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Multi-model adaptive control of a simulated pH neutralization process

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Abstract

A multi-model adaptive PID controller is developed and evaluated in a simulation study for a nonlinear pH neutralization process. The performance and robustness characteristics of the multi-model controller are compared to those for conventional PID controllers and an alternative "multi-model interpolation" controller.

Keywords: Adaptive control; Switching control; Multiple models; PID control

1. Introduction

Many industrial processes inevitably change over time for a variety of reasons that include: equipment changes, different operating conditions, or changing economic conditions. Consequently, a fundamental control problem is how to provide effective control of complex processes where significant process changes can occur, but cannot be measured or anticipated. The conventional solution is conservative controller tuning for worst case conditions. However, this approach can result in poor control system performance for more typical conditions. Alternatively, adaptive control strategies are available where the controller parameters and/or control structure are modified online as conditions change (Åström & Wittenmark, 1995).

This paper is concerned with a special class of adaptive control strategies referred to as *switching control* or *multi-model adaptive control* (Angeli & Mosca, 2002; Freidovich & Khalil, 2003; Hespanha, 2001; Johansen & Murray-Smith, 1997; Morse, 1996; Narendra & Balakrishnan, 1997). The motivation for multi-model control is that for many complex technical processes, the local behavior can be captured at least approximately by a set of relatively simple models. Also, a corresponding feedback controller can be designed for each individual model. For these situations, an adaptive control approach based on selecting the best model (and controller) for the current conditions provides a promising approach. Selection of the performance criterion and switching strategy are key design issues.

In multi-model adaptive control, a bank of candidate models (and/or controllers) are specified a priori. Then a supervisory controller selects the most appropriate model (or controller) for the current conditions. For each model, a suitable controller can be designed off-line. The online controller switching is based on the performance evaluation of the bank of models (and/or controllers). Control problems involving transitions between known operating regimes are readily handled by a multi-model approach (Johansen & Murray-Smith, 1997). Multi-model control is also applicable to more general control problems where operating regimes cannot be determined a priori (Hespanha, 2001; Narendra & Balakrishnan, 1997). For example, the capabilities of multi-model control have been successfully demonstrated for drug infusion control where variability and unpredictability are key issues (He, Kaufman, & Roy, 1986; Schott & Bequette, 1997). Other reported applications include control of pH (Dougherty & Cooper, 2003b; Galán, Romagnoli, & Palazoglu, 2004), distillation columns (Dougherty & Cooper, 2003a; Rodriguez, Romagnoli, & Goodwin, 2003), power systems

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(Chadouri, Majumder, & Pal, 2004), and chemical reactors (Tian & Hoo, 2005). In (Rodrigues, Theilliol, Adam-Medina, & Sauter, 2006) multiple models are used for fault detection and isolation, with the models representing normal or faulty situations.

2. Multi-model adaptive PID control

In this paper, a multi-model adaptive strategy for PID controllers that is based on a set of simple linear dynamic models is considered. Each model has the same structure but different values of the model parameters. Grids of parameter values are assigned based on an assumed range for each model parameter. The ranges can be determined from a priori knowledge of expected operating conditions. For example, ranges for process gains and time constants can be specified based on physical knowledge such as the maximum and minimum values of temperatures and product flow rate. The grid spacing does not have to be constant.

In Section 3 the multi-model strategy is compared to a novel adaptive control strategy where the controller is automatically re-tuned after poor performance is detected (Wojsznis & Blevins, 2002; Wojsznis, Blevins, & Wojsznis, 2003). The re-tuning is based on re-estimating model parameters from recent input/output data.

A block diagram for the multi-model control strategy considered in this paper is shown in Fig. 1, where u is the input, y is the output, d is the unmeasured disturbance, and y_{sp} is the setpoint. The model parameters $\hat{\theta}$ for the current conditions are determined by calculating a performance index π_i for each model i. For example, the following low-pass filtered squared prediction error can be used as such an index:

$$\pi_i(k) = \lambda\pi_i(k-1) + (1-\lambda)\varepsilon_i^2(k), \quad i = 1, 2, \ldots, M. \tag{1}$$

Here M is the number of models in the model bank, and $\varepsilon_i(k) = y(k) - y_i(k)$ denotes the one-step prediction error for model i at time k. The filter constant $0 \leqslant \lambda \leqslant 1$ can be interpreted as a forgetting factor, as will be discussed later.

At each time k, the performance index of the currently chosen model, $\pi_c(k)$, is compared with the values for the other models. If

$$\pi_c(k) > (1+h)\min_i \pi_i(k) \tag{2}$$

the model with the smallest value of π_i is selected and the corresponding controller is implemented. In Eq. (2), $h > 0$ is a hysteresis parameter that prevents excessive switching. Both in theory and in practice, it is important that excessive switching be avoided. The use of a hysteresis term is a convenient approach for fulfilling this requirement.

2.1. Process model and unmeasured disturbances

Unmeasured disturbances can be a significant problem for adaptive control strategies, including multi-model control. For example, additive disturbances can result in adaptation of model parameters when the parameters have not actually changed. The resulting incorrect model parameters can produce very poor control. For multi-model control applications, the unknown disturbance can be approximated as a bias term for either the input or the output. Consequently, the bank of models can include different disturbance magnitudes, as well as different values of the other model parameters.

In this paper, the multi-model control strategy is based on a single-input, single-output model, namely, a first-order plus time-delay model with an additive input disturbance:

$$y_i(k+1) = a_i y(k) + b_i(u(k-\ell_i) + d_i), \quad i = 1, \ldots, M. \tag{3}$$

The model parameters are a_i, b_i, d_i and time delay ℓ_i, where the subscript i denotes the model index. For the simulation examples in Section 3, the parameters a_i and ℓ_i are assumed to be known. Thus, the model bank consists of models in the form of (3) with different values of b_i and d_i. When an unmeasured disturbance occurs, it is mapped to an approximately equivalent disturbance d_i in the model bank. Although the disturbance estimate is only a rough

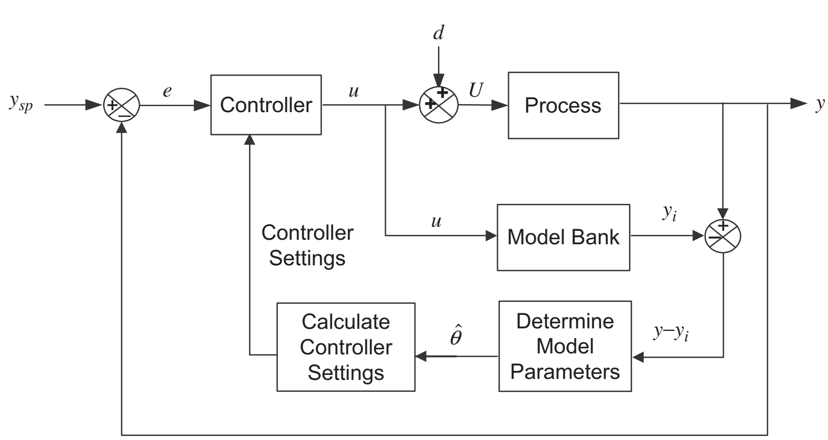

Fig. 1. Block diagram of multi-model adaptive control.

approximation of the actual disturbance, it can result in excellent switching control as demonstrated by the simulations in Section 3.

A straightforward method to build up a model bank consists of specifying ranges for the parameter values, and selecting models by using grids over these ranges. For disturbance d_i , which appears additively, a constant grid spacing is reasonable. However, for the gain parameter b_i, which appears multiplicatively, it is more appropriate to use a logarithmically spaced grid.

As noted by Tan, Marquez, Chen, and Liu (2004) and Arslan, Camurdan, Palazoglu, and Arkun (2004), several options can be used to select how many and which models should be used. Here a heuristic procedure that constructs the model set by taking into account the models already in the set and the corresponding closed-loop controllers is used.

The procedure works as follows:

(1) For each model parameter, the range of parameter values, the number of grid points, and either a linear or logarithmic spacing, were selected. Each possible combination of parameters corresponds to one candidate model.
(2) For each nominal model G, the IMC design method is used to select a controller G_c to shape the complementary sensitivity, $G_T = G_cG/(1 + G_cG)$. The details of this design will be discussed shortly.
(3) Then each controller is tested for "adequate" robust performance for *every* model in the parameter space that is adjacent to the candidate model that was used to design the IMC controller. Adjacent means here the models that are next to the candidate model, but has a different value of one parameter. If this was not the case, the number of grid points for the parameter at hand (and therefore the number of candidate models) is increased.

The following criteria were used to determine if "adequate" robust performance was met, where G is the candidate model with the corresponding controller G_c, and G_k is a model adjacent to G.

(3.1) *Robust stability*—For every G_k, the multiplicative uncertainty $\Delta := G_kG^{-1} - 1$ associated with the mismatch between G_k and G must satisfy:

$$|\Delta(j\omega)| < G_T(j\omega) \quad \forall\omega,$$

where $G_T = G_cG/(1 + G_cG)$ denotes the complementary sensitivity.

(3.2) *Robust performance*—For every model G_k adjacent to candidate model G, with corresponding controller G_c, the norm of the actual complementary sensitivity $G_cG_k/(1 + G_cG_k)$ for model G_k has a peak value less than 1.15. This corresponds to a second-order damping factor of 0.5 and prevents excessive overshoot and oscillations.

(3.3) *Closeness to nominal behavior*—For every G_k and for all frequencies ω,

$$0.5 \leqslant \frac{|G_c(j\omega)G_k(j\omega)/(1 + G_c(j\omega)G_k(j\omega))|}{|G_T(j\omega)|} \leqslant 2.$$

This requirement penalizes significant differences between the closed-loop behavior for the (nominal) candidate model and that of another model when the same controller is employed. The factor of two is somewhat arbitrary.

2.2. Forgetting of past data

In adaptive control applications, past data must be discounted (i.e., *forgotten*) in order to have the adaptive controller respond in a timely manner to process changes. Typically, a *forgetting factor* is employed such as λ in (1), where $0 \leqslant \lambda \leqslant 1$. The specification of λ involves an inherent tradeoff. If λ is too large, the adaptation is too sluggish while if λ is too small, the adaptation is overly aggressive resulting in loss of relevant information and excessive adaptation. In this paper, a heuristic forgetting of past data is employed. The basic premise is that when there is little input excitation, forgetting of past data is suspended by setting $\lambda = 1$. On the other hand, when there is sufficient excitation, λ is set equal to a specified constant, λ_0. Two metrics are considered as measures of the degree of process excitation:

(1) The prediction error, $\varepsilon = y - y_c$.
(2) The control error, $e = y_{sp} - y$.

Here y_c is the output predicted by the currently chosen model. During a period where the metric exceeds a specified threshold, λ is set equal to a constant $\lambda_0 < 1$. Otherwise, λ has the nominal value of one and no forgetting of past data occurs.

Both of these alternatives are evaluated in Section 3. The threshold for each metric was chosen to be 10 times larger than the expected value for normal operating conditions. However, the normal operating conditions might be difficult to specify in practice. In the simulations, normal operating conditions correspond to no inputs other than the measurement noise. In practice, the multiplication factor should be significantly larger than one. The choice of 10 is somewhat arbitrary, but the results do not depend significantly on the choice of this parameter.

2.3. Controller design

The PID controller design included a low pass filter of the error signal:

$$G_c(s) = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)\left(\frac{1}{\tau_f s + 1}\right). \quad (4)$$

The controller was designed using a first-order-plus-time-delay (FOPTD) model that corresponds to (3) with no disturbance:

$$\frac{y(s)}{u(s)} = \frac{K e^{-Ls}}{\tau s + 1}. \qquad (5)$$

The controller parameters were calculated using an IMC design procedure (Morari & Zafiriou, 1989), resulting in the following parameters:

$$K_c = \frac{\tau + 0.5L}{K(\tau_c + L)}; \quad \tau_I = \tau + 0.5L; \quad \tau_D = \frac{0.5L\tau}{\tau + 0.5L};$$
$$\tau_f = \frac{0.5L\tau_c}{\tau_c + L}.$$

Here, the closed-loop time constant τ_c is a design parameter. If the model is perfect, the IMC procedure results in a complementary sensitivity magnitude $|G_T| = |1/(\tau_c s + 1)|$. The PID is implemented as the discrete-time version obtained with the bilinear transform.

3. Simulations

In this section, the proposed switching PID (SPID) controller is evaluated in simulation studies for two examples: (i) a physical nonlinear model of the UCSB pH neutralization process (Hall & Seborg, 1989), and (ii) an approximate FOPTD model of the process. The stirred-tank neutralizer has three dilute inlet streams: base (NaOH), acid (HNO_3), and buffer ($NaHCO_3$). The exit stream pH is controlled by adjusting base flow rate Q_3 while the liquid level is regulated by manipulating acid flow rate Q_1. The buffer flow rate Q_2 is the major unmeasured disturbance. The pH neutralization process is highly nonlinear as indicated by the static map shown in Fig. 2, where the steady-state gain between y and u is shown as a function of pH.

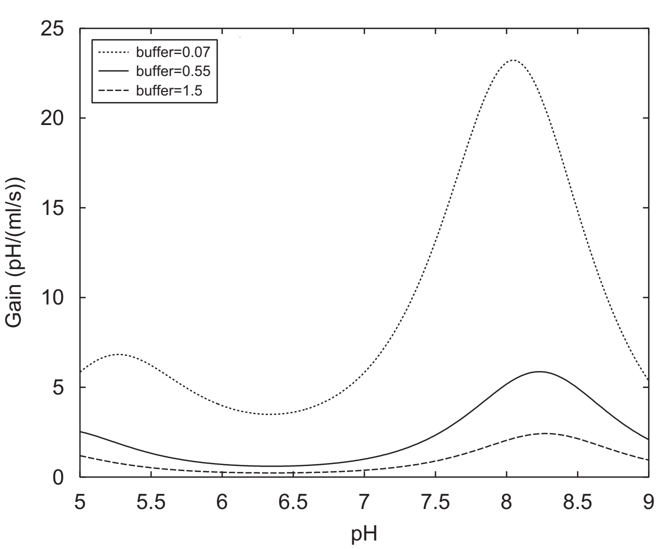

Fig. 2. pH gain variation.

The linear model serves as a simple approximate model for the nonlinear pH process. This model is in the form of Eq. (5) and the nominal parameter values are $K = 1$, $\tau = 3.6$ min, and $L = 0.75$ min. For the pH process, only the process gain K varies significantly for the normal range of operating conditions. Thus, in order to keep the simulation relatively simple and illustrative, the time constant τ and the delay L were assumed to be constant. Only the process gain K and the assumed input disturbance d were varied.

The parameter ranges and grid spacing of the parameters were based on a reasonable range of operating conditions for the physical process. The gains in the model bank were chosen using the criteria suggested in Section 2.1, resulting in 11 different values between 0.3 and 25.6, according to a geometric sequence with a ratio of 1.56. The input disturbances in the model bank, which does not have any impact in the robust performance test, were arbitrarily parameterized as 59 equally spaced values in the interval between -2.9 and $+2.9$. Thus, the model set consisted of a total of 649 models for each simulation. The hysteresis parameter h in Eq. (2) was set equal to one, but the simulation results do not depend critically on this parameter.

In the simulation studies, five PID control strategies were evaluated:

(1) A nominal controller designed for the nominal model in (5) with IMC parameter, $\tau_c = 1.5$ min.
(2) A conservatively tuned controller designed for a "high gain condition" of $K = 3.5$. This controller was designed using $\tau_c = 0.75$ min.
(3) A multi-model (switching) controller with the forgetting factor based on the prediction error.
(4) A multi-model (switching) controller with the forgetting factor based on the control error.
(5) An adaptive controller based on "multiple model interpolation (MMI)" and intermittent controller re-tuning.

In the MMI adaptive control strategy developed by Emerson Process Management (Wojsznis & Blevins, 2002; Wojsznis et al., 2003), after poor controller performance is detected, data are collected for a specified period of time (e.g., the open-loop settling time). Then the model parameters are re-estimated and the corresponding model-based controller is updated. Various criteria can be used to detect poor controller performance. In this application, data collection is initiated when the control error exceeds 100 times the expected value for nominal conditions. Then data were collected for a period of 10 min. The FOPTD model and IMC controller design for the SPID approach were also used for the re-tuning method. However, an output disturbance, rather than an input disturbance, was assumed.

Initially, the five controllers were evaluated for the linear system in Eq. (5). In this study the gain K was assumed

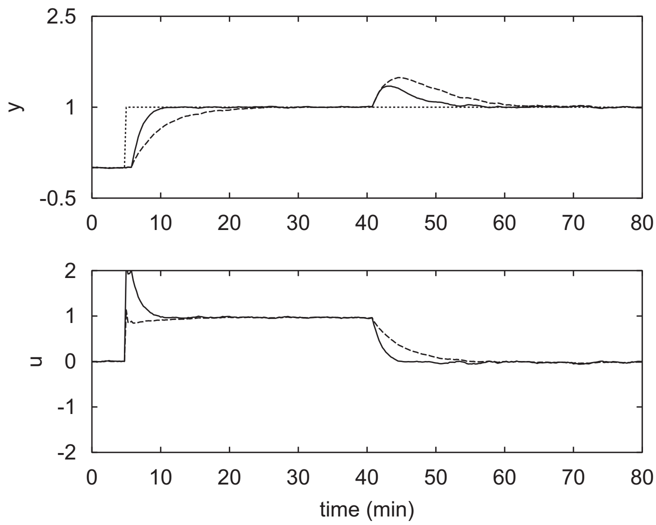

Fig. 3. Control of nominal linear model: nominal controller (—), conservative controller (- - -), and setpoint (···).

unknown and it was also assumed that the input was perturbed by an additive input disturbance. The results are summarized in Figs. 3–8 and Table 1. The controllers were compared for a setpoint change at $t = 5$ min, followed by a unit step disturbance at $t = 40$ min. For the high gain model ($K = 3.5$), the nominal controller produces the very oscillatory response in Fig. 6. However, the switching controllers in Figs. 7 and 8 readily adapt to the changing conditions without excessive oscillations. The forgetting factor and estimated model parameters are also shown in Figs. 4,5,7, and 8. Furthermore, the steady-state values of the true parameters are also given in these four figures.

3.1. The nonlinear pH model simulation

The five controllers were further evaluated by having the nonlinear pH model serve as the "process". Initially, the controllers were tested for a series of three step changes in

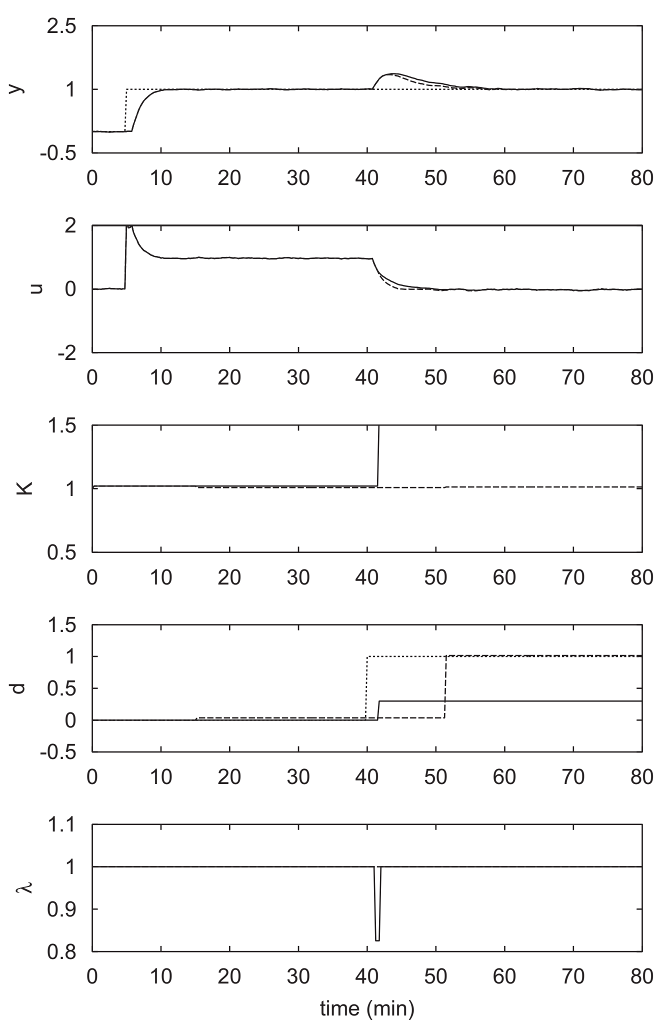

Fig. 4. Control of nominal linear model: switching controller (—), MMI re-tuning controller (- - -), setpoint and true values (···). Prediction error used in forgetting.

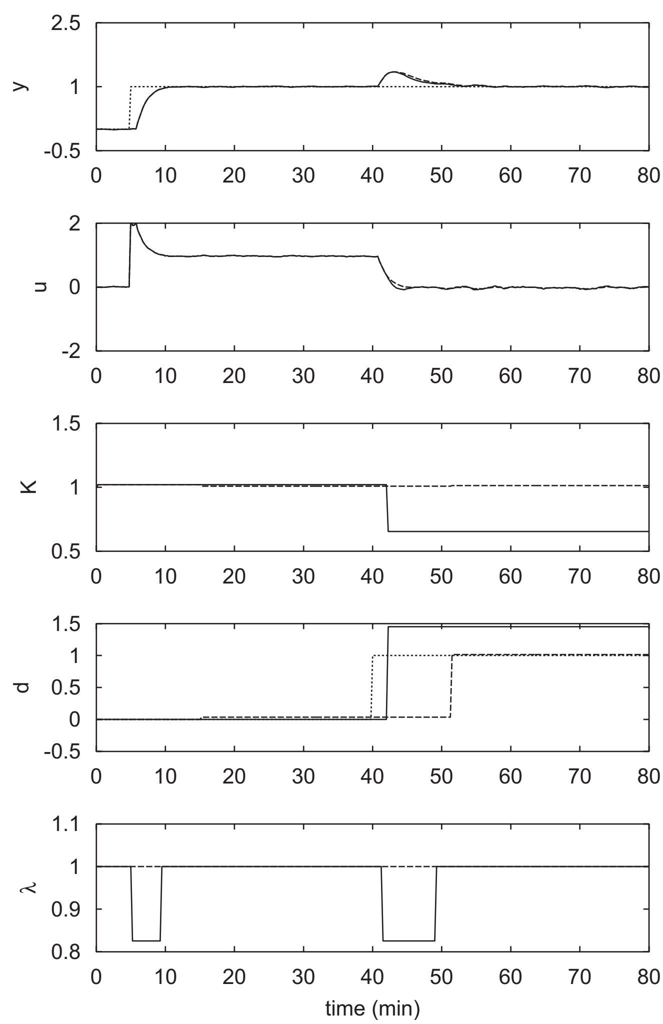

Fig. 5. Control of nominal linear model: switching controller (—), MMI re-tuning controller (- - -), setpoint and true values (···). Control error used in forgetting.

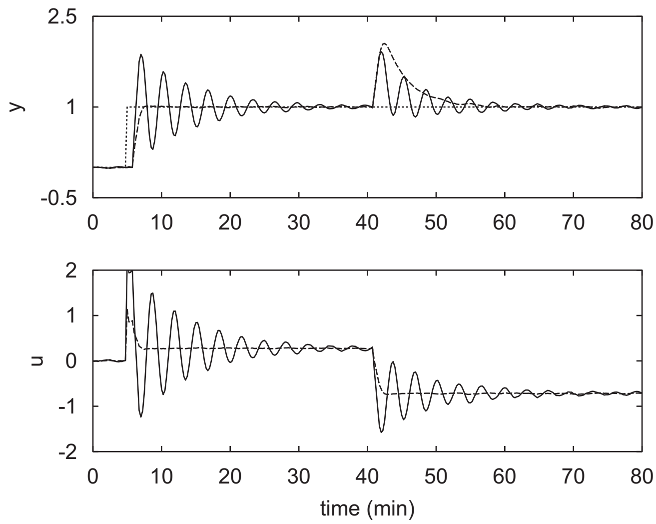

Fig. 6. Control of high-gain linear model: nominal controller (—), conservative controller (- - -), and setpoint (···).

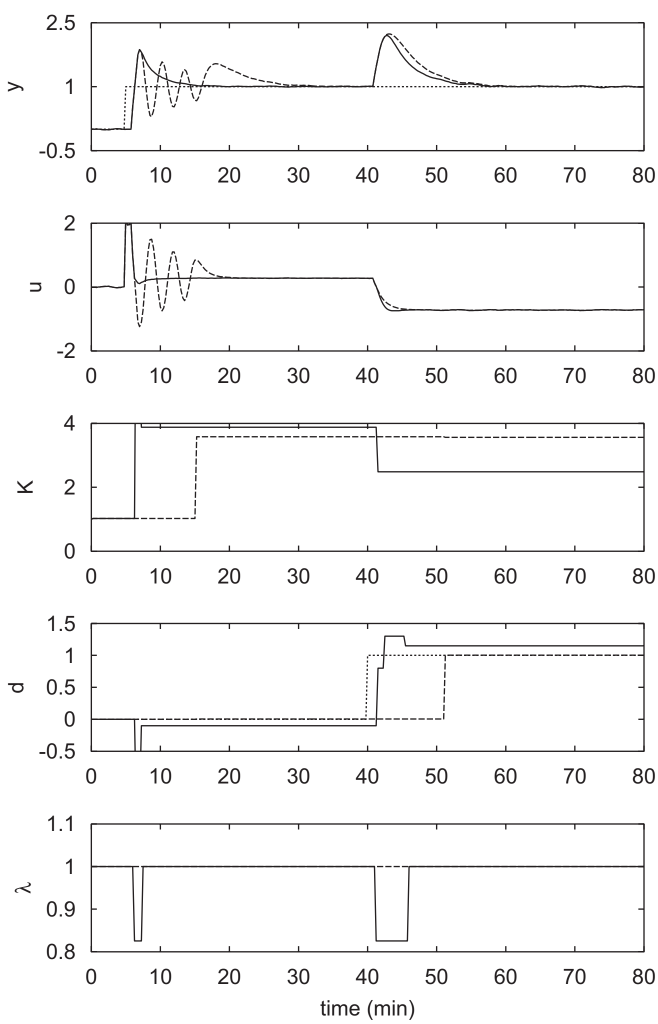

Fig. 7. Control of high-gain linear model: switching controller (—), MMI re-tuning controller (- - -), setpoint and true values (···). Prediction error used in forgetting.

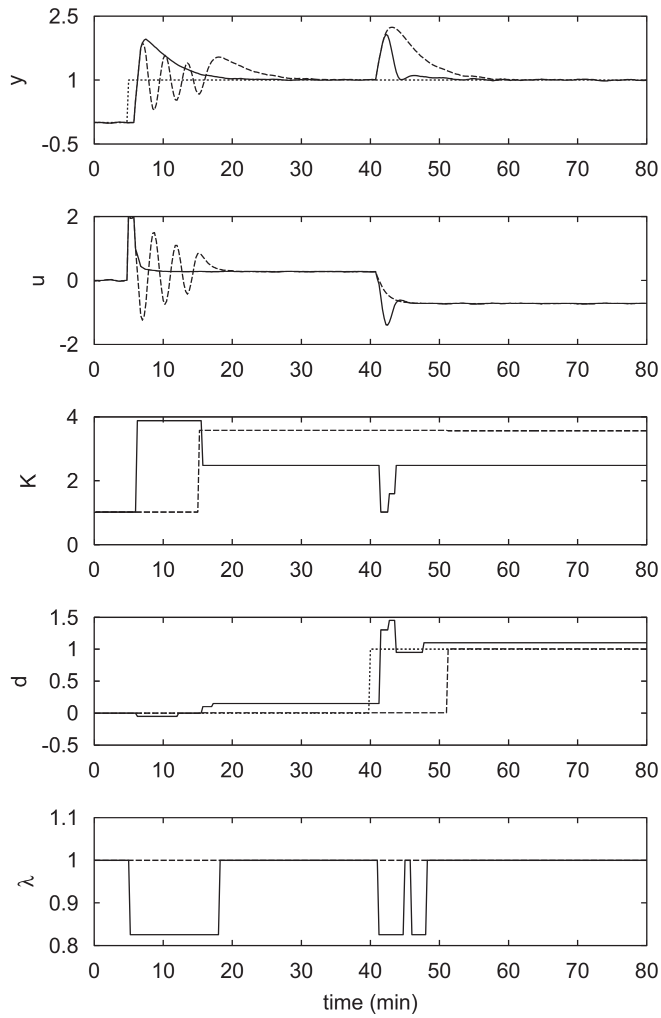

Fig. 8. Control of high-gain linear model: switching controller (—), MMI re-tuning controller (- - -), setpoint and true values (···). Control error used in forgetting.

the buffer flow rate Q_2: a decrease from 0.55 to 0.07 ml/s at $t = 5$ min, then an increase to 1.5 ml/s at $t = 30$ min, and finally a return to the initial value at $t = 75$ min. Note that the process gain increases as Q_2 decreases, as shown in Fig. 2.

Fig. 9 indicates that the nominal PID controller resulted in an unstable response after the first Q_2 disturbance, while the conservatively tuned controller is stable but rather sluggish for the other two disturbances. The two switching controllers in Figs. 10 and 11 provided satisfactory control for all three disturbances. Their performance is about the same, regardless of whether the control error or the prediction error is used to specify the forgetting factor.

Since the re-tuning adaptive controller was initialized with the nominal controller setting, its initial response to the first disturbance is also oscillatory. However, after the 10 min data collection period finishes at $t = 20$ min, the re-tuned controller is quite satisfactory. Note that the "true

Table 1
The Integral Absolute Errors for the linear model simulations

	n-PID	c-PID	ε-sw.	*e*-sw.	re-tune
Nominal linear system					
Setpoint	9.2	20.4	9.2	9.2	9.2
Disturb.	9.6	21.0	12.6	8.3	9.5
Total	18.8	41.4	21.8	17.5	18.7
High-gain linear system					
Setpoint	24.7	6.6	14.7	25.0	32.1
Disturb.	15.1	21.3	25.8	11.0	32.5
Total	39.8	27.9	40.5	36.0	64.6

The best value for each disturbance is highlighted with boxes.

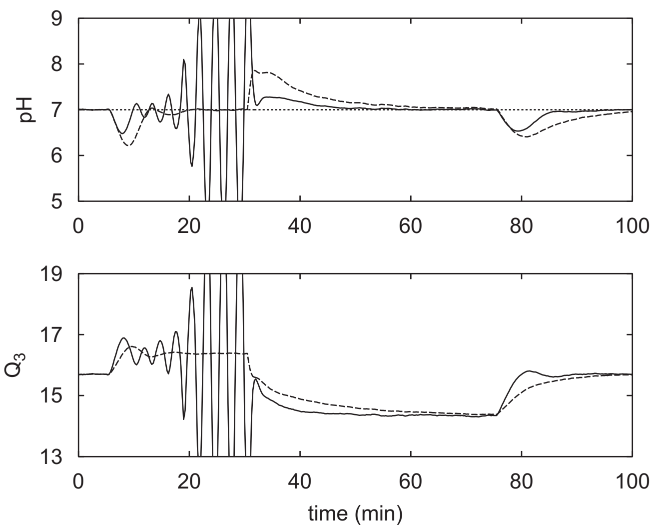

Fig. 9. Control pH system, buffer changes: nominal controller (—), conservative controller (- - -), and setpoint (···).

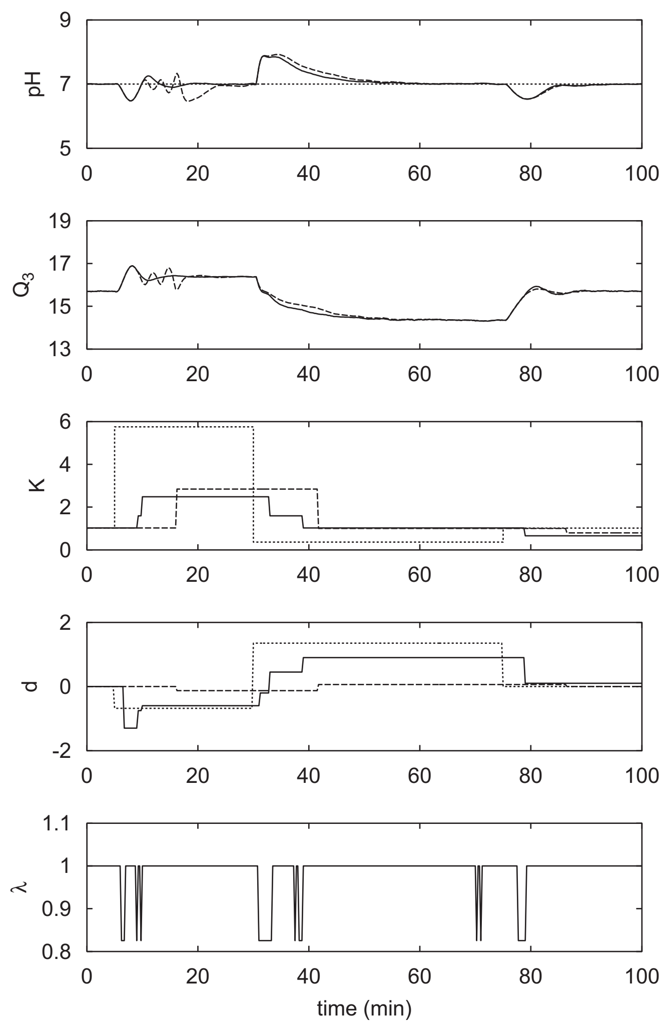

Fig. 10. Control of pH system, buffer changes: switching controller (—), MMI re-tuning controller (- - -), setpoint and true values (···). Prediction error used in forgetting.

values" of gain K and disturbance d in Fig. 9 and subsequent figures are local values for the current value of Q_2. Similar results were obtained for the setpoint changes in Figs. 12–14. The process gain changes by over a factor of seven during these setpoint changes, as is apparent from Fig. 2.

The values of the Integral Absolute Error performance index for the nonlinear simulation examples are reported in Table 2. Some observations from the simulations are given below:

(1) When the process to be controlled is linear (cf. Table 1) the setpoint response is optimal when one uses the controller that was designed for the actual process: the nominal PID works the best for the nominal process and the conservative PID works best for the high-gain process. However, in either case the multi-model switching controller with the forgetting factor based on the prediction error achieves a close second-best performance.

(2) When the process is linear, the best disturbance rejection responses are achieved by the multi-model switching controllers.

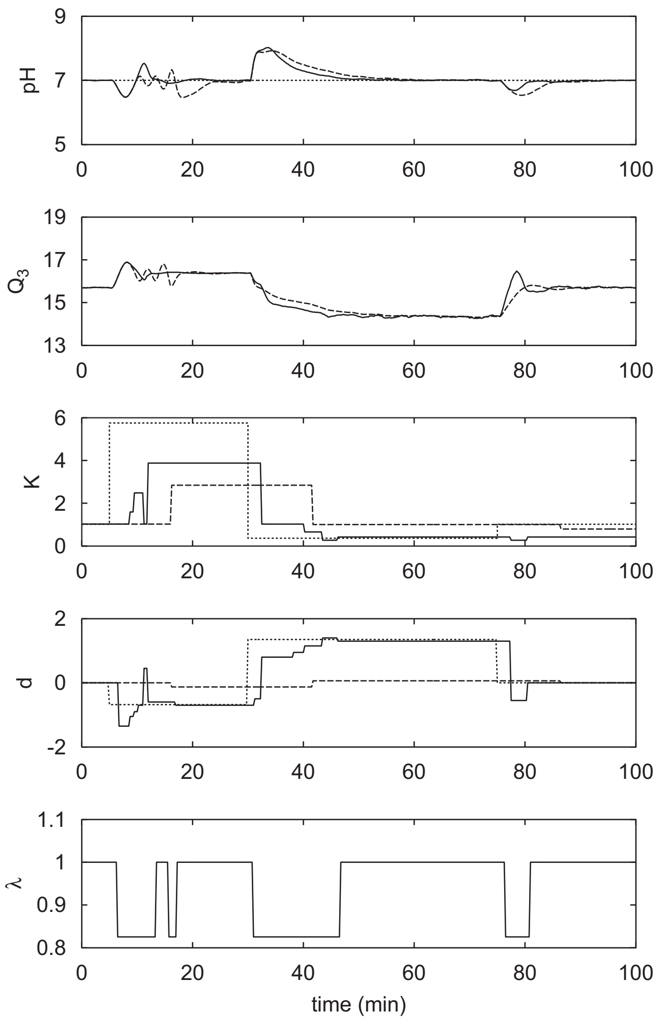

Fig. 11. Control of pH system, buffer changes: switching controller (—), MMI re-tuning controller (- - -), setpoint and true values (···). Control error used in forgetting.

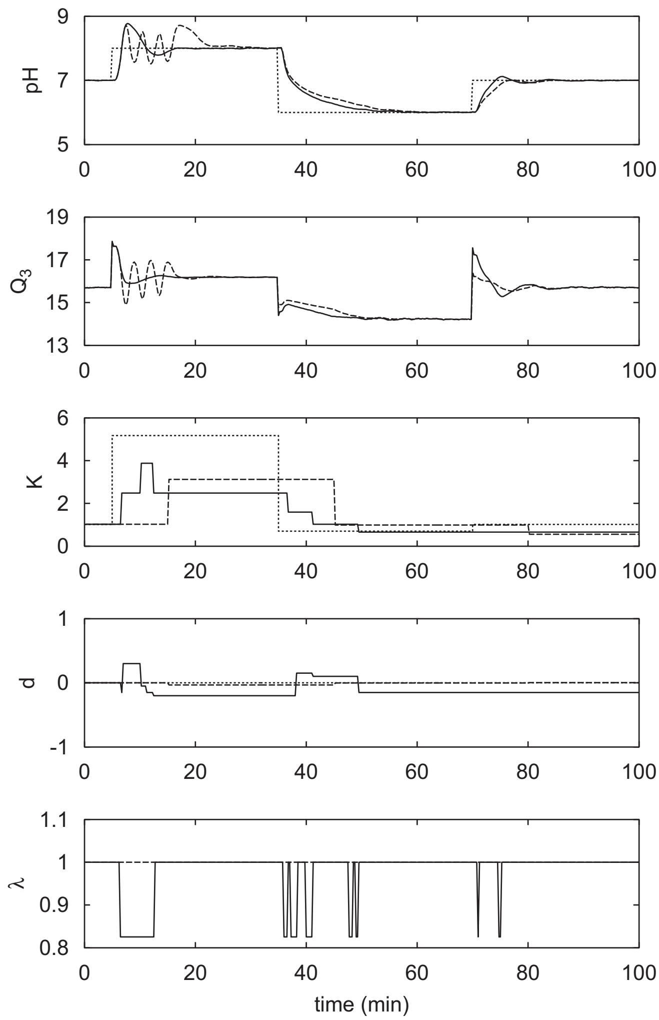

Fig. 13. Control of pH system, setpoint changes: switching controller (—), MMI re-tuning controller (- - -), setpoint and true values (···). Prediction error used in forgetting.

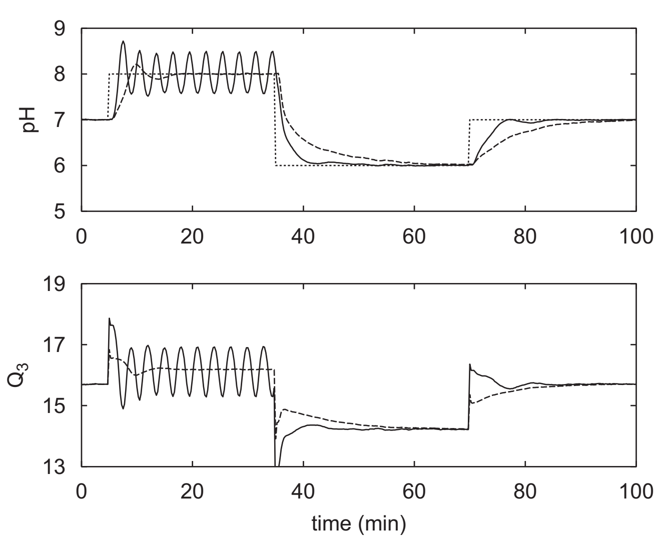

Fig. 12. Control of pH system, setpoint changes: nominal controller (—), conservative controller (- - -), and setpoint (···).

(3) When the process is nonlinear (cf. Table 2), the nominal and the conservative PIDs continue to give good results for some specific sets in disturbances or setpoints, but none of these provides acceptable results for all conditions. However, the multi-model switching controller based on the control error performs consistently good for every disturbance and change in setpoint.

4. Conclusions

A multi-model PID control strategy has been evaluated in two simulation studies that included comparisons with three other PID controllers: a re-tuning adaptive controller and two nonadaptive controllers. The simulations indicated that the multi-model controller was quite effective over wide ranges of unmeasured disturbances and process changes. The re-tuning strategy also performed very well, but was slower to respond to sudden disturbances.

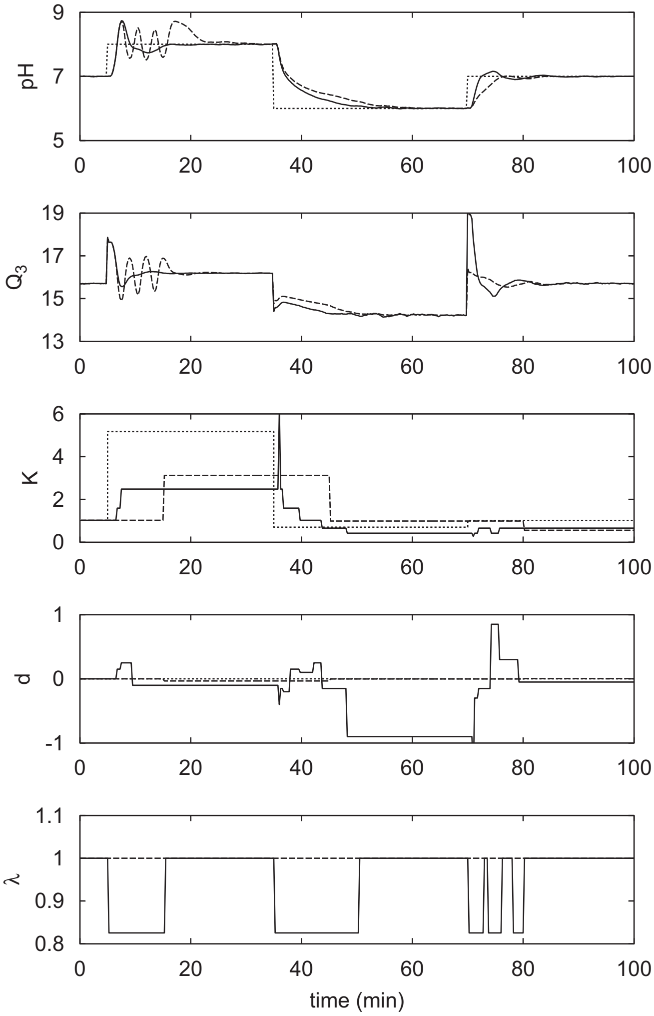

Fig. 14. Control of pH system, setpoint changes: switching controller (—), MMI re-tuning controller (- - -), setpoint and true values (···). Control error used in forgetting.

Table 2
The Integral Absolute Errors for the pH simulations

	n-PID	c-PID	ε-sw.	*e*-sw.	re-tune
pH system					
Q_2 0.55 → .07	81.4[a]	14.8	**9.0**	10.9	19.2
Q_2 0.07 → 1.5	**25.3**[b]	41.7	33.5	31.5	42.3
Q_2 1.5 → 0.55	12.5	26.5	11.7	**5.4**	12.1
Total buffer	119.2	83.0	54.2	**48.8**	73.6
Setpoint 7 → 8	40.7[a]	**13.5**	17.3	14.9	31.5
Setpoint 8 → 6	**19.2**[b]	40.7	36.2	34.4	46.3
Setpoint 6 → 7	13.7	30.4	11.1	**9.5**	13.3
Total setpoint	73.6	84.6	64.6	**58.8**	91.1
Total pH	192.8	167.6	118.8	**107.6**	164.7

The best value for each disturbance is highlighted with boxes.
[a]Unstable.
[b]Average value, depends on the times of the setpoint changes.

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