Optimal Control with Input Constraints applied to Internal Combustion Engine Test Benches

Thomas E. Passenbrunner * **Mario Sassano** † **Luigi del Re** *

* *Institute for Design and Control of Mechatronical Systems, Johannes Kepler University Linz, 4040 Linz, Austria (e-mail: {thomas.passenbrunner,luigi.delre}@jku.at)*
† *Dipartimento di Ingegneria Civile e Ingegneria Informatica, Università di Roma "Tor Vergata", Rome, Italy (e-mail: mario.sassano@uniroma2.it)*

Abstract: Optimal control of nonlinear systems provides a major challenge in control engineering. Constraints on the input signals are common to many real-world applications and render the problem to be tackled even more complicated. This paper proposes a method to map the input constraints by nonlinear functions to the state equations of the system, afterwards an approximation of the solution of the resulting optimization problem is calculated by means of a dynamic extension to the state of the system. The approach is applied in the control of test benches for internal combustion engines, where speed and torque references need to be tracked at the crankshaft of the engine. Simulation results using a high-quality simulator, also regarding effects that have not been included in the model for controller design, show the performance of the proposed approach.

1. INTRODUCTION

An optimal control problem consists in determining a control input such that a desired cost functional is minimized, or maximized, along the trajectories of the resulting closed-loop system. A standard approach to the solution of the problem hinges upon the solution of a first-order nonlinear partial differential equation, see e.g. Anderson and Moore [1989], Bertsekas [2005], Bryson and Ho [1975]. The explicit solution of the Hamilton-Jacobi-Bellman (HJB) partial differential equation may be hard or even impossible to determine in practical cases. Therefore, several methodologies to approximate the solution of the HJB partial differential equation in a neighborhood of the origin with a desired degree of accuracy have been proposed, see e.g. Hunt and Krener [2010], Lukes [1969], McEneaney [2007]. A novel approach to approximate the solution of the HJB partial differential equation by means of a dynamic extension is presented in Sassano and Astolfi [2012] and successfully applied to an internal combustion engine test bench in Passenbrunner et al. [2011] and the air path of a turbocharged Diesel engine in Sassano et al. [2012].

The problem is exacerbated in many real-world applications by input constraints. While the problem of control of systems with bounded inputs has been extensively addressed in the past (see e.g. Aangenent et al. [2012], Lin [1998]), optimal control of such systems has been rarely approached. In this paper the limitations of the actuators are mapped by nonlinear functions in such a way that the state equations exhibit, after further manipulations, the structure required for the application of the technique presented in Sassano and Astolfi [2012]. This technique allows to approximate the solution of the arising HJB partial differential equation.

In particular, this approach is applied to develop a control for an internal combustion engine test bench. The operation of an internal combustion engine in a vehicle is simulated at a test bench without this vehicle. Test benches are advantageous because of reproducibility and often reduced time required for development and configuration and therefore reduced costs. Indeed, both the load – in a vehicle, being the direct consequence of the road and vehicle conditions – as well as the engine speed have to be computed and enforced by a dynamometer at a test bench. Usually, the dynamometer is an electric dynamometer and two separate control loops are employed to control the actuators in industry.

The significance of experiments at a test bench is a direct consequence of the precision of the control system. Therefore, the subject has received attention in different ways. A digital controller for a turbocharged Diesel engine as well as a direct current dynamometer using a closed-loop pole assignment technique was developed in Tuken et al. [1990]. The model reference adaptive control approach using Lyapunov stability theory to derive the parameters udpate law is applied to the engine speed and torque control problem in Yanakiev [1998]. Multivariable controls of the engine-dynamometer system have become increasingly popular in recent times. The closed loop reference tracking is maximized by balancing the bandwidths of the loop transfer functions in Bunker et al. [1997]. In Gruenbacher and del Re [2008] a robust inverse tracking method is applied to control an internal combustion engine test bench achieving a high tracking performance. The inverse optimal control problem – which consists in fixing the structure of the solution of the HJB partial differential equation and then computing the actual cost that is optimized by the resulting control law – is solved in Gruenbacher et al. [2008]. In Passenbrunner et al. [2011] an approximation of the solution of the optimization problem is calculated for a test bench, however without considering input constraints.

The input constraints are mppaed to the state equations and subsequently a multi-input multi-output controller taking these constraints into account for an internal combustion engine test bench is proposed in this work. The control law is implemented on an accurate model of the test bench showing good performance in simulation. Although, the description of the system used for control design is linear in this paper and the optimal control problem for a linear system with box constraints might also be addressed in a different way, it is apparent that a linear system description is not necessary and the consideration of nonlinearities in the system is straightforward.

The paper is organized as follows: Section 2 directs attention to internal combustion engine test benches and the problem formulation, Section 3 deals with the dynamic control. Simulation results using a test bench simulator taking e.g. measurement noise, combustion oscillations and limitations into account are presented in Section 4. The paper is concluded with comments on the proposed methodology and a future outlook in Section 5.

2. PROBLEM FORMULATION

The typical setup of an internal combustion engine test bench is depicted in Figure 1. The engine under test is connected via a flexible shaft to a second actuator, the dynamometer. The accelerator pedal position α and the set value $T_{D,set}$ of the dynamometer torque T_D provide the inputs of the test bench. The engine speed ω_E, the dynamometer speed ω_D and the dynamometer torque T_D can be measured, the shaft torque T_{ST} can either be measured or estimated, the engine torque T_E can be estimated from available measurements, see e.g. Passenbrunner et al. [2012].

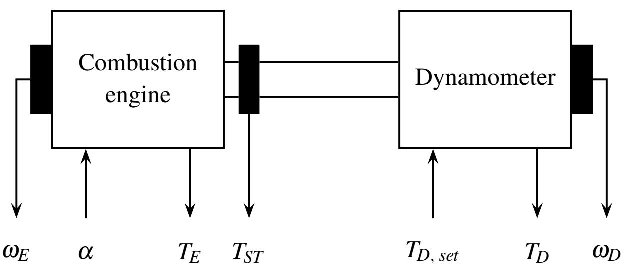

Fig. 1. Typical setup of an internal combustion engine test bench.

The entire mechanical part of the test bench can be modeled by a two-mass-oscillator as follows

$$\begin{aligned} \theta_E \dot{\omega}_E &= T_E - T_{ST}, \\ \theta_D \dot{\omega}_D &= T_{ST} - T_D, \\ \Delta\dot{\varphi} &= \omega_E - \omega_D \end{aligned} \tag{1}$$

with the shaft torque T_{ST} determined as

$$T_{ST} = c\Delta\varphi + d\left(\omega_E - \omega_D\right), \tag{2}$$

where θ_E and θ_D are the inertias of the internal combustion engine and the dynamometer, respectively. The inertias of the connecting shaft and the measurement flange have been already included in these values. The constant c is the stiffness of the connecting shaft and d denotes the damping of the connecting shaft.

In the following we assume that the intrinsic nonlinearities of the internal combustion engine can be compensated. This compensation can be achieved, for instance, by the parametrization of the engine control unit or by an inversion of the nonlinearities. The dynamic relation between the accelerator pedal position α and the engine torque T_E can be modeled, in the Laplace domain, as a first order low-pass filter

$$G_{\alpha\to T_E}(s) = \frac{k_E}{s\delta_E + 1}$$

with δ_E being the time constant of the filter, k_E the stationary gain and s the Laplace variable.

Assume additionally that a controller is available that enforces a certain desired behavior induced by the choice of the reference value $T_{D,set}$ to the electric dynamometer. This can be guaranteed by introducing the following transfer function from the desired dynamometer torque $T_{D,set}$ to the actual dynamometer torque T_D

$$G_{T_{D,set}\to T_D}(s) = \frac{1}{s\delta_D + 1},$$

where δ_D denotes the time constant of the resulting first order transfer function. Usually, the electric dynamometer builds up torque much faster than an internal combustion engine, hence $\delta_E \approx 4\delta_D$.

The main objective of the simulation of an internal combustion engine on a test bench is to track given profiles of engine speed ω_E and shaft torque T_{ST}. Therefore, a coordinate transformation (see (2)) is done and in the state-space representation of the system the shaft torque T_{ST} is consequently used instead of the torsion $\Delta\varphi$ of the shaft. The resulting dynamical system is described by equations of the form

$$\begin{aligned} \dot{x} &= Ax + B\tilde{u}, \\ y &= Cx \end{aligned} \tag{3}$$

where the state $x = \begin{bmatrix}\omega_E & \omega_D & T_{ST} & T_E & T_D\end{bmatrix}^\top = \begin{bmatrix}x_1 & x_2 & x_3 & x_4 & x_5\end{bmatrix}^\top$ is a vector in $\mathbb{R}^5$, the input $\tilde{u} = \begin{bmatrix}\alpha & T_{D,set}\end{bmatrix}^\top = \begin{bmatrix}\tilde{u}_1 & \tilde{u}_2\end{bmatrix}^\top$ belongs to $\mathbb{R}^2$ and the system matrices are defined as

$$A = \begin{bmatrix} 0 & 0 & -\frac{1}{\theta_E} & \frac{1}{\theta_E} & 0 \\ 0 & 0 & \frac{1}{\theta_D} & 0 & -\frac{1}{\theta_D} \\ c & -c & -d\frac{\theta_E+\theta_D}{\theta_E\theta_D} & \frac{d}{\theta_E} & \frac{d}{\theta_D} \\ 0 & 0 & 0 & -\frac{1}{\delta_E} & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{\delta_D} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & 0 & \frac{k_E}{\delta_E} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\delta_D} \end{bmatrix}^\top,$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Due to physical limitations of the available actuators, constraints on the control actions must necessarily be taken into account during the control design process. In particular, these constraints can be expressed as

$$\begin{aligned} 0 \le \tilde{u}_1 &\le \overline{u}_1, \\ -\overline{u}_2 \le \tilde{u}_2 &\le \overline{u}_2. \end{aligned} \tag{4}$$

The specific values of the constraints, namely $\overline{u}_1$ and $\overline{u}_2$, may vary for different test benches or settings. For a Diesel passenger car coupled with an electric dynamometer of similar power, the relation $\overline{u}_1 k_E \approx \overline{u}_2$ can be considered as an accurate approximation to determine these values. The interpretation of the constraints in (4) is given as follows: On one hand, the accelerator pedal position α ranges from 0% to 100%. Moreover, the engine torque T_E is positive and increasing with α and it may only get negative for very small values of α. On the other hand, the dynamometer torque T_D can be positive and negative within some boundaries which are mainly governed by thermal losses. Therefore, the electric machine can indifferently brake as well as drive the internal combustion engine.

As a first step towards the design of an optimal control law, a nonlinear transformation of the inputs of the system is considered to deal with input constraints and asymptotically stable and sufficiently fast filters are introduced. The previous approximations are described in details and are properly motivated in the remaining part of this section.

As far as the first source of approximation is concerned, we introduce the following nonlinear functions of the control inputs, namely

$$\tilde{u}_1 = \frac{\overline{u}_1+\varepsilon}{2}\frac{2}{\pi}\arctan\left\{u_1 - \tan\left\{\frac{\pi}{2}\frac{\overline{u}_1-\varepsilon}{\overline{u}_1+\varepsilon}\right\}\right\} + \frac{\overline{u}_1-\varepsilon}{2},$$
$$\tilde{u}_2 = \overline{u}_2\frac{2}{\pi}\arctan\{u_2\}$$

and, conversely,

$$u_1 = \tan\left\{\frac{2}{\overline{u}_1+\varepsilon}\frac{\pi}{2}\left(\tilde{u}_1 - \frac{\overline{u}_1-\varepsilon}{2}\right)\right\} + \tan\left\{\frac{\pi}{2}\frac{\overline{u}_1-\varepsilon}{\overline{u}_1+\varepsilon}\right\},$$
$$u_2 = \tan\left\{\frac{1}{\overline{u}_2}\frac{\pi}{2}\tilde{u}_2\right\},$$

respectively. Note that the arc tangent function has been chosen as it is well suited to approximate a saturation. Other functions may be employed as well, however, a case distinction depending on the input might be needed. As detailed above, the constraints on the input u_2 can easily be taken into account. As a consequence, the input $\tilde{u}_2$ of the system is bounded and reaches the limits only if the transformed input u_2 diverges to infinity.

The situation is slightly different for the input $\tilde{u}_1$. The mapping is somewhat more complicated, as it must be possible to set this input to zero. The constant ε generates a very small offset that allows to adapt the transformed input to the constraints and to enforce $\tilde{u}_1 = 0$ whenever $u_1 = 0$.

In order to obtain a more tractable partial differential equation when dealing with an optimal control problem, we restrict our attention to input-affine nonlinear systems. Towards this end, the state x of system (3) is extended by the additional state z as follows

$$\begin{bmatrix}\dot{x}\\ \dot{z}\end{bmatrix} = f(x,z) + g(x,z)\,u \qquad y = C\,x \tag{5}$$

with

$$f_1(x,z) = \frac{1}{\theta_E}(-x_3+x_4)$$
$$f_2(x,z) = \frac{1}{\theta_D}(x_3-x_5)$$
$$f_3(x,z) = c(x_1-x_2) - d\frac{\theta_E+\theta_D}{\theta_E\theta_D}x_3 + \frac{d}{\theta_E}x_4 + \frac{d}{\theta_D}x_5$$
$$f_4(x,z) = \frac{k_E}{\delta_E}\frac{\overline{u}_1+\varepsilon}{\pi}\arctan\left\{z_1 - \tan\left\{\frac{\pi}{2}\frac{\overline{u}_1-\varepsilon}{\overline{u}_1+\varepsilon}\right\}\right\} + \frac{k_E}{\delta_E}\frac{\overline{u}_1-\varepsilon}{2} - \frac{1}{\delta_E}x_4$$
$$f_5(x,z) = \frac{1}{\delta_D}\overline{u}_2\frac{2}{\pi}\arctan\{z_2\} - \frac{1}{\delta_D}x_5$$
$$f_6(x,z) = -\frac{1}{\varepsilon_E}z_1$$
$$f_7(x,z) = -\frac{1}{\varepsilon_D}z_2$$

and

$$g(x,z) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{\varepsilon_E} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\varepsilon_D}\end{bmatrix}^{\top},$$

where $x(t) \in \mathbb{R}^5$, $z(t) \in \mathbb{R}^2$ and ε_E and ε_D describe the time constants of asymptotically stable and sufficiently fast filters. In other words, we suppose that the control inputs do not act instantaneously on system (3) but *via* additional dynamics.

Before formally presenting the control problem approached herein, a cost functional is associated to system (5), namely

$$J(u) = \frac{1}{2}\int_0^\infty \Big(q(x(t),z(t)) + u(t)^\top u(t)\Big)\,\mathrm{d}t, \tag{6}$$

where $q\colon \mathbb{R}^7 \to \mathbb{R}_+$ is a positive semi-definite continuous function. As it will appear evident in the following the choice of limiting the approach to input-affine systems is motivated by the possibility of easily computing the minimum of the HJB partial differential equation with respect to the control input. In fact, a different choice would lead to a different, and possibly more complicated, expression for the HJB partial differential equation.

Moreover, the quadratic penalty term $u^\top u$ in (6) hinders the input u from reaching the boundaries of the constraints (4) imposed on the control inputs, hence ruling out bang-bang solutions from the set of admissible optimal solutions. In our practical framework, however, this approximation is reasonable as a bang-bang solution would force the actuators to work continuously at the limit of their possibilities.

Finally, the filters introduced above describe the effect of unmodeled dynamics and delays that are always present in real world applications. As a matter of fact, an optimal solution imposing instantaneous changes of the control action would be of limited practical use.

The following problem is tackled in this paper:

Problem 1. Given the dynamical system (5), determine a control law u such that the cost functional (6) is minimized along the trajectories of the resulting closed-loop system.

The interest of Problem 1 lies in the fact that, as explained and motivated above, the optimal solution u of Problem 1 provides, via inverse transformations, an accurate approximation of the optimal control law $\tilde{u}$ for the original system (3) with input constraints (4).

3. DYNAMIC CONTROL

Consider a nonlinear system, affine in the control, described by equations of the form

$$\dot{x} = f(x) + g(x)\,u \tag{7}$$

with $f\colon \mathbb{R}^n \to \mathbb{R}^n$ and $g\colon \mathbb{R}^n \to \mathbb{R}^{n\times m}$ smooth mappings, where $x(t) \in \mathbb{R}^n$ denotes the state of the system and $u(t) \in \mathbb{R}^m$ the input. The task of the control is to minimize a cost functional similar to (6), namely

$$J(u(t)) = \frac{1}{2}\int_0^\infty \left(q(x(t)) + u(t)^\top u(t)\right) \mathrm{d}t, \tag{8}$$

where $q\colon \mathbb{R}^n \to \mathbb{R}_+$ is a positive semi-definite continuous function, subject to the dynamical constraint (7), the initial condition $x(t_0) = x_0$ and the requirement that the zero equilibrium of the closed-loop system be locally asymptotically stable. Herein we consider the problem of approximating the solution of the regional dynamic optimal control problem, the definition of which is given in the following statement.

Problem 2. Consider system (7) and the cost functional (8). The regional dynamic optimal control problem consists in determining an integer $\tilde{n} \geq 0$, a dynamic control law of the form

$$\begin{aligned}\dot{\xi} &= \alpha(x,\xi),\\ u &= \beta(x,\xi)\end{aligned}$$

with $\xi(t) \in \mathbb{R}^{\tilde{n}}$, $\alpha\colon \mathbb{R}^n \times \mathbb{R}^{\tilde{n}} \to \mathbb{R}^{\tilde{n}}$, $\beta\colon \mathbb{R}^n \times \mathbb{R}^{\tilde{n}} \to \mathbb{R}^m$ and a set $\hat{\Omega} \subset \mathbb{R}^n \times \mathbb{R}^{\tilde{n}}$ containing the origin of $\mathbb{R}^n \times \mathbb{R}^{\tilde{n}}$ such that the closed-loop system

$$\begin{aligned}\dot{x} &= f(x) + g(x)\beta(x,\xi),\\ \dot{\xi} &= \alpha(x,\xi)\end{aligned} \tag{9}$$

has the following properties:

(i) The zero equilibrium of system (9) is asymptotically stable with region of attraction containing $\hat{\Omega}$.

(ii) For any $\hat{u}(t)$ and any (x_0, ξ_0) such that the trajectory of system (9) remains in $\hat{\Omega}$

$$J(\beta) \leq J(\hat{u}).$$

It is well-known that if the scalar function $V\colon \mathbb{R}^n \to \mathbb{R}_+$ is a solution of the HJB partial differential equation

$$V_x f(x) - \frac{1}{2}V_x g(x) g(x)^\top V_x^\top + \frac{1}{2}q(x) = 0, \tag{10}$$

then the static state feedback $u_o = -g(x)^\top V_x^\top$ solves the regional dynamic optimal control with $\tilde{n} = 0$. Unfortunately, the explicit solution of the HJB partial differential equation may be hard or impossible to find. Therefore, we consider herein a different notion of the solution of (10).

Definition 1. Consider system (7). Let $\sigma(x) \triangleq x^\top \Sigma(x) x > 0$, for all $x \in \mathbb{R}^n \setminus \{0\}$, with $\Sigma(x)\colon \mathbb{R}^n \to \mathbb{R}^{n\times n}$. A $\mathscr{C}^1$ mapping $P(x)\colon \mathbb{R}^n \to \mathbb{R}^{1\times n}$, $P(0) = 0$, is said to be an algebraic $\overline{P}$ solution of (10) if

$$P(x) f(x) - \frac{1}{2}P(x) g(x) g(x)^\top P(x)^\top + \frac{1}{2}q(x) + \sigma(x) = 0,$$

and P is tangent at $x = 0$ to the symmetric positive definite solution of the algebraic Riccati equation associated with the linearized problem, i.e.

$$\left.\frac{\partial P(x)^\top}{\partial x}\right|_{x=0} = \overline{P}.$$

$\circ$

Proposition 1. Sassano and Astolfi [2012] Let P be an algebraic $\overline{P}$ solution with $\Sigma(0) > 0$. Then, there exist a matrix $R > 0$, a neighborhood of the origin $\Omega \subseteq \mathbb{R}^{2n}$ and $\overline{k}$ such that for all $k > \overline{k}$ the function

$$V(x,\xi) = P(\xi)x + \frac{1}{2}|x - \xi|_R^2,$$

is positive definite and satisfies the partial differential inequality

$$V_x f(x) + V_\xi \dot{\xi} - \frac{1}{2}V_x g(x) g(x)^\top V_x^\top + \frac{1}{2}q(x) \leq 0, \tag{11}$$

with $\dot{\xi} = -kV_\xi^\top$, for all $(x,\xi) \in \Omega$. $\diamond$

Clearly, the approximation with respect to the optimal solution stems from the fact that a partial differential inequality is solved in place of an equation. However, the explicit solution of the partial differential inequality is avoided by allowing for a dynamic state feedback.

Considering the problem formulated in the previous section, Proposition 1 entails that the computation of an algebraic $\overline{P}$ solution is enough to obtain the dynamic control law

$$\begin{aligned}\dot{\xi} &= -kV_\xi^\top(e,\xi),\\ u &= -B^\top V_\xi^\top(e,\xi),\end{aligned} \tag{12}$$

where e_i denotes the tracking error for the variable x_i with respect to the corresponding reference value, namely $e_i = x_i - x_{i,set}$, $i = 1,\ldots,7$, hence approximating the solution of Problem 1. Note that a closed-form expression of the algebraic solution can easily be determined, since no constraints, e.g. integrability or positivity, are imposed.

4. RESULTS

The method summarized in the previous section has been used to design a controller suited for an internal combustion engine test bench modelled as in (5) in a simulation environment. The test bench simulator has e.g. already been used to design a robust inverse control (see Gruenbacher and del Re [2008]) or an observer for torque estimation (see Passenbrunner et al. [2012]) and takes in addition to the dynamics described in (5) measurement noise, combustion oscillations and limitations of the actuators into account. The sampling time is 1 ms, this value is also used for measurements at the test bench. Note that a real-time calculation of the control law is possible for this sampling time on a rapid prototyping enviroment.

In many multi-input multi-output controls of internal combustion engine test benches the engine speed ω_E and the engine torque T_E are tracked. In stationary operating points the engine torque T_E and the shaft torque T_{ST} are equal. However, in transient operation the shaft torque T_{ST} is the torque that is transfered via clutch, transmission and the wheels to the road, while the engine torque T_E also includes the torque necessary to accelerate the inertia. Therefore, we focus on the design of a control for this configuration. If the references $\omega_{E,ref}$ of the engine speed ω_E and $T_{ST,ref}$ of the shaft torque T_{ST} jumps from one operating point to another one, the references of the states can be calculated from (3) with $\dot{x} = 0$. If the references change continuously, it is recommended to determine the references of the states from (3) using an integral control with high gain.

The operating points have to be considered at the inputs and the outputs in simulation. Furthermore, the states of the test bench are observed, see for example Passenbrunner et al. [2012] or Ortner et al. [2008].

The proposed controller taking the input constraints into account is compared with a Linear Quadratic Regulator (LQR) not taking any constraint into account (called LQR1 in the following) and an other LQR (called LQR2) which has been tuned such that the input constraints are not violated. Note that both LQR have been designed for system (3) also extended with the additional input filters, compare (5).

The proposed controller has been adjusted due to classical requirements on test bench control. LQR1 is tuned such that characteristic quantities like the amplitude of overshoots and rise times are comparable to the proposed controller. The weights of the first and the third state – the engine speed ω_E and the shaft torque T_{ST}, respectively – have been set to large values compared to the weights of the other states in all three cases, in addition the weighting of the inputs has been increasing for LQR2 with respect to the tuning of LQR1. Furthermore, note that LQR2 is a common implementation for the used test bench and shows an improved performance with respect to two separate controllers, which are frequently used in industry.

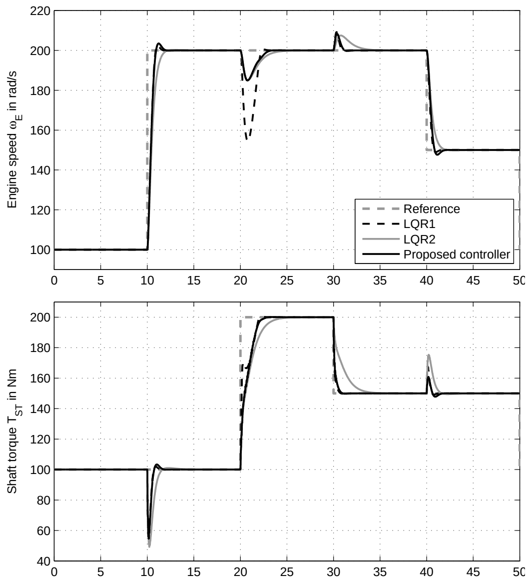

Fig. 2. Comparison of tracking of engine speed ω_E (top graph) and shaft torque T_{ST} (bottom graph) for a simple experiment using two differently tuned LQR and the proposed controller.

In Figure 2 a comparison of the tracking of the engine speed ω_E and the shaft torque T_{ST} for all three controls is shown, Figure 3 shows a cutout. The amplitude and the duration of overshoots and the rise times using LQR1 and the proposed controller are comparable. Only small differences can be found in the tracking performance. For example, when increasing the shaft torque T_{ST} while keeping the engine speed ω_E constant at $t = 20$ s, the coupling causes much larger disturbances at the engine speed ω_E. This is caused by a very aggressive control law which has been designed without a consideration of input constraints.

Comparing with LQR2 a slower tracking of the engine speed ω_E and the shaft torque T_{ST} as well as larger amplitudes and longer durations of disturbances due to couplings have to be noticed.

Figure 4 shows the time histories of the inputs – accelerator pedal position α and the set value $T_{D,set}$ of the dynamometer torque T_D. The inputs calculated by the proposed controller are within the input constraints for all times, the same also holds for this reference trajectories for LQR2. However, for general trajectories this must not necessarily be the case. The LQR1 does not care about input constraints, on real test benches such rapid changes of the inputs up to the limits can cause a variety of effects and therefore distort the validity of the results of the experiments.

Figure 5 shows the time histories of the states $x_i(t)$, $i = 1, \ldots, 5$ of system (3). Engine speed ω_E (first graph of Figure 5) and dynamometer speed ω_D (second graph of Figure 5) show very similar time histories. In stationary operation they are the same, while during transients they differ slightly. The action of the engine torque T_E (fourth graph of Figure 5) and the dynamometer torque T_D (fifth graph of Figure 5) are opposite. For example, when the engine speed is increased, the internal combustion engine gets accelerated by a greater torque, while the dynamometer additionally reduces the load and therefore also contributes to the acceleration of the engine. The third graph of Figure 5 shows the shaft torque T_{ST}.

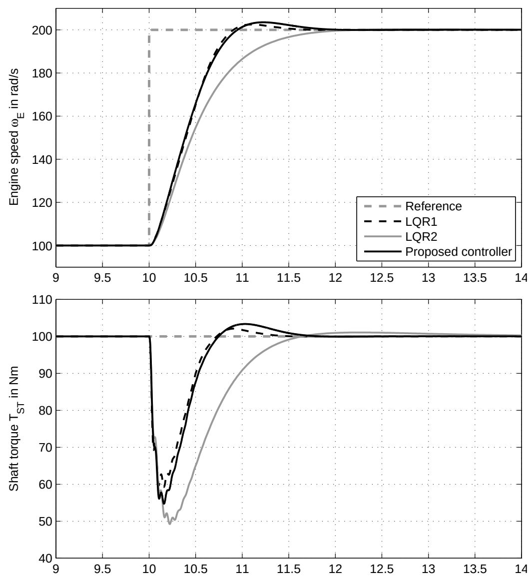

Fig. 3. Comparison of tracking of engine speed ω_E (top graph) and shaft torque T_{ST} (bottom graph) for a simple experiment using two differently tuned LQR and the proposed controller.

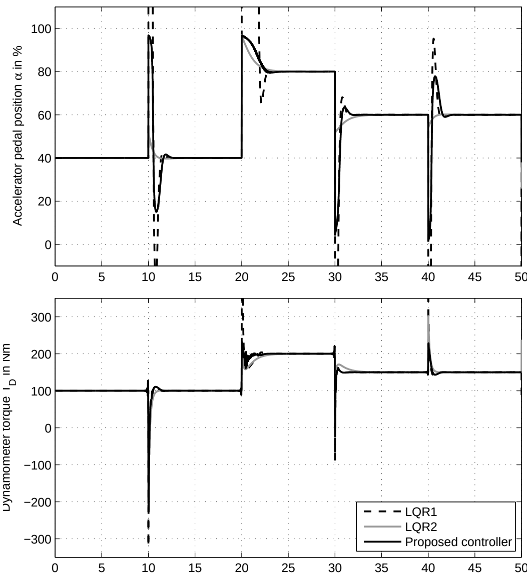

Fig. 4. Time histories of the accelerator pedal position α and the set value $T_{D,set}$ of the dynamometer torque T_D for a simple experiment using two differently tuned LQR and the proposed controller.

5. CONCLUSION AND OUTLOOK

The optimal control problem of an internal combustion engine test bench is discussed in this paper. The input constraints are

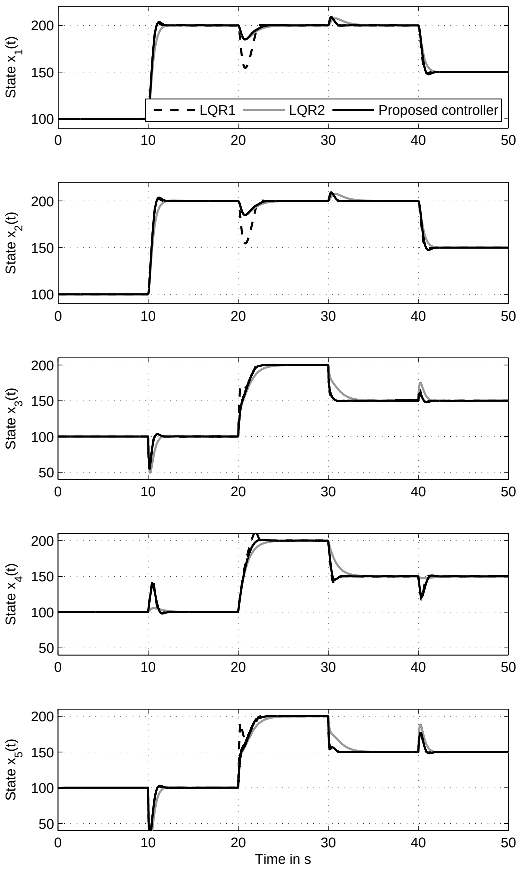

Fig. 5. Time histories of the states $x_i(t)$, $i = 1, \ldots, 5$ of the system (3) during a simple experiment using two differently tuned LQR and the proposed controller.

mapped to the state equations, furthermore the state is extended to avoid the calculation of the explicit solution of the HJB partial differential equation.

The proposed controller shows a good tracking performance compared to other controls. Both, the set values of the engine speed as well as the shaft torque are achieved quickly with almost limited overshoots. The effects of couplings are limited.

In contrast to other multi-input multi-output controller, the calculation and the tuning of the proposed controller is simple, easily adopted and extended to other test benches and test benches with different setup.

Measurements applying the developed multi-input multi-output controller will be performed on the test bench for which the simulator has been created. Note that a real-time implementation of the dynamic control is already available and allows a sampling time of 1 ms that is common in control of internal combustion engine test benches. The design of the controller will also be adopted to a test bench setup with a truck engine loaded by a hydrodynamic dynamometer. In this setup the dynamometer can only load the internal combustion engine, driving the engine is not possible and both actuators have a non-symmetrical limitation.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the sponsoring of this work by the COMET K2 Center "Austrian Center of Competence in Mechatronics (ACCM)".

REFERENCES

W. H. T. M. Aangenent, W. P. M. H. Heemels, M. J. G van de Molengraft, D. Henrion, and M. Steinbuch. Linear control of time-domain constrained systems. *Automatica*, 48(5):736–746, 2012.

B. D. O. Anderson and J. B. Moore. *Optimal Control: Linear Quadratic Methods*. Prentice Hall, 1989.

D. P. Bertsekas. *Dynamic Programming and Optimal Control – Volume I, third edition*. Athena Scientific, 2005.

A. E. Bryson and Y. C. Ho. *Applied Optimal Control: optimization, estimation, and control*. Taylor and Francis, New York, 1975.

Byron J. Bunker, Matthew A. Franchek, and Bruce E. Thomason. Robust multivariable control of an engine-dynamometer system. *IEEE Transaction on Control Systems Technology*, 5 (2), March 1997.

Engelbert Gruenbacher and Luigi del Re. Robust inverse control for combustion engine test benches. In *2008 American Control Conference*, June 2008. Seattle, Washington, USA.

Engelbert Gruenbacher, Patrizio Colaneri, and Luigi del Re. Guaranteed robustness bounds for matched-disturbance nonlinear systems. *Automatica*, 44(9):2230–2240, September 2008.

T. Hunt and A. J. Krener. Improved patchy solution to the Hamilton-Jacobi-Bellman equations. In *49th IEEE Conference on Decision and Control*, Atlanta, Georgia, USA, December 2010.

Zongli Lin. Global control of linear systems with saturating actuators. *Automatica*, 34:897–905, July 1998.

D. L. Lukes. Optimal regulation of nonlinear dynamical systems. *SIAM Journal on Control and Optimization*, 7(1):75–100, 1969.

W. M. McEneaney. A curse-of-dimensionality-free numerical method for solution of certain HJB pdes. *SIAM Journal on Control and Optimization*, 46(4):1239–1276, 2007.

Peter Ortner, Engelbert Gruenbacher, and Luigi del Re. Model based nonlinear observers for torque estimation on a combustion engine test bench. In *2008 IEEE Conference on Control Applications*, Hagen, Germany, August 2008.

Thomas Passenbrunner, Mario Sassano, and Luigi del Re. Approximate optimal control of internal combustion engine test benches. In *50th IEEE Conference on Decision and Control and European Control Conference*, Orlando, Florida, USA, December 2011.

Thomas Passenbrunner, Hannes Trogmann, and Luigi del Re. Unknown input high-gain observer for internal combustion engine test benches. In *2012 American Control Conference*, Montreal, Quebec, Canada, June 2012.

Mario Sassano and Alessandro Astolfi. Dynamic approximate solutions of the HJ inequality and of the HJB equation for input-affine nonlinear systems. *IEEE Transactions on Automatic Control*, 57(10):2490–2503, 2012.

Mario Sassano, Thomas Passenbrunner, Markus Hirsch, Luigi del Re, and Alessandro Astolfi. Approximate optimal control of the air path of a diesel engine. In *2012 American Control Conference*, Montreal, Quebec, Canada, June 2012.

Taner Tuken, R. Rees Fullmer, and Jon VanGerpen. Modeling, identification, and torque control of a diesel engine for transient test cycles. In *SAE Technical Paper Series*, February 1990. paper 900235.

Diana Yanakiev. Adaptive control of diesel engine-dynamometer systems. In *37th IEEE Conference on Decision and Control*, Tampa, Florida, USA, December 1998.