Perturbed Iterative Feedback Tuning

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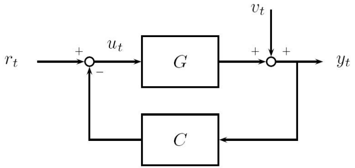
Outline of the presentation:

Given the feedback loop with a plant G and a controller C



Criterion based control performance optimization, seek the set of control parameters, ρ, which fulfills

$$\bar{\rho} = \arg\min_{\rho} J(\mathbf{y}, \mathbf{u})$$

The general performance cost used here is

$$J(\rho) = \frac{1}{2N} \sum_{t=1}^{N} y_t(\rho)^2 + \lambda\, u_t(\rho)^2$$

A local minimizer can be found by iterating in

$$\rho_{i+1} = \rho_i - \gamma_i \mathbf{R}_i^{-1} \frac{\partial J(\rho_i)}{\partial \rho}$$

where $\mathbf{R}_i$ is some positive definite matrix in order to insure a step in a descent direction. The gradient of the performance cost is

$$\frac{\partial J(\rho_i)}{\partial \rho} = \frac{1}{N} \sum_{t=1}^{N} y_t(\rho_i) \frac{\partial y_t(\rho_i)}{\partial \rho} + \lambda u_t(\rho_i) \frac{\partial u_t(\rho_i)}{\partial \rho}$$

From the block diagram one get for the disturbance rejection problem

$$\frac{\partial y_t(\rho_i)}{\partial \rho} = -\frac{\partial C(\rho_i)}{\partial \rho} GS^2(\rho_i) v_t$$

which is a function of the true system.

Iterative Feedback Tuning is a tuning method which generate a unbiased cost function gradient estimate based on closed loop data from two experiments.

- Collect data $\{y_t^1(\rho_i)\}_{t=1,\dots,N}$ where $r_t^1 = 0$
- Collect data $\{y_t^2(\rho_i)\}_{t=1,\dots,N}$ where $r_t^2 = -y_t^1$

which is used as:

$$\widehat{\frac{\partial J(\rho_i)}{\partial \rho}} = \frac{1}{N}\sum_{t=1}^{N} y_t^1(\rho_i)\widehat{\frac{\partial y_t(\rho_i)}{\partial \rho}}$$

where

$$\widehat{\frac{\partial y_t(\rho_i)}{\partial \rho}} = \frac{\partial C(\rho_i)}{\partial \rho} y_t^2 = \frac{\partial y_t(\rho_i)}{\partial \rho} + \frac{\partial C(\rho_i)}{\partial \rho} S v_t^2$$

A convergence analysis of the IFT method has shown that the control parameters converge in distribution to an zero mean normal distribution (Hildebrand et al., 2005a).

$$\sqrt{n}(\rho_n - \bar{\rho}) \xrightarrow{D} \mathcal{N}(0, \mathbf{\Sigma})$$

$$\mathbf{\Sigma} = a^2 \int_0^\infty e^{\mathbf{A}t} \mathbf{R}^{-1} \mathrm{Cov}\left[\widehat{\frac{\partial J(\bar{\rho})}{\partial \rho}}\right] \mathbf{R}^{-1} e^{\mathbf{A}^T t} dt$$

hence we can define the following term for the quality of our current loop with the iterate ρ_n.

$$\Delta J_n \equiv \mathrm{E}[J(\rho_n)] - J(\bar{\rho})$$

When tuning the loop for disturbance rejection IFT may exhibit a rate of convergence which is too slow for practical applications.

Attempts to improve the convergence properties of IFT include design of optimal prefilters in the cost function (Hildebrand et al., 2005b) and Perturbed Iterative Feedback Tuning (Huusom et al., 2008a)

This contribution will pursue the latter of the two and apply external perturbation signals to the process in order to insure informative data.

Perturbed Iterative Feedback Tuning include external perturbations in order to shape the curvature of the cost function. It still provide an unbiased gradient estimate of the cost function.

- Collect data $\{y_t^1(\rho_i)\}_{t=1,\ldots,N}$ where $r_t^1 = r_t^p$
- Collect data $\{y_t^2(\rho_i)\}_{t=1,\ldots,N}$ where $r_t^2 = -y_t^1$

which is used as:

$$\frac{\widehat{\partial J(\rho_i)}}{\partial \rho} = \frac{1}{N}\sum_{t=1}^{N} y_t^1(\rho_i)\frac{\widehat{\partial y_t(\rho_i)}}{\partial \rho}$$

where

$$\frac{\widehat{\partial y_t(\rho_i)}}{\partial \rho} = \frac{\partial C(\rho_i)}{\partial \rho} y_t^2 = \frac{\partial y_t(\rho_i)}{\partial \rho} + \frac{\partial C(\rho_i)}{\partial \rho} S v_t^2$$

The effect of this external perturbation signal is evident if we break up the expression for the gradient estimate of the cost function.

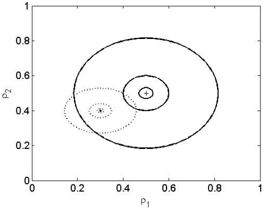
$$\widehat{\frac{\partial J(\rho_i)}{\partial \rho}} = \frac{1}{N}\sum_{t=1}^{N} y_t^1(\rho_i)\widehat{\frac{\partial y_t(\rho_i)}{\partial \rho}} = S_N(\rho_i) + E_N(\rho_i)$$

where S_N and E_N is the analytic and the variance component respectively.

$$S_N(\rho_i) = \frac{1}{N}\sum_{t=1}^{N}\left(S(\rho_i)(Gr_t^p + v_t^1)\right)\left(-\frac{\partial C(\rho_i)}{\partial \rho} GS(\rho_i)^2(Gr_t^p + v_t^1)\right)$$

$$E_N(\rho_i) = \frac{1}{N}\sum_{t=1}^{N}\left(S(\rho_i)(Gr_t^p + v_t^1)\right)\left(\frac{\partial C(\rho_i)}{\partial \rho} S(\rho_i) v_t^2\right)$$

Introducing an external perturbation signal in the experiment will affect the performance cost function $J(\rho, \vartheta)$.



Error associated with the tuning

$$\Delta(\vartheta) \triangleq \mathrm{E}\left[(\rho_n(\vartheta) - \bar{\rho}(\vartheta))(\rho_n(\vartheta) - \bar{\rho}(\vartheta))^T\right]$$

Hence there is a potential using perturbations if

$$\mathrm{E}[J(\rho_n(\vartheta), \vartheta^0)] < \mathrm{E}[J(\rho_n(\vartheta^0), \vartheta^0)]$$

By Taylor expansion it can be shown that:

$$
\begin{aligned}
&\mathrm{E}\left[J(\rho_n(\vartheta),\vartheta^0)\right] - J(\bar{\rho}(\vartheta^0),\vartheta^0) = \Delta J_n(\vartheta) \\
&\approx \frac{1}{2}\mathrm{Tr}\left\{\frac{\partial^2 J(\bar{\rho}(\vartheta^0),\vartheta^0)}{\partial\rho^2}\left(\bar{\rho}(\vartheta)-\bar{\rho}(\vartheta^0)\right)\left(\bar{\rho}(\vartheta)-\bar{\rho}(\vartheta^0)\right)^T\right\} + \\
&\quad \frac{1}{2}\mathrm{Tr}\left\{\frac{\partial^2 J(\bar{\rho}(\vartheta^0),\vartheta^0)}{\partial\rho^2}\Delta(\vartheta)\right\}
\end{aligned}
$$

which gives a clear distinction in bias and variance error.

For minimum variance control an optimal model based design exists. The perturbed process output is:

$$y_t = GS(\rho_i) r_t^p + S(\rho_i) v_t$$

Using the perturbation signal $r_t^p = \sqrt{\alpha}/G\, v_t$

$$y_t = GS(\rho_i) \frac{\sqrt{\alpha}}{G} v_t + S(\rho_i) v_t = (1 + \sqrt{\alpha}) S(\rho_i) v_t$$

which is a scaled expression for the output of the unperturbed case.

In reality the noise signal which affect the process is unknown but the specturm Φ_v may be known and used in the design.

For the general case where the cost function include both penalty on the output and the control an optimal unbiased design do not exist.

An optimal perturbation signal can be found by a numerical minimization algorithm using the quality index

$$\Delta J_n(\vartheta) \triangleq \mathrm{E}[J(\rho_n(\vartheta),0)] - J(\bar{\rho}(0),0)$$

This optimization may include constraints. The solution requires a plant model estimate.

An extensive analysis and a full algorithm is given in Huusom et al. (2008b)

Given the following system:

$$\text{Plant model:} \quad G(q) = \frac{q^{-1} - 0.5q^{-2}}{1 - 0.3q^{-1} - 0.28q^{-2}}$$

$$\text{Noise model:} \quad H(q) = \frac{1}{1 + 0.9q^{-1}}$$

$$\text{Controller:} \quad C(q) = \rho_1 + \rho_2 q^{-1}$$

where

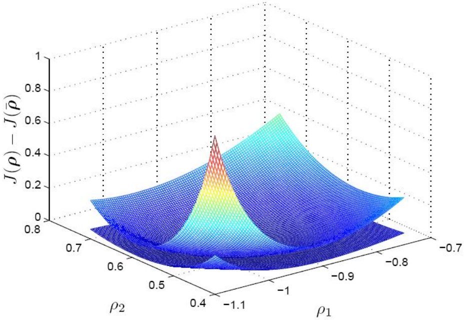
$$y_t = G(q)u_t + H(q)e_t, \quad e_t \in \mathcal{N}_{iid}(0,1)$$

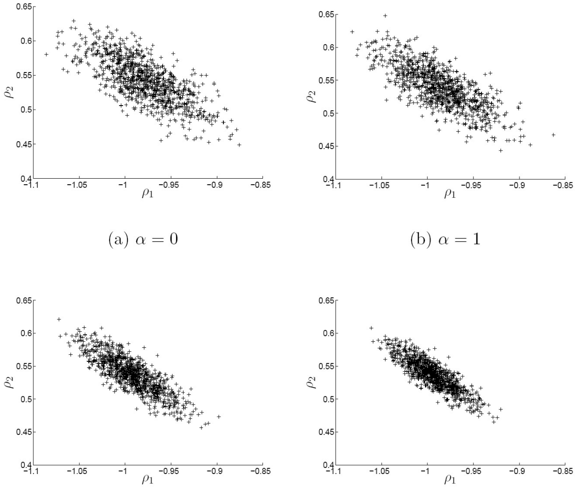
$$u_t = C(q)(r_t - y_t)$$

For the minimum variance control problem the optimal perturbation design $r_t^p = \sqrt{\alpha}/G v_t$ is used.

$$J(\rho, r^p) = (\sqrt{\alpha}+1)^2 J(\rho, 0)$$

In the figure $\alpha \in \{0; 1\}$





(a) $\alpha = 0$

(b) $\alpha = 1$

(c) $\alpha = 5$

(d) $\alpha = 10$

- Convergence of the IFT and the PIFT methods for the disturbance rejection problem has been discussed.
- The role of the perturbation is to shape the cost function hence balance its effect on bias and variance of the control quality index.
- It is shown that for a minimum variance control design an optimal design for the perturbations signal exists which will not introduce bias
- For a more general control design the optimal perturbation signal can be achieved by optimization

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