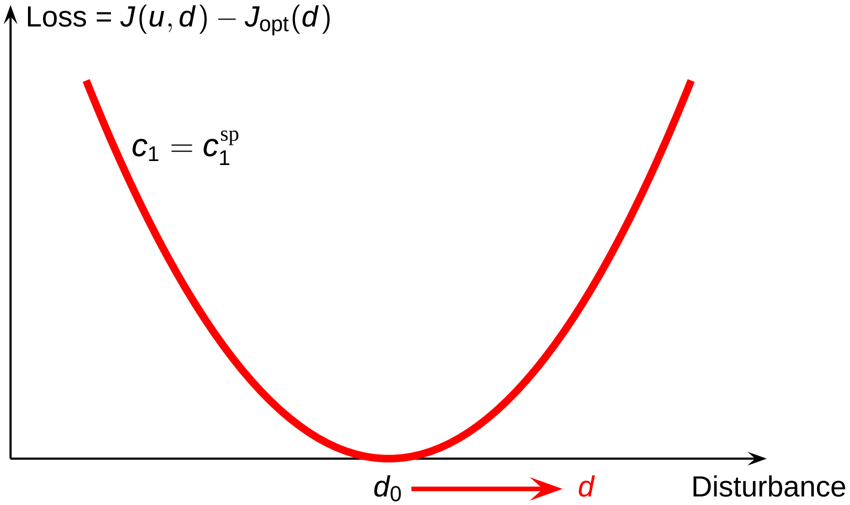

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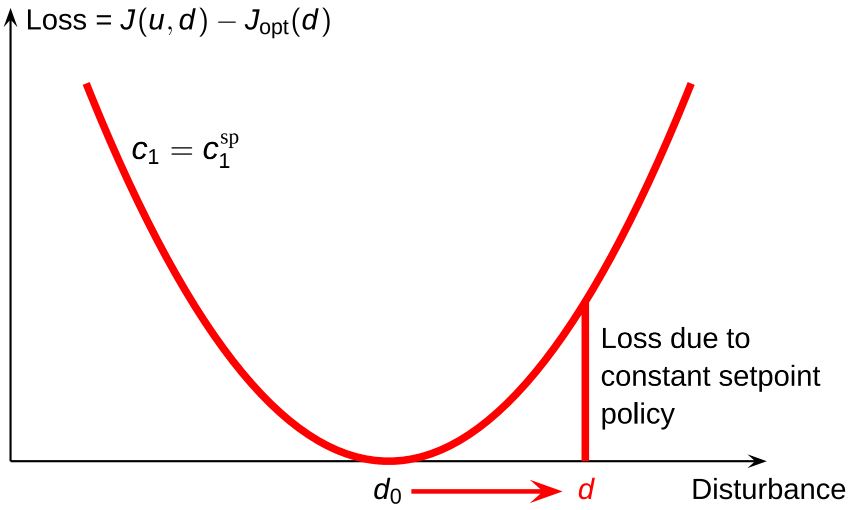

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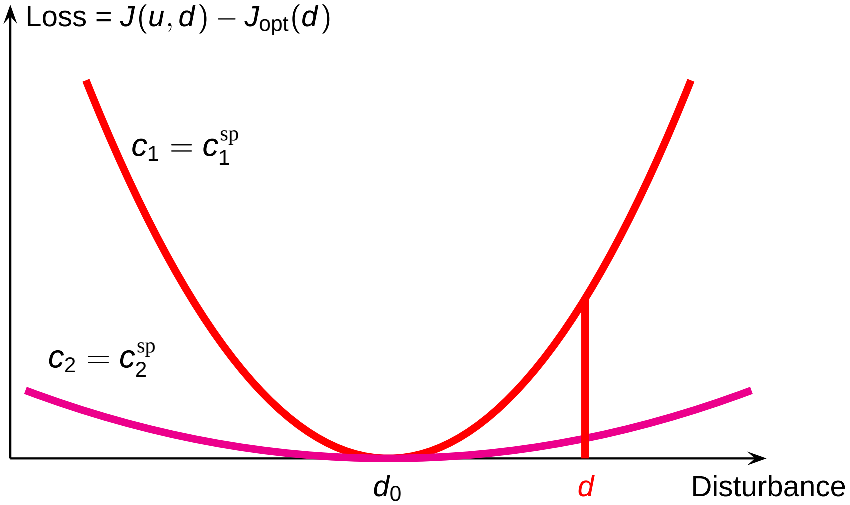

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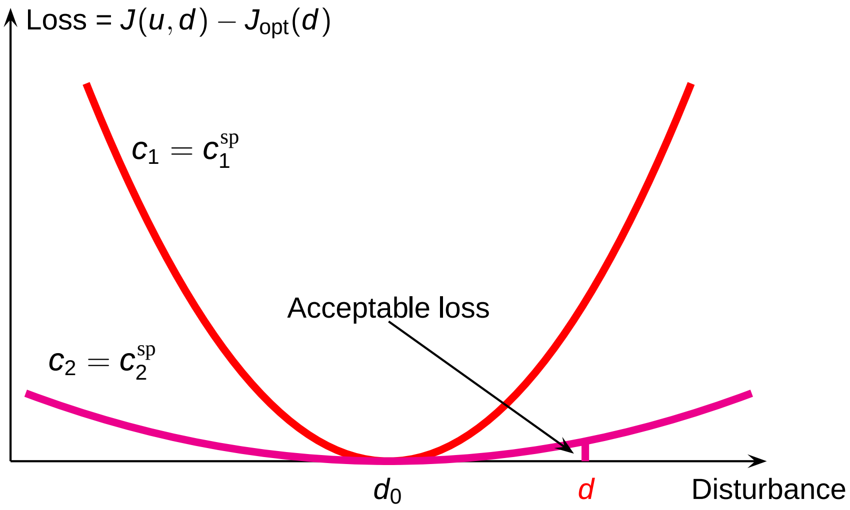

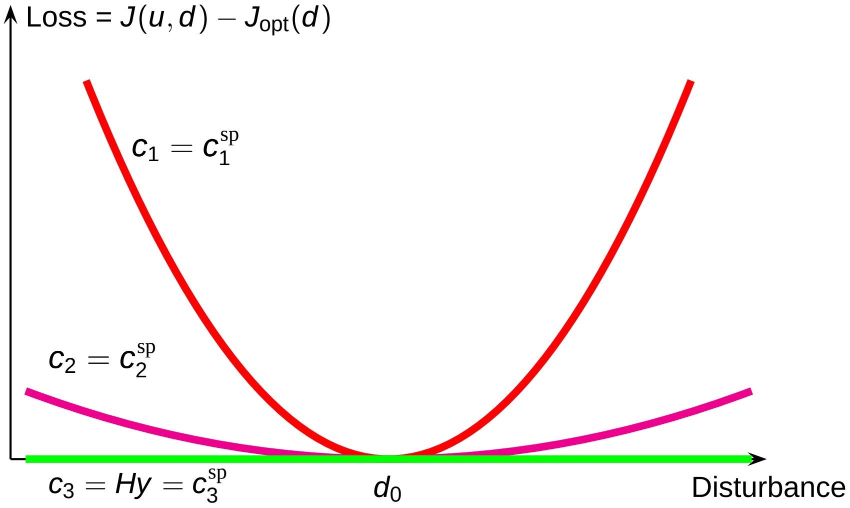

Loss = $J(u,d) - J_{opt}(d)$
$c_1 = c_1^{sp}$
$c_2 = c_2^{sp}$
$c_3 = Hy = c_3^{sp}$
d_0
Disturbance

Nullspace method for QP problems

Theorem (Nullspace method for QP)

- *Consider the quadratic problem*

$$\min_u J = \begin{bmatrix} u & d \end{bmatrix} \begin{bmatrix} J_{uu} & J_{ud} \\ J_{ud}^{\mathrm{T}} & J_{dd} \end{bmatrix} \begin{bmatrix} u \\ d \end{bmatrix} \qquad (1)$$

  If there exist $n_y \geq n_u + n_d$ independent measurements, then the optimal solution to (1) *has the property that there exist variable combinations $c = Hy$ that are invariant to the disturbances d.*[a]

- *H may be found from $HF = 0$, where $F = \frac{\partial y^{opt}}{\partial d^{\mathrm{T}}}$*

[a] Alstad and Skogestad *Ind. Eng. Chem. Res.* 2007

For a given $x(t)$, one solves the quadratic problem

$$\min_{U=(u_0,u_1,\cdots,u_{N-1})} J(U,x(t)) = x_N^{\mathrm{T}} P x_N + \sum_{k=0}^{N-1} \left[x_k^{\mathrm{T}} Q x_k + u_k^{\mathrm{T}} R u_k\right]$$

subject to

$$\begin{aligned} x_0 &= x(0) \\ x_{k+1} &= A x_k + B u_k, \quad k = 0, 1, \cdots, N-1 \\ y_k &= C x_k, \quad k = 0, 1, \cdots, N \end{aligned}$$

Link between linear-quadratic control and self-optimizing control

Let

$$d = x_0 \quad \text{and} \quad y = \begin{bmatrix} u \\ x \end{bmatrix}$$

The optimal combination

$$c = Hy$$

can be written as the feedback law

$$c = u - (Kx + g)$$

and H (or K) can be obtained from nullspace method

Exact local method

- Objective: $\min_u \; J(u,d) = \begin{bmatrix} u \\ d \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} J_{uu} & J_{ud} \\ J_{ud}^{\mathrm{T}} & J_{dd} \end{bmatrix} \begin{bmatrix} u \\ d \end{bmatrix}$
- Measurements: $y = G^y u + G_d^y d$, $y_m = y + n^y$
- Add constraints $c = Hy = c_s$ to the problem
- Resulting loss: $L = (u,d) - J_{\mathrm{opt}}(d)$
- The H that minimizes L may be found by[1]

$$\min_H \|H\tilde{F}\|$$

$$\text{subject to } HG^y = J_{uu}^{1/2}$$

Here $\tilde{F} = [FW_d \; W_{n^y}]$ and $F = -(G^y J_{uu}^{-1} J_{ud} - G_d^y)$

[1]Alstad et. al. *Journal of Process Control* 2009

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Nullspace method/LQR: open loop solutions

Full information:

$$\begin{aligned} \text{Invariant 1:} \quad & u_0 = K_0 x_0 \\ \text{Invariant 2:} \quad & u_1 = K_1 x_0 \\ \vdots \quad & \quad \vdots \\ \text{Invariant } n\text{:} \quad & u_{N-1} = K_{N-1} x_0 \end{aligned}$$

For implementation $u_k = K_0 x_k$ is OK, constant optimal feedback gain:

$$u_1 = K_1 x_0 = \underbrace{K_1 (A + BK_0)^{-1}}_{=K_0} x_1$$

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For implementation $u_k = K_0^y y_k$ not OK, optimal feedback gain not constant:

$$u_1 = \underbrace{K_1^y \left(C (A + BK_0^y C)^{-1} C^\dagger \right)}_{\neq K_0^y} y_1$$

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$K_0^y, K_1^y, \cdots$ minimizes the open-loop distance from the LQR controller.

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$$\begin{aligned} \min_{K^y} x_N^{\mathrm{T}} P x_N + \sum_{i=0}^{N-1} x_i^{\mathrm{T}} Q x_i + u_i^{\mathrm{T}} R u_i \\ \text{subject to } x_0 = x(0) = d \\ x_{k+1} = Ax_k + Bu_k, \quad y_k = Cx_k \\ u_k = -K^y y_k \end{aligned}$$

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- Solution is general a function of x_0. Therefore consider impluse response and minimze $\mathcal{H}_2$-norm.

- Impulse-response objective function:

$$J = \sum_{i=1}^{n_x} e_i^{\mathrm{T}} M(K) e_i,$$

where $M(K)$ is from subsituting $x_i = (A - BKC)^i x_0$ and $u_k = -KCx_k$ into the objective function.

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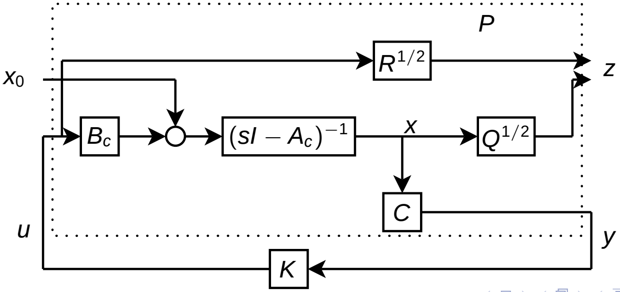
- Correspondning problem: Minimize the $\mathcal{H}_2$ norm of



Example 1: Second order process

Want to control $g(s) = \frac{k}{\tau^2 s^2 + 2\tau\zeta s + 1}$ with a LQ-optimal PI controller.

- Write system on state-space form $\dot{x} = ax + bu, y = cx + du$ and augment the plant with integrated output:

$$\begin{bmatrix} \dot{x} \\ \dot{\sigma} \end{bmatrix} = \begin{bmatrix} a & 0 \\ c & 0 \end{bmatrix} \begin{bmatrix} x \\ \sigma \end{bmatrix} + \begin{bmatrix} b \\ d \end{bmatrix} u$$

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- Objective function: $\sum_{i=0}^{\infty} x_i^{\mathrm{T}} Q x_i + u_i^{\mathrm{T}} R u_i$ with $Q = \mathrm{diag}(0, 0, 1)$ and $R = 1$
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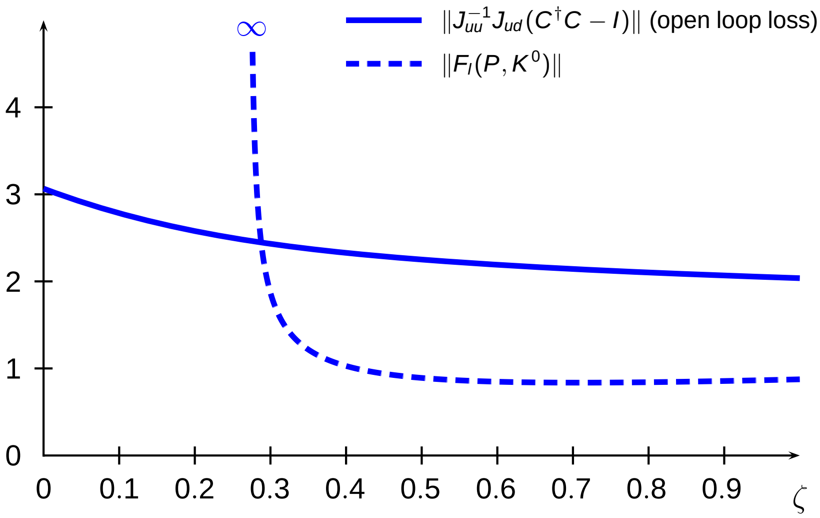
Example 1 cont'd

In Matlab, solve

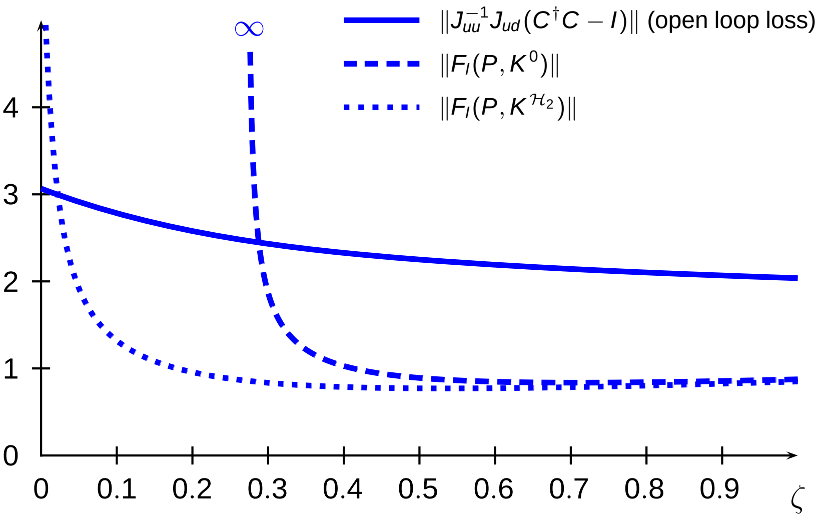
```
f = inline('norm(lft(P,-[K(1) K(2)]))','K','P')
K0 = Klqr*pinv(C);
Kyopt = fminunc(@(K) f(K,P),K0)
```

Resulting controller is "fixed-structure" $\mathcal{H}_2$ optimal.

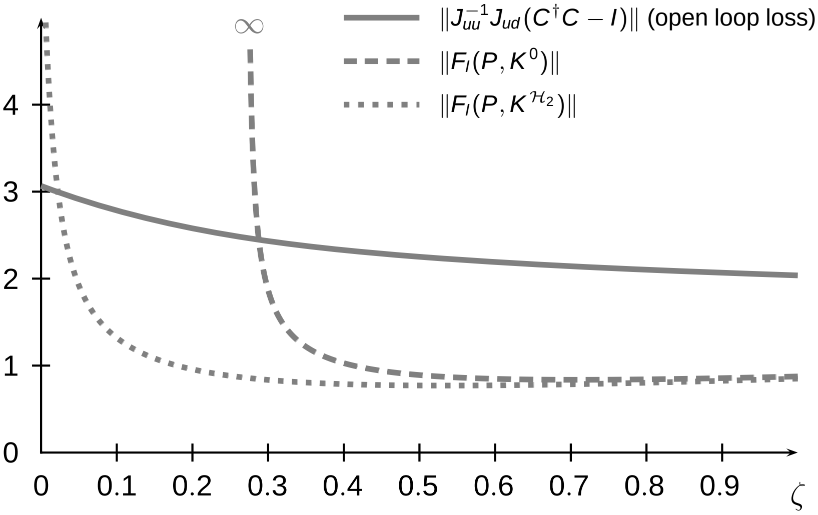
Example 1 cont'd

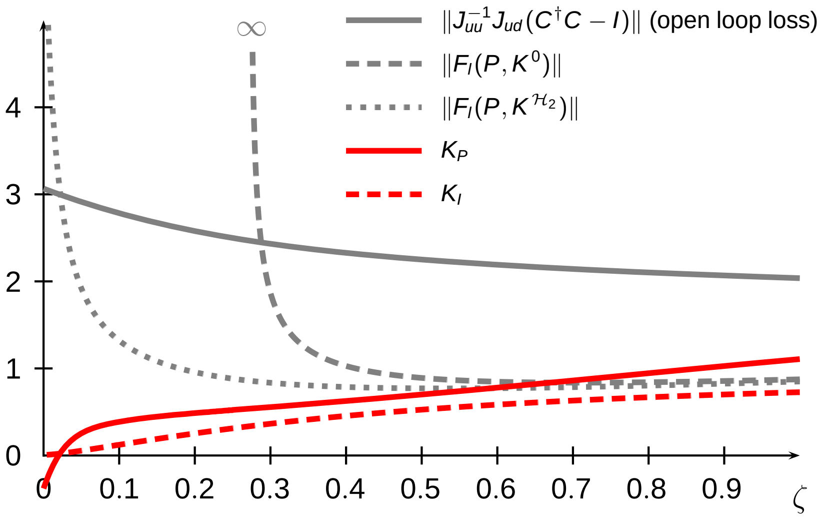


Example 1 cont'd

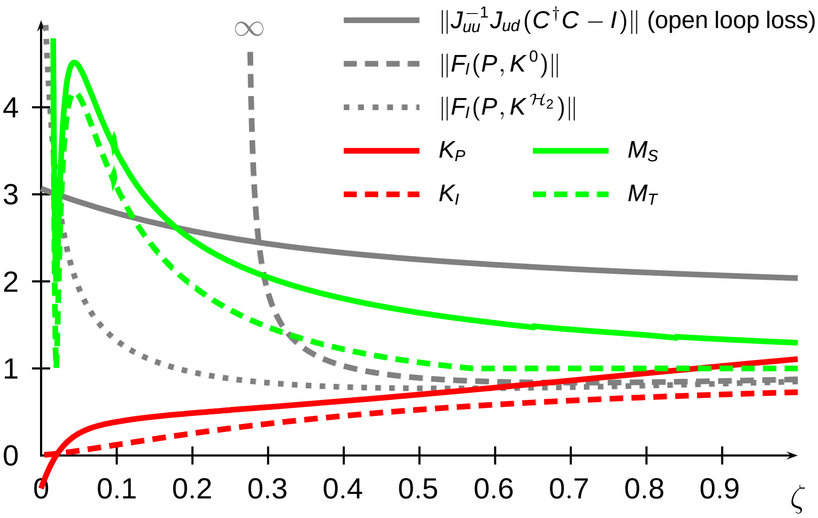


Example 1 cont'd

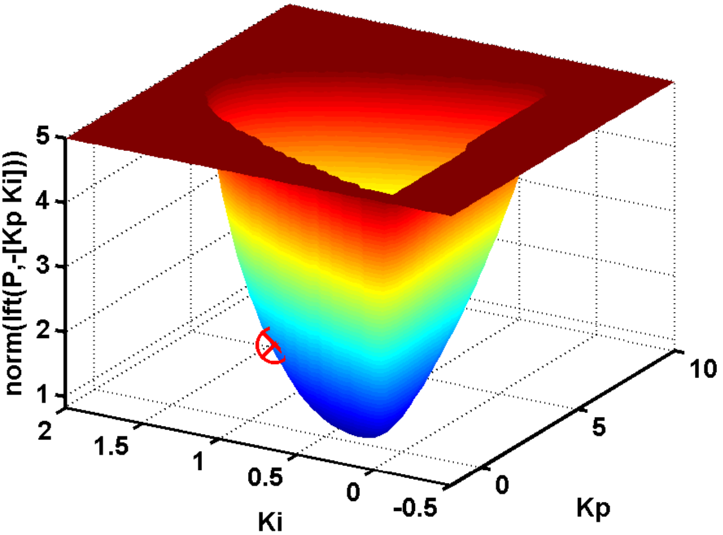



∞
$\|J_{uu}^{-1}J_{ud}(C^{\dagger}C - I)\|$ (open loop loss)
$\|F_l(P, K^0)\|$
$\|F_l(P, K^{\mathcal{H}_2})\|$
K_P
K_I
4
3
2
1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
ζ

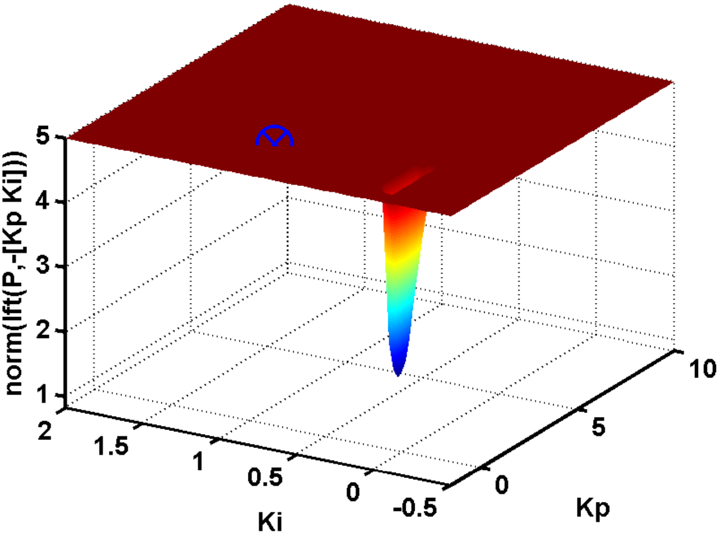
Example 1 cont'd



Objective function, $\zeta = 0.3$



Objective function, $\zeta = 0.05$



Example 2: Distillation column

Distillation model from Skogestad, 1997:

- Binary distillation column
- 41 stages
- Each stage at equilibrium, constant relative volatility of 1.5
- Linearized flow dynamics
- Negligible vapor dynamics
- Constant pressure
- LV-configuration (levels closed).

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- MIMO-PID controller, $c(s) = K_P + \frac{1}{s}K_I + \frac{s}{\tau_d s+1}K_D$

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Disturbances:

- At $t = 100$ (and $t = 300$) minutes F steps up (and down) by 1%.
- At $t = 400$ minutes z_F changes by 10%.

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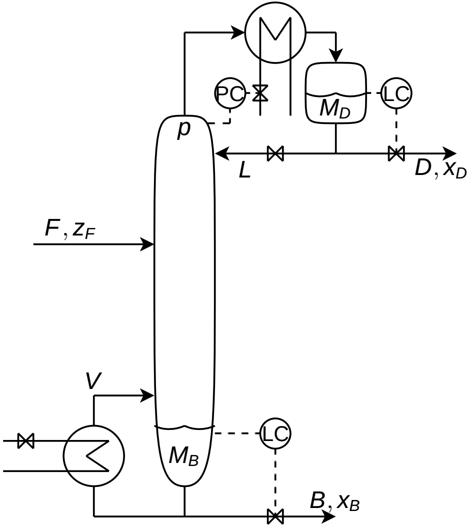

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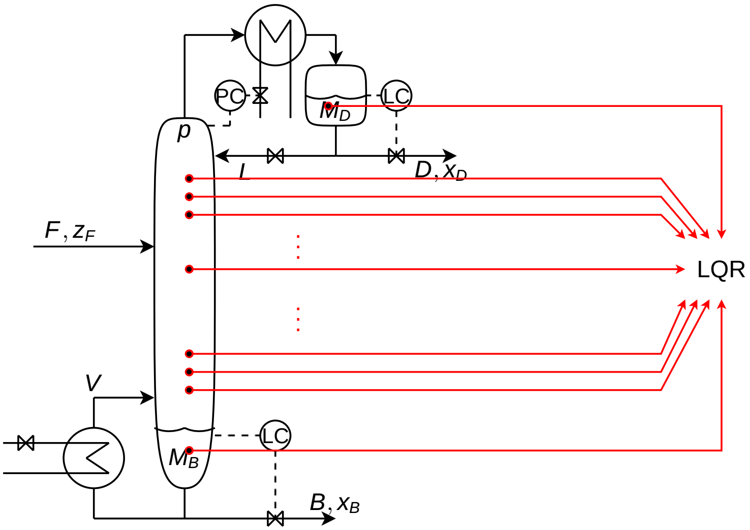
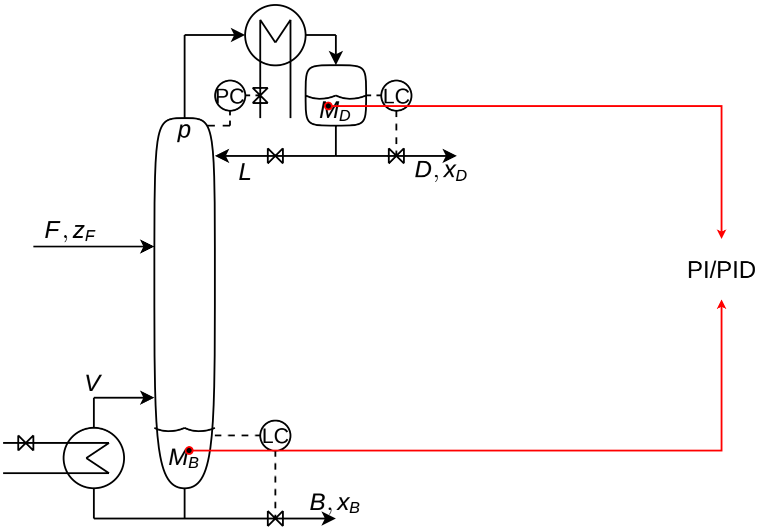
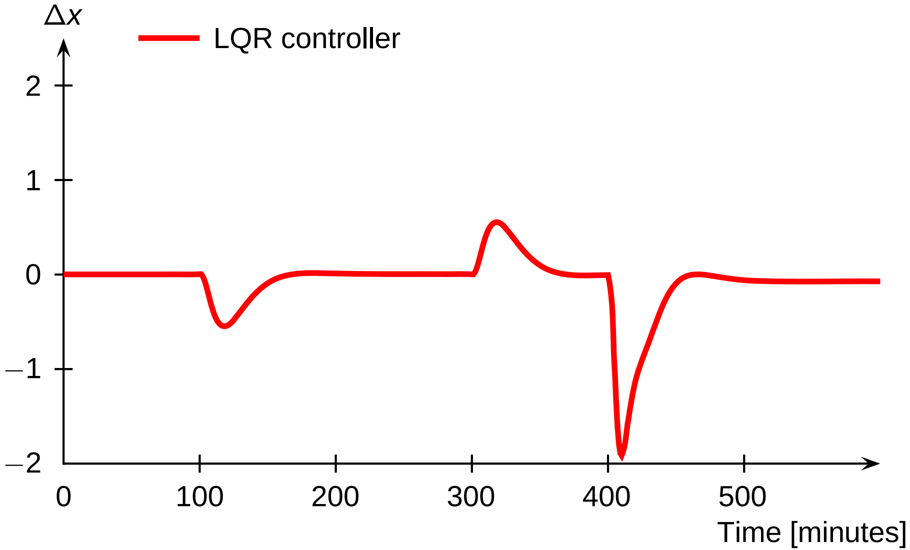


Example 2: Distillation column



Example 2: Simulation results



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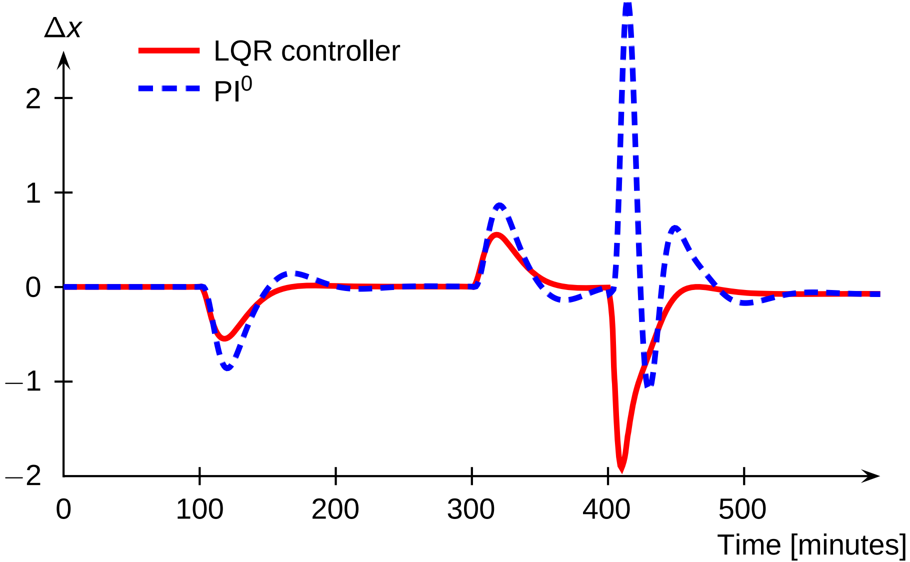

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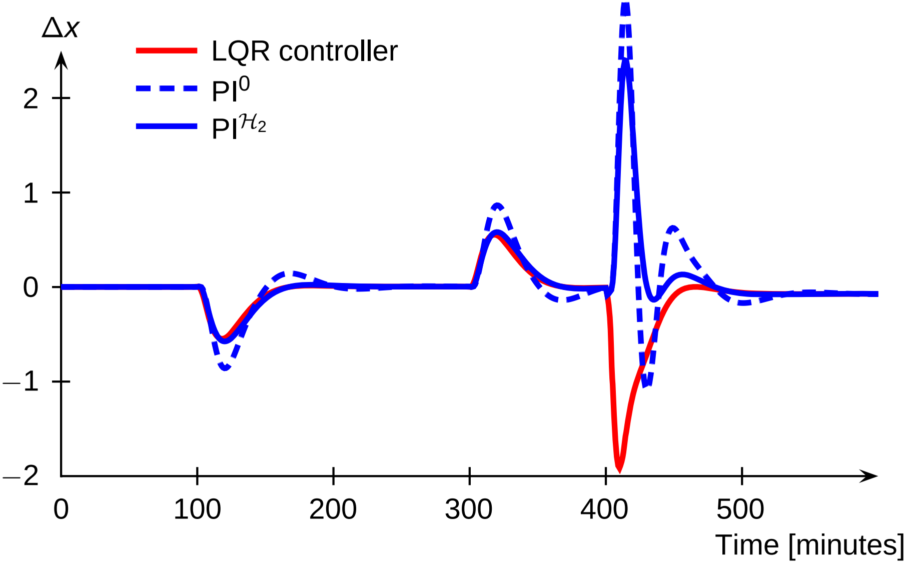

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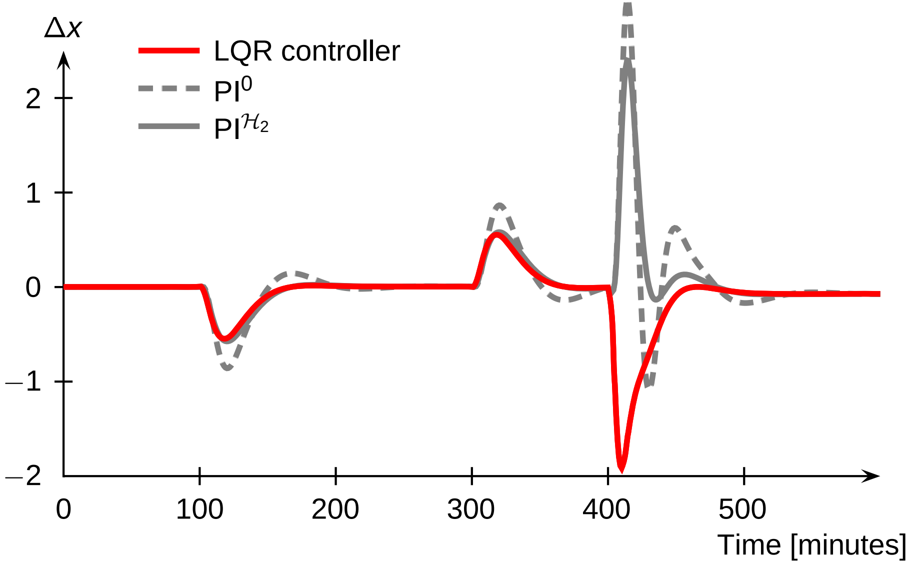

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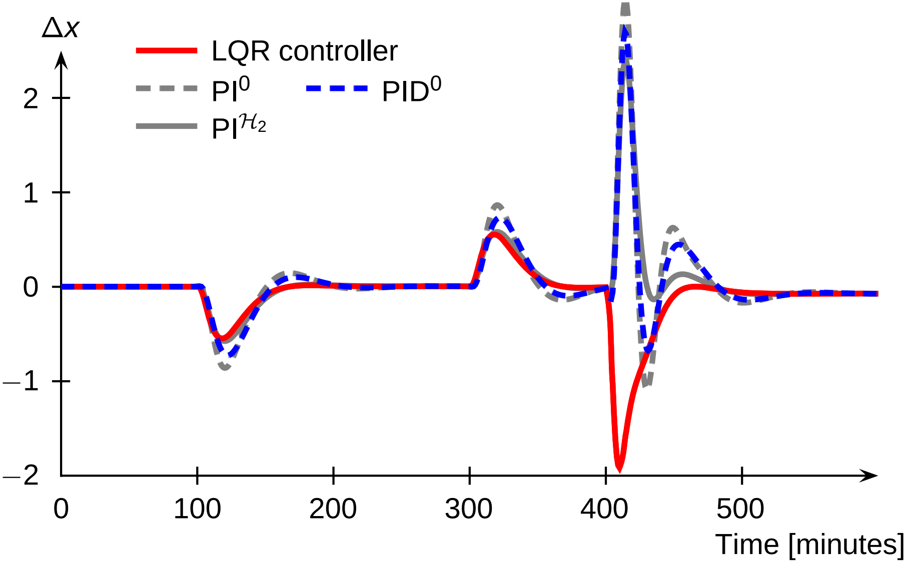

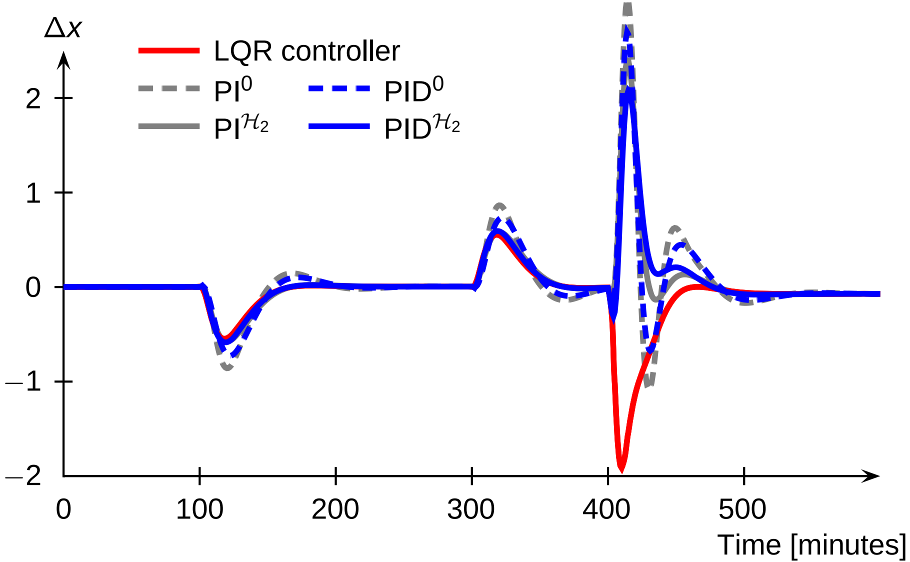
Δx
LQR controller
PI^0
PID^0
PI^H2
PID^H2
2
1
0
−1
−2
0
100
200
300
400
500
Time [minutes]

Conclusions

- Showed link between linear-quadratic control and self-optimizing control
- Full information: $u = -Kx$ exists (as an implementation to the open-loop LQR problem)
- Does not hold for output feedback
- Our method can be used to initalize fixed-structure $\mathcal{H}_2$-optimal design. (Use $K_0^y = KC^\dagger$ if $D = 0$, else solve a convex program to get K_0^y.)
    - This problem seems to be convex, but most of the time $J = '\texttt{inf}'$

Static output feedback

The static output feedback problem is the problem of deciding for given matrices A, B and C whether there exists a matrix K such that $A + BKC$ has all its eigenvalues in the left half plane.

Source:
`http://www.inma.ucl.ac.be/~blondel/books/openprobs/`