On the existence of unknown input observers for state affine systems up to output injection

M. Sahnoun * **H. Hammouri** *

* *Laboratoire d'Automatique et de Génie des Procédés LAGEP, UMR CNRS 5007, Université Claude Bernard Lyon I, Bat. 308G, 43, Bd du 11 Novembre 1918, 69622 Villeurbanne Cedex, France, (e-mail: sahnoun-at-lagep.univ-lyon1.fr, hammouri-at-lagep.univ-lyon1.fr)*

Abstract: In this paper, the design of an unknown input observer is considered. The main contribution consists in the obtention of a sufficient condition to design an observer which estimates a part of the state independently of the knowledge of some inputs. Based on the geometric approach, a sufficient condition for the existence of an unknown input observer for state affine systems up to output injection is given. This approach is illustrated in a numerical example.

Keywords: State affine systems, unknown input observer.

1. INTRODUCTION

The dynamical system which permits to obtain an asymptotic estimation of an unknown state function in the presence of an unknown input is called an unknown input observer (abbreviated as UIO hereafter). The problem of designing an UIO has received many attentions over the past four decades. It has been started by Basile and Marro [1969] and after that, many contributions have been proposed see for instance, Bhattacharyya [1978], Hautus [1983] and Darouach et al. [1994].

The design of observers for systems with unknown inputs plays an important role in fault detection. In the linear case, a solution based on the geometric approach has been proposed in Massoumnia et al. [1989] which consists in designing an observer that can detect and uniquely identify a component failure, first for the case where components can fail simultaneously, and then for the case where they fail only one at a time. Bokor and Balas [2004] also gave a necessary and sufficient condition for the existence of an unknown input observer for lpv systems using a geometric approach. On the other side, several contributions have been proposed in the nonlinear case. The Fault Detection and Isolation (FDI) for nonlinear systems was introduced by Seliger and Frank [1991]. They have proposed a nonlinear fault-detection observer which is robust to the disturbances as well as to the model uncertainties. Then, a necessary and sufficient condition to solve the so-called Fundamental Problem of Residual Generation (FPRG) through a differential geometric approach has been proposed by Hammouri et al. [2000] for state affine systems and by Hammouri et al. [2001] for bilinear systems.

The design of UIO has been considered for only subclasses of nonlinear systems see for instance Liu et al. [2006], Moreno and Dochain [2008] has proposed a methodology to make a global analysis of observability and detectability of reaction systems, with a particular concern about the design of robust observers and gave sufficient conditions to construct a robust observer for state affine systems up to output injection.

In the present paper, an original contribution is addressed to give a sufficient condition which allows the design of a stable unknown input observer for state affine systems up to output injection based on the geometric approach. The theory behind these developments is inspired from recent results in Hammouri and Tmar [2010] in which a necessary and sufficient condition to the existence of an unknown input observer for state affine systems is considered.

The paper is organized as follows: in Section 2, the problem under consideration is formalized and a mathematical formulation of the UIO for state affine systems up to output injection is given. Theorem 1 which gives a sufficient condition for the existence of the UIO and the proof are given in Section 3. Section 4 is devoted to illustrate the proposed approach on a numerical example. Finally, Section 5 contains the conclusion.

2. PROBLEM STATEMENT

2.1 Problem formulation

Consider the following state affine system up to output injection:

$$\begin{cases} \dot{x}(t) &= A(u(t), y(t))x(t) + B(u(t), y(t)) + K(x(t))v(t) \\ y(t) &= Cx(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^p$ is the known input, $v(t) \in \mathbb{R}^l$ is the unknown one, $y(t) \in \mathbb{R}^m$ represents the measured output vector and $K(x(t))$ is the matrix which depends on x. $A(u, y)$ and $B(u, y)$ are matrices which depend smoothly on (u, y), and C is a constant matrix of rank m.

Let $\mathcal{U} \subset \mathcal{L}^\infty(\mathbb{R}^+, \mathbb{R}^p)$ be a class of known inputs $u(.)$ that we specify below.

Given a linear map Γ from $\mathbb{R}^n$ into $\mathbb{R}^r$, our aim in this paper is to obtain sufficient conditions permitting to estimate $\Gamma x(t)$ independently on the unknown inputs $v(.)$ by using a dynamical filter. A such filter is called an unknown input observer. More precisely, the mathematical formulation of the unknown observer that we consider in order to estimate $\Gamma x(t)$ is given as follows:

Definition 1.

Let Ω be an open subset of $\mathbb{R}^n$. Let Γ and P be linear maps from $\mathbb{R}^n$ into $\mathbb{R}^r$ and $\mathbb{R}^q$ respectively.

With regard to (Γ, P), an $\mathcal{U}-$unknown input observer for system (1) which estimate $\Gamma x(t)$ for every $x(0) \in \Omega$, is a dynamical system of the form:

$$\begin{cases} \dot{z}(t) &= A_1(u(t), y(t))z(t) + E_1(u(t), y(t))y(t) \\ &\quad + B_1(u(t), y(t)) + E_2(h(t))y(t) + E_3(h(t))z(t) \\ \dot{h}(t) &= H(h(t), u(t), y(t)) \\ \gamma(t) &= Fz(t) + Dy(t) \end{cases} \tag{2}$$

where $z(t) \in \mathbb{R}^q$ and $h(t) \in \mathcal{S}$ an open subset of some $\mathbb{R}^N$. F and D are constant matrices, $A_1(u, y), B_1(u, y), E_1(u, y)$, $E_2(h), E_3(h)$ and $H(h, u, y)$ are analytic, and such that for every $u(.) \in \mathcal{U}$, the following properties hold:

i) $\forall x(0) \in \Omega$; $Px(0) = z(0)$ implies $Px(t) = z(t)$ and $\Gamma x(t) = \gamma(t)$ $\forall t \geq 0$.
ii) For every $x(0) \in \Omega$ and $z(0) \in \mathbb{R}^q$, $\lim_{t\to\infty}(Px(t) - z(t)) = 0$

Remark 1.

- From i), ii) of the definition, we deduce that: $\lim_{t\to\infty} \|\Gamma x(t) - \gamma(t)\| = 0$.
- Using the fact that $\gamma(t) = Fz(t) + Dy(t)$, $z(t) = Px(t)$ and condition i) of the definition, we deduce that $FP + DC = \Gamma$.

In the following section we will give some mathematical tools which will be used to design an $\mathcal{U}$-unknown input observer.

2.2 Preliminary results

The basic concepts and tools of the differential geometric approach to nonlinear control can be find in the classical literatures (see for instance Isidori [1995]). In order to state our main result, the following notions will be required.

In the sequel $A(u(t), y(t))$ will be replaced by $A(w(t))$ where $w(t) \in \mathbb{R}^{m+p}$ is a free signal which does not depend on $y(.)$.

Consider the following system:

$$\begin{cases} \dot{x}(t) &= A(w(t))x(t) \\ Y(t) &= Cx(t) \end{cases} \tag{3}$$

Definition 2.

A subspace V of $\mathbb{R}^n$ is said to be a $(C, A(w))$-invariant subspace, if one of the following equivalent conditions holds:

1) there exists a smooth matrix $E(.)$ such that $\forall w$, $(A(w) + E(w)C)V \subset V$.
2) $\forall w$, $A(w)(Ker(C) \cap V) \subset V$

Remark 2.

Given a subspace W, the set of all $(C, A(w))$-invariant subspaces containing W admits a unique minimum with respect to the inclusion. This minimum will be denoted by W^*.

Let $E(w)$ be a $n \times p$ smooth matrix and N a $l \times p$ constant one and consider the following system:

$$\begin{cases} \dot{x}(t) &= (A(w(t)) + E(w(t))C)x(t) \\ s(t) &= NCx(t) \end{cases} \tag{4}$$

Denoting by $\mathcal{O}(NC, E(w))$ the unobservable space of system (4); namely the space containing all x such that $NCx = 0$ and for every $w_1, \ldots, w_k$, $NC(A(w_1) + E(w_1)C)\ldots(A(w_k) + E(w_k)C)x = 0$.

Definition 3.

V is said to be a unobservability subspace if there exists matrices N and $E(.)$ such that $V = \mathcal{O}(NC, E(w))$.

Proposition 1.

The set of all $(C, A(w))-$unobservability subspaces containing W admits a minimum W^{**} with respect to $\subset$. Moreover W^{**} is the limit of the following decreasing sequence:

$$\begin{cases} W_0 &= Ker(C) + W^* \\ W_{i+1} &= W^* + Ker(C) \cap [\bigcap_{j=1}^{k} (A_j)^{-1} W_i] \end{cases} \tag{5}$$

where $(A_1, \ldots, A_k)$ is any basis of the smallest vector space containing $\{A(w);\ w \in \mathbb{R}^{m+p}\}$. Consequently, there exist matrices N^{**} and an analytic matrix $E^{**}(w)$ such that $W^{**} = \mathcal{O}(N^{**}C, E^{**}(w))$. Namely, W^{**} is the unobservable space of:

$$\begin{cases} \dot{x}(t) &= (A(w(t)) + E^{**}(w(t))C)x(t) \\ y^{**}(t) &= N^{**}Cx(t) \end{cases} \tag{6}$$

This algorithm can be found in Hammouri et al. [2000], for state affine systems and in Persis and Isidori [2000] for general nonlinear systems.

So, W^{**} is a $(C, A(w))-$invariant subspace $((A(w) + E^{**}(w)C)W^{**} \subset W^{**})$, and therefore the following quotient system is well defined:

$$\begin{cases} \dot{\varepsilon}(t) &= \overline{(A(w(t)) + E^{**}(w(t))C)}^{W^{**}} \varepsilon(t) \\ y^{**}(t) &= \overline{N^{**}C}\varepsilon(t) \\ \varepsilon \in \mathbb{R}^n / W^{**} \end{cases} \tag{7}$$

Remark 3.

By construction, system (7) is observable in the rank sense, and therefore it is observable in the sense that it admits an input which distinguishes every two different initial states. As a consequence, the set of inputs which make system (7) completely uniformly observable is not empty. In the sequel, this set will be denoted by $\mathcal{U}_W$.

3. MAIN RESULT

3.1 Sufficient condition for the existence of unknown input observer

Our main result that we will establish below requires the following notion:

Definition 4.
Let V be a subspace of $\mathbb{R}^n$. V is $\mathcal{U}$-externally detectable, if there exist smooth matrices $E_1(w), E_2(h)$ such that the following conditions hold:

a. $(A(w) + E_1(w)C)V \subset V,\ \forall w \in \mathbb{R}^{m+p}$
b. $E_2(h)CV = \{0\},\ \forall h \in \mathcal{S}$
c. The following quotient system:

$$\Sigma : \begin{cases} \dot{\xi}(t) = \overline{(A(w(t)) + E_1(w(t))C + E_2(h(t))C)}^V \xi(t) \\ \dot{h}(t) = H(h(t), w(t)),\ (\xi, h) \in (\mathbb{R}^n/V) \times \mathcal{S}. \end{cases} \tag{8}$$

is such that the subset $\{0\} \times \mathcal{S}$ of $\mathbb{R}^n/V \times \mathcal{S}$ attracts every trajectory of system (8).

Let $\mathcal{D}_{ext}(\mathcal{U}_W, W)$ be the set of all $\mathcal{U}_W$-externally detectable subspaces containing a subspace W and admits a smallest element $\hat{W}$ with respect to $\subset$, then we have:

Lemma 1.
Given a subspace W of $\mathbb{R}^n$, $\mathcal{D}_{ext}(\mathcal{U}_W, W)$ is not empty.

Now we can state our main result:

Theorem 1.
Let $\Gamma : \mathbb{R}^n \to \mathbb{R}^r$ be a linear map and let $K(x)$ be the matrix associated to the unknown inputs of system (1). Let W be the vector subspace of $\mathbb{R}^n$ spanned by the family $\{K^j(x);\ 1 \le j \le l;\ x \in \mathbb{R}^n\}$, where $K^j(x)$ is the jth column of $K(x)$. Let $\widehat{W}$ be the smallest element of $\mathcal{D}_{ext}(\mathcal{U}_W, W)$, and P a linear projection from $\mathbb{R}^n$ onto $\mathbb{R}^q$ such that $Ker(P) = \widehat{W}$. Let Ω be an open subset of $\mathbb{R}^n$, and assuming that $\mathcal{U} = \{u(.)/(u(.), y(.)) \in \mathcal{U}_W; \forall x(0) \in \Omega\}$ is not empty, where $y(.)$ is the output of system (1) associated to $(x(0), u(.))$, then:

If $\hat{W} \cap Ker(C) \subset Ker(\Gamma)$ then system (1) admits an $\mathcal{U}$-unknown input observer of the form (2) which exponentially estimates $\Gamma(x(t))$ for every $x(0) \in \Omega$.

Proof of lemma 1:
It suffices to show that $W^{**} \in \mathcal{D}_{ext}(\mathcal{U}_W, W)$. From above, we know that $(A(w)+E^{**}(w)C)W^{**} \subset W^{**}$. Thus using a simple change of coordinate, we can transform the system:

$$\begin{cases} \dot{x}(t) = (A(w(t)) + E^{**}(w(t))C)x(t) \\ y^{**}(t) = N^{**}Cx(t) \end{cases} \tag{9}$$

into the form:

$$\begin{cases} \dot{x}_1(t) = A_{11}(w(t))x_1(t) \\ \dot{x}_2(t) = A_{21}(w(t))x_1(t) + A_{22}(w(t))x_2(t) \\ y^{**}(t) = C_2x_2(t),\ (x_1, x_2) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \end{cases} \tag{10}$$

So that system (7) becomes equivalent to:

$$\begin{cases} \dot{x}_2(t) = A_{22}(w(t))x_2(t) \\ y^{**}(t) = C_2x_2(t) \end{cases} \tag{11}$$

¿From remark 3, $\mathcal{U}_W$ is then the set of inputs which renders system (11) completely uniformly observable. Now using the observer stated in Bornard et al. [1989], Hammouri and Morales [1990], we deduce that:

$$\begin{cases} \dot{\widehat{x}}_2(t) = A_{22}(w(t))\widehat{x}_2(t) - S^{-1}(t)C_2^TR(C_2\widehat{x}_2(t) - y^{**}(t)) \\ \dot{S}(t) = -\theta S(t) - A_{22}^T(w(t))S(t) - S(t)A_{22}(w(t)) \\ \quad +C_2^TRC_2 \\ S(0),\ R \text{ are SPD matrices} \\ \theta > 0 \ \text{ is a scalar parameter} \end{cases} \tag{12}$$

forms an exponential observer which converges for every input which renders system (11) completely uniformly observable. Namely, for every $w \in \mathcal{U}_W$, the following system:

$$\begin{cases} \dot{e}(t) = (A_{22}(w(t)) - S^{-1}(t)C_2^TRC_2)e(t) \\ \dot{S}(t) = -\theta S(t) - A_{22}^T(w(t))S(t) - S(t)A_{22}(w(t)) \\ \quad +C_2^TRC_2 \end{cases} \tag{13}$$

is such that $\{0\} \times \mathbb{R}^{n_2}$ attracts every trajectory of (13). By construction, system (13) can be seen as a quotient space of the form (8). This ends the proof of the lemma.

The following proposition plays a significant role in the construction of the unknown input observer and shows the relationship between the notion of unknown input observer and the external detectability one.
In the sequel $\mathcal{U}_V$ is the set of inputs which renders system completely uniformly observable.

Proposition 2.
Let P and Γ be linear maps from $\mathbb{R}^n$ into $\mathbb{R}^q$ and $\mathbb{R}^r$ respectively. Assume that there exist constant matrices F and D such that $\Gamma = FP + DC$, and that $\forall x \in \mathbb{R}^n$, $PK(x) = 0$, where $K(x)$ is the matrix associated to unknown inputs of system (1).
Assuming that $V = Ker(P)$ is $\mathcal{U}_V$-externally detectable and that $\mathcal{U} = \{u(.)/(u(.), y(.)) \in \mathcal{U}_V,\ \forall x(0) \in \Omega\}$ is not empty. Then system (1) admits an $\mathcal{U}-$unknown input observer of the form (2).

Proof of Proposition 2
Let $V = Ker(P)$ is $\mathcal{U}$-externally detectable. Then, from definition 4, there exist $E_1(w), E_2(h)$ and $H(h, w)$ satisfying:

a) $(A(w) + E_1(w)C)V \subset V, \forall w \in \mathbb{R}^{m+p}$
b) $E_2(h)CV = \{0\}, \forall h \in \mathcal{S}$
c) The following quotient system:

$$\Sigma : \begin{cases} \dot{\xi}(t) = \overline{(A(w(t)) + E_1(w(t))C + E_2(h(t))C)}^V \xi(t) \\ \dot{h}(t) = H(h(t), w(t)),\ (\xi, h) \in (\mathbb{R}^n/V) \times \mathcal{S}. \end{cases} \tag{14}$$

is such that $\{0\} \times \mathcal{S}$ attracts every trajectory of system (14).

¿From a) above, we know that V is invariant under $(A(w) + E_1(w)C)$ and we have $PK(x(t)) = 0$. So, after a simple linear change of coordinates, system (1) takes the following form:

$$\begin{cases} \dot{x}_1(t) = A_{11}(u, y)x_1(t) + A_{12}(u, y)x_2(t) + B_1(u, y) \\ \quad +k_1(x(t))v \\ \dot{x}_2(t) = A_{22}(u, y)x_2(t) + B_2(u, y) \\ y(t) = y_1(t) + y_2(t) \\ y_1(t) = C_1x_1(t); \quad y_2(t) = C_2x_2(t) \end{cases} \tag{15}$$

where $(x_1, x_2) \in \mathbb{R}^{n-q} \times \mathbb{R}^q$, and P can be seen as the projection $P(x_1, x_2) = x_2$. Moreover in the coordinates system (x_1, x_2) , system (15) in which w is replaced by (u, y) can be represented by the following system:

$$\Sigma : \begin{cases} \dot{e}(t) &= (A_{22}(u,y) + E_1^2(u,y)C_2 + E_2^2(h)C_2)e(t) \\ \dot{h}(t) &= H(h,u,y) \end{cases} \tag{16}$$

The candidate $\mathcal{U}-$unknown input observer which estimates $\Gamma(x(t))$ takes the following form:

$$\Sigma : \begin{cases} \dot{z}(t) &= (A_{22}(u,y) + E_1^2(u,y)C_2 + E_2^2(h)C_2)z(t) \\ & \quad +B_2(u,y) - E_1^2(u,y)y_2 - E_2^2(h)y_2 \\ \dot{h}(t) &= H(h,u,y) \\ \gamma(t) &= Fz(t) + Dy(t) \end{cases} \tag{17}$$

Indeed, setting $e(t) = z(t) - Px(t) = z(t) - x_2(t)$, it follows that $e(t)$ satisfies equations (16), and from above $e(t)$ converges to 0 for every $(u(t), y(t)) \in \mathcal{U}_V$. Finally, using expression $\Gamma = FP + DC$, we deduce that $\Gamma(x(t)) - \gamma(t)$ converges to 0.

Proof of Theorem 1

Let $\hat{W}$ be the smallest element of $\mathcal{D}_{ext}(\mathcal{U}_W, W)$, Γ and P be a linear map from $\mathbb{R}^n$ onto $\mathbb{R}^r$ and $\mathbb{R}^q$ respectively such that $Ker(P) = \hat{W}$.

Assume that $\hat{W} \cap Ker(C) \subset Ker(\Gamma)$ and let us show that system (1) admits an $\mathcal{U}-$unknown input observer.

Clearly, $\hat{W} \cap Ker(C) \subset Ker(\Gamma)$, implies the existence of constant matrices F and D such that $\Gamma = FP + DC$.

Now assuming that $\mathcal{U} = \{u(.)/(u(.), y(.)) \in \mathcal{U}_W; \forall x(0) \in \Omega\}$ is not empty and using the fact that $\hat{W}$ is an $\mathcal{U}_W$-externally detectable subspace containing W and using the fact that $PK(x) = 0$, $\forall x$ (since $Ker(P) = \hat{W}$ and $W \subset \hat{W}$), then from proposition 2, system (3) admits an $\mathcal{U}-$unknown input observer.

4. NUMERICAL EXAMPLE

Consider the following system:

$$\begin{cases} \dot{x}(t) &= A(u(t), y(t))x(t) + B(u(t), y(t)) + Kx(t)v(t) \\ y(t) &= Cx(t) \end{cases} \tag{18}$$

where $A(u,y) = \begin{pmatrix} uy_1 & uy_1 & 0 & -1 & uy_1 \\ 0 & 0 & -1 & uy_1 & 0 \\ uy_1 & 0 & 0 & uy_1 & 0 \\ -uy_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & uy_1 & -1 \end{pmatrix}$,

$B(u,y) = \begin{pmatrix} -(1+y_1^4)y_1 \\ 0 \\ -(1+y_2^4)y_2 + uy_1 \\ u \\ 0 \end{pmatrix}$,

$C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$ and $K = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$. u and v are respectively the known and unknown inputs.

Setting $\Gamma(x) = (x_1, x_3, x_4, x_5)$, in the sequel, we will show that $\Gamma(x(t))$ can be estimated. First, we will show that assumption of theorem 1 is satisfied, next, we give the filter which will be able to estimate $\Gamma(x(t))$.

Let W be the vector subspace spanned by the columns of K.

− Computation of W^* and $E^*(u, y)$:

Applying 2) of definition 2, it is easy to see that $W^* = W$. To calculate $E^*(u, y)$, such that $(A(u, y) + E^*(u, y)C)W^* \subset W^*$, it suffices to solve $[W^{*\perp}]^T(A(u, y) + E^*(u, y)C)[W^*] = 0$, $\forall(u, y)$, $[.]^\perp$ stands from the perpendicular space of [.]. A simple calculation gives: $E^*(u, y) = \begin{pmatrix} e_{11}^* & e_{12}^* \\ e_{21}^* & e_{22}^* \\ -uy_1 & e_{32}^* \\ uy_1 & e_{42}^* \\ 0 & e_{52}^* \end{pmatrix}$.

− Computation of W^{**}, $E^{**}(u, y)$ and N^{**} which appear in system (6):

Applying algorithm (5), we get:

$$[W^{**}] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

As above, $E^{**}(u, y)$ can be obtained by solving $[W^{**\perp}]^T(A(u, y) + E^{**}(u, y)C)[W^{**}] = 0$ $\forall(u, y)$:

$$E^{**}(u,y) = \begin{pmatrix} e_{11}^{**} & e_{12}^{**} \\ e_{21}^{**} & e_{22}^{**} \\ -uy_1 & e_{32}^{**} \\ uy_1 & e_{42}^{**} \\ e_{51}^{**} & e_{52}^{**} \end{pmatrix}.$$

Thus, we can choose for instance $E^*(u, y) = E^{**}(u, y) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -uy_1 & 0 \\ uy_1 & 0 \\ 0 & 0 \end{pmatrix}$.

− Computation of N^{**}:

We solve $N^{**}C[W^{**}] = 0$, which yields to $N_1^{**} = 0$. Thus, one can take $N^{**} = (0\ 1)$, or $N^{**}C = (0\ 0\ 1\ 0\ 0)$.

Hence $(A(u, y) + E^*(u, y)C)$ takes the following form:

$$\begin{pmatrix} uy_1 & uy_1 & 0 & -1 & uy_1 \\ 0 & 0 & -1 & uy_1 & 0 \\ 0 & 0 & 0 & uy_1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & uy_1 & -1 \end{pmatrix}$$

In our case $E^{**} = E^*$, now, it is easy to see that the following quotient system:

$$\begin{cases} \dot{\varepsilon} = \overline{(A(u,y) + E^{**}(u,y)C)}^{W^*} \varepsilon \\ \varepsilon \in \mathbb{R}^5/W^* \equiv \mathbb{R}^3 \end{cases} \quad (19)$$

$$\begin{cases} \dot{\xi} &= \overline{(A(u,y) + E^{**}(u,y)C)}^{W^{**}} \xi \\ Y &= \overline{N^{**}C}\xi \\ \xi &\in \mathbb{R}^5/W^{**} \equiv \mathbb{R}^2 \end{cases} \quad (20)$$

can be respectively represented by:

$$\dot{\varepsilon} = \begin{pmatrix} 0 & uy_1 & 0 \\ 0 & 0 & 0 \\ 1 & uy_1 & -1 \end{pmatrix} \varepsilon \quad (21)$$

$$\begin{cases} \dot{\xi} &= \begin{pmatrix} 0 & uy_1 \\ 0 & 0 \end{pmatrix} \xi \\ Y &= [1 \quad 0]\xi \end{cases} \quad (22)$$

Setting $\mathcal{U}$ the set of bounded inputs $u(.)$ such that $u(t)y_1(t)$ renders system (22) completely uniformly observable. Setting $\bar{A}(u,y) = \begin{pmatrix} 0 & uy_1 \\ 0 & 0 \end{pmatrix}$ and $\bar{C} = (1 \quad 0)$, and consider the following system:

$$\begin{cases} \dot{\xi} &= \begin{pmatrix} 0 & uy_1 \\ 0 & 0 \end{pmatrix} \xi - S^{-1}\bar{C}^T\bar{C}\xi \\ \dot{S} &= -\theta S - \bar{A}^T(u,y)S \\ &\quad -S\bar{A}(u,y) + \bar{C}^T\bar{C} \end{cases} \quad (23)$$

where $S(0)$ is a SPD matrix and θ is a constant parameter. ¿From Bornard et al. [1989], Hammouri and Morales [1990] (see system (12)), if $u(.) \in \mathcal{U}$, then $\xi(t)$ exponentially converges to 0.

In what follows, we will show that $\widehat{W} = W^*$. To do so, let us calculate $E_1(w)$, $E_2(h)$, $H(h,w)$ satisfying conditions of definition 4:

- $E_1(w) = E^{**}(w)$, (here $w = (u,y)$).
- Setting $h = S$, and $E_2(h) = \begin{pmatrix} 0 \\ 0 \\ -S^{-1}C^T(N^{**})^T \\ 0 \end{pmatrix}$.
- $H(h,w) = -\theta S - \bar{A}^T(w)S - S\bar{A}(w) + \bar{C}^T\bar{C}$, moreover, if $S(0)$ is SPD, then the solution $S(t)$ remains SPD for every t. In the sequel, we set $\mathcal{S}$ to be the set of 2×2 SPD matrices (it is an open subset of the vector space of symmetric matrices).

Clearly, the quotient system:

$$\Sigma : \begin{cases} \dot{x} = \overline{(A(w) + E_1(w)C + E_2(h)C)}^{\widehat{W}} x \\ \dot{h} = H(h,w), \ (x,h) \in (\mathbb{R}^5/\widehat{W}) \times \mathcal{S}. \end{cases} \quad (24)$$

can be represented as follows:

$$\begin{cases} \dot{\varepsilon} = \begin{pmatrix} 0 & uy_1 & 0 \\ 0 & 0 & 0 \\ 1 & uy_1 & -1 \end{pmatrix} \varepsilon + \begin{pmatrix} -S^{-1}\bar{C}^T\varepsilon_1 \\ 0 \end{pmatrix} \\ \dot{S} = -\theta S - \bar{A}^T(w)S - S\bar{A}(w) + \bar{C}^T\bar{C} \end{cases} \quad (25)$$

On one hand, the two first equations of (25) are the same of those of (23). Hence, $(\varepsilon_1(t), \varepsilon_2(t))$ converges to 0. On the other hand, $u(.)y_1(.)$ is bounded and the third equation of (25) is stable with respect to ε_3. Consequently, $\lim_{t\to\infty} \varepsilon(t) = 0$.

Obviously, $\widehat{W} = W^* = W$ is the smallest $\mathcal{U}-$externally detectable subspace containing W.

Moreover, it is obvious to see that $\hat{W} \cap Ker(C) \subset Ker(\Gamma)$. Finally, setting P to be the projection $P(x) = (x_3, x_4, x_5)$, thus we have $\hat{W} = Ker(P)$, and the assumption of theorem 1 is then satisfied.

The $\mathcal{U}$-unknown input observer which estimates $\Gamma(x(t))$ takes the form:

$$\begin{cases} \dot{z} &= \begin{pmatrix} 0 & uy_1 & 0 \\ 0 & 0 & 0 \\ 1 & uy_1 & -1 \end{pmatrix} z + \begin{pmatrix} -(1 + y_2^4)y_2 + uy_1 \\ u \\ 0 \end{pmatrix} \\ &\quad - \begin{pmatrix} S^{-1} & 0 \\ 0 & 0 \end{pmatrix} \tilde{C}^T(\tilde{C}z - y_2) \\ \dot{S} &= -\theta S - \bar{A}^T(w)S - S\bar{A}(w) + \bar{C}^T\bar{C} \\ \gamma(t) &= Fz(t) + Dy(t) = \begin{pmatrix} y_1(t) \\ z_1(t) \\ z_2(t) \\ z_3(t) \end{pmatrix} \end{cases} \quad (26)$$

with $F = \begin{pmatrix} \underline{0} \\ I_3 \end{pmatrix}$, $D = \begin{pmatrix} [1 \quad 0] \\ \underline{0} \end{pmatrix}$ and $\tilde{C} = [1 \quad 0 \quad 0]$ where $\underline{0}$ means the zero matrix of adequate dimension, and I_3 the 3×3 identity matrix.

Recall that $P(x) = (x_3, x_4, x_5)$, from the above construction, it is clear that conditions i), ii) of definition 1 are fulfilled.

The performances of the proposed observer are evaluated in simulation. We have considered a disturbance on the two first states of (18) and we have estimated the states x_3, x_4 and x_5.

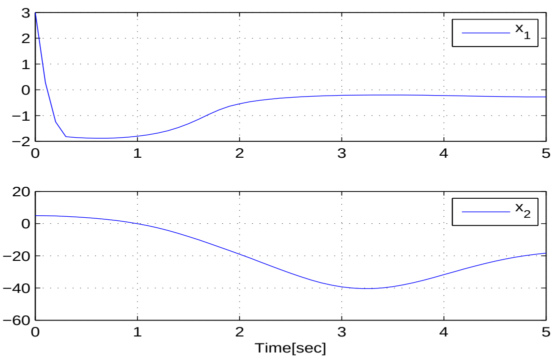

Fig. 1. States evolution of x_1 and x_2

Fig.1 shows the evolution of states x_1 and x_2 which are affected by the unknown inputs.

The results of the observer are depicted on Fig.2 which shows that such observer provides an appropriate state estimator despite of the presence of the unknown inputs.

5. CONCLUSIONS

In this paper, the design of an unknown input observer for state affine systems up to output injection was considered. The proposed observer is stable and estimates the unknown state or a part. A sufficient condition for the existence was given using a geometric approach. An analysis has been performed using the concept of external detectability to construct the largest detectable system which is not affected by the unknown inputs.

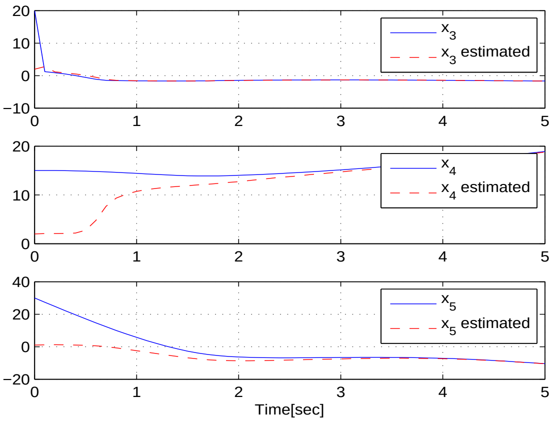

Fig. 2. Evolution of states x_3, x_4 and x_5 with their estimates.

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