Dual adaptive control for non-minimum phase systems with functional uncertainties

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Abstract: A dual control for a nonlinear system with non-minimum phase based on the bicriterial approach is proposed and discussed. A particular class of the nonlinear input/output recursive model is composed of linear and nonlinear blocks, the latter being implemented with a multi-layered perceptron neural network. The unknown parameters of the model are estimated in real-time by the extended Kalman filter. The chosen nonlinear model with the affine structure in inputs together with the certainty equivalence principle utilization allow to obtain an analytical solution to control based on generalised minimum variance method. Behaviour of the system based on the enforcement of the certainty equivalence can negatively be affected, especially in a presence of disturbances and functional uncertainties. For that, the control action is enhanced about dual property based on the bicriterial approach that uses two separate criteria to introduce one of the opposing aspects between estimation and control.

1. INTRODUCTION

In the last decade, the area of adaptive control of nonlinear stochastic systems has acquired increased attention [Fabri and Kadirkamanathan, 2001, Liu, 2001, Herzallah, 2007, Král and Šimandl, 2011b]. It is mainly due to a requirement to control complex nonlinear systems. It is often assumed, that nonlinear functions, describing the system are unknown, and hence adaptive control is called functional [Fabri and Kadirkamanathan, 2001]. It is in a contrast to the classical adaptive control where only parameters of the functions, both linear or nonlinear, are unknown.

Control of the systems subject to functional uncertainties demands an adequate adaptive control strategy. Optimal control signal based on stochastic control principles should simultaneously optimise control performance and reduce an uncertainty [Fel'dbaum, 1965]. That means takes into consideration the degree of uncertainty present in the estimates of the systems unknown features and probes system input to actively reduce this uncertainty in the future as well. Unfortunately, an optimal solution entails the use of dynamic programming to solve the Bellman recursive relations and cannot mostly be found either numerically or analytically even for a class of linear systems. Therefore, many suboptimal solutions providing dual properties have been proposed [Wittenmark, 1975, Milito et al., 1982, Filatov and Unbehauen, 2000] leading to a tractable, albeit less optimal solution, but which somewhat retains the two desirable properties of dual control: caution and probing.

This paper is concerned with suboptimal dual adaptive control of a nonlinear class of stochastic systems with a functional uncertainty. A basic concept of this problem was originally considered in Fabri and Kadirkamanathan [2001], where a model based on neural networks (NN) as appropriate tool [Nørgaard et al., 2000, Sarangapani, 2006] for the unknown nonlinear function approximation was considered. Unknown parameters of the nonlinear model were estimated by the extended Kalman filter (EKF) and the innovations dual control (IDC) criterion [Milito et al., 1982] was used as the cost function. This was followed by Bugeja et al. [2009] where the dual adaptive control was successfully applied to trajectory tracking control of a mobile robot. In Šimandl et al. [2005], control quality was improved by utilization of the bicriterial dual control (BDC) originally proposed in [Filatov and Unbehauen, 2004] instead of the IDC, and by a Gaussian Sum Filter (GSF) for parameter estimation. In Král and Šimandl [2011b], functional adaptive control was extended for a general MIMO class of nonlinear systems. In Král and Šimandl [2011a], BDC was newly extended to a long-term system performance where the predictors of the future behaviour of the system are based on the affine neural network predictor model.

Despite these partial achievements in functional adaptive control, there are still open problems. One of them is the limitation of existing solutions for systems with minimum phase, which significantly reduces their use. This assumption is often very difficult to guarantee as discretized system cannot ensure the minimum phase system, even if the original system is minimum phase. Although there are suggestions for nonlinear control systems with minimum phase, they are either designed for deterministic systems, for systems with known parameters (i.e. non-adaptive frame) or the resulting controllers do not have the property of duality [Zhu et al., 1999, Talebi et al., 2000, Zayed et al., 2006, Campi and Weyer, 2010]. It can be said that an adequate solution for systems with functional uncertainties does not yet exist.

Based on the motivation point, the general goal of the paper is an extension of the functional adaptive controllers developed in Šimandl et al. [2005] and Král and Šimandl [2011a] for nonlinear stochastic systems with non-minimum phase behaviour [Chen and Khalil, 1995]. More specifically, the purpose is to define an appropriate nonlinear model in such a way that the control design will not require a minimum phase assumption and, in addition, will have the advantage of a dual control based

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on the bicriterial approach. It makes the control system more amenable and practical for real-time implementation. This will differ from Šimandl et al. [2005] and Král and Šimandl [2011a] in two main aspects: (a) The nonlinear model of the system consists of a linear and a nonlinear submodel. Although such a representation need not guarantee the exact model achievement of the general nonlinear system, it can often provide a sufficient approximation for control purposes. (b) The novel controller proposed in this paper inherently circumvents problems with minimum phase behaviour by avoiding the structural invertibility property.

The rest of the paper is organised as follows: Section 2 is focused on a description of the problem statement. The nonlinear model based on the NN and the nonlinear estimation method are specified in Section 3. Control design based on the dual modification of the non-dual controller using CE assumption is derived in Section 4. A numerical example demonstrating promising control quality results is contained in Section 5 and finally, Section 6 concludes the paper.

2. PROBLEM STATEMENT

The dynamical system to be controlled is a nonlinear stochastic discrete time-invariant system given in input-output representation as

$$\mathscr{S}: \quad y_{k+1} = f(\mathbf{y}_k, \mathbf{u}_k) + e_{k+1}, \tag{1}$$

where $f: \mathbb{R}^{n_y+n_u} \to \mathbb{R}$ is an unknown nonlinear function, $\mathbf{y}_k = [y_{k-n_y+1}, \ldots, y_k]^T \in \mathbb{R}^{n_y}$, $\mathbf{u}_k = [u_{k-n_u+1}, \ldots, u_k]^T \in \mathbb{R}^{n_u}$, u_k and y_k are input and output signals at discrete time instants $k \in 0, 1, \ldots, N-1$ and $\{e_k\}$ is an additive noise and the following assumptions are considered:

As. 1: The nonlinear function $f(\mathbf{y}_k, \mathbf{u}_k) \in C^\infty$.
As. 2: The structural parameters n_y and n_u of the system are known.
As. 3: $\{e_k\} \in \mathbb{R}$ is a zero-mean white Gaussian sequence with a known variance σ^2.

In order to formulate the optimal (dual) control problem, it is necessary to specify an appropriate criterion

$$J = E\left\{\sum_{k=0}^{N-1} \mathscr{L}(y_{k+1}, u_k, \mathbf{\Theta}_k)\right\}, \tag{2}$$

where $\mathscr{L}(\cdot)$ is a cost functional, $\mathbf{\Theta} \in \mathbb{R}^{\Theta_n}$ represents an unknown vector parameter of the model. The conditional expectation operator $E\{\cdot\}$ is taken over all underlying random quantities, that would rate the quality of the control process.

As was already mentioned, it is usually impossible to find a closed form solution to the minimization of (2) for the complex system such as (1). Therefore, the general optimal control must be simplified. A common simplification is to reduce the control horizon to only one step ahead and to enforce the certainty equivalence principle on the problem. The resulting control law is often denoted as the heuristic certainty equivalence controller (HCE) because all random variables are assumed to be equal to their expectations. But without any further modification these suboptimal solutions lead to the non-dual control. Behaviour of such controlled system is negatively affected due to the total omission of the uncertainties [Wittenmark, 1975, Filatov and Unbehauen, 2000]. That can bring an insufficient control quality.

Nevertheless, the control performance of the non-dual solutions could be improved in a presence of disturbances and parameter uncertainties, especially for smooth startup of the process. For that, the obtained non-dual control will be subsequently enhanced about the dual property. One of the few options that allow such a modification represents the bicriterial approach successfully applied by Filatov and Unbehauen [2004] in various control techniques for linear stochastic systems. The key idea of this method consists in the cost function which exploits two separate criteria. Each criterion introduces one of the opposing aims between estimation and control; *probing* and *caution*. The final control low will be obtained by a sequent minimization of the criteria (3) and (5).

The first criterion in the bicriterial approach is suggested in the following form

$$J_k^c = E\left\{(\hat{y}_{k+1}^{nom}(\mathbf{y}_k, \mathbf{u}_k, \mathbf{\Theta}) - y_{k+1})^2 | \mathbf{I}^k\right\}, \tag{3}$$

where $\boldsymbol{I}^k$ is the information state containing all measurable inputs and outputs available up to time instant k. The nominal output $\hat{y}_{k+1}^{nom}$ is defined as the response of the system to the input signal u_k^{CE} that is generated by the arbitrary non-dual controller as described above. The system output y_{k+1}^{nom} hence should provide the desired system dynamics according to the bounded reference signal r_{k+1}.

The criterion (3) evaluates quality of the control and involves minimization of the expected value of the tracking error. The resulting control

$$u_k^c = \operatorname*{argmin}_{\mathrm{u_k}} J_k^c \tag{4}$$

respects the uncertainties in knowledge of the unknown function of the system (1), and is equal to *cautious control*.

The second criterion in the bicriterial approach is chosen as

$$J_k^a = -E\left\{(y_{k+1} - \hat{y}_{k+1}(\mathbf{y}_k, \mathbf{u}_k, \mathbf{\Theta}))^2 | \mathbf{I}^k\right\}, \tag{5}$$

where $\hat{y}_{k+1}$ is a one step prediction of the output of the controlled system. This criterion evaluates the estimation quality. It should accelerate the parameter estimation process for future control improvement by increasing the predictive error value. The controller provides an optimal excitation added to the cautious control and determines magnitude of the *probing signal*.

Finally, the dual predictive control u_k is then searched by

$$u_k = \operatorname*{argmin}_{\mathrm{u_k} \in \Omega_\mathrm{k}} J_k^a, \tag{6}$$

where Ω_k defines the symmetrically distributed region around the caution control as

$$\Omega_k = [u_k^c - \delta_k, u_k^c + \delta_k]. \tag{7}$$

The choice of the parameter δ_k stems from the reasoning that it is necessary to enrich the caution control with probing proportionally to the uncertainty of the unknown parameter vector $\mathbf{\Theta}$ which describes unknown function $f(.)$ in the system (1). A common choice for δ_k is

$$\delta_k = \eta \operatorname{tr}\{\mathbf{P}_{k+1}\}, \tag{8}$$

where tr is trace operator and $\eta \geq 0$ provides the amplitude of the probing signal. The matrix $\mathbf{P}_{k+1}$ describes the rate of uncertainty of the parameter estimate $\hat{\mathbf{\Theta}}_{k+1}$ conditioned by $\mathbf{I}^k$ and can be obtained using a nonlinear estimation method.

The design of the functional adaptive dual control will proceed in the following way. First, a suitable representation of the system (1) will be determined. The chosen model will be seen as a combination of a linear and a nonlinear submodel which has been shown to be particularly useful in an adaptive control framework Zhu et al. [1991]. The nonlinear submodel will be approximated by the MLP NN as a suitable compromise between the number of the parameters and the model complexity. Then, an attention will be focused on an important issue of the nonlinear model parameter estimation which will be solved as a nonlinear optimization problem. Finally, the dual control law derivation will be finished by a sequent minimization of the criteria (3) and (5), where the nominal output $\hat{y}_k^{nom}$ will be generated using a non-dual controller based on a certainty equivalence principle.

3. NEURAL NETWORK BASED MODEL

In this section, an appropriate affine model structure of the system (1) is specified and the searching process for the optimal parameter values of the model is described.

To control a nonlinear system (1), a generalised parametric model structure is used

$$\mathscr{M}: \quad A(z^{-1})y_{k+1} = B(z^{-1})u_k + f^o(\boldsymbol{y}_k, \boldsymbol{u}_{k-1}) + e_{k+1}, \quad (9)$$

where $A(z^{-1}) = 1 + a_1 z^{-1} + \dots a_{n_y} z^{-n_y}$ and $B(z^{-1}) = b_0 + b_1 z^{-1} + \dots b_{n_u} z^{-n_u}$ are n_y and n_u, respectively, order polynomials, z^{-1} is a one step backward shift operator and $f^o(\boldsymbol{y}_k, \boldsymbol{u}_{k-1})$ is a nonlinear function which accounts for any unknown time-delays, uncertainty and non-linearity in the complex model and is approximated by an MLP NN with nf neurons in a single hidden layer given as

$$f^0(\mathbf{x}_k, \mathbf{w}, \mathbf{c}) = (\mathbf{c})^T [(\phi(\mathbf{x}_k, \mathbf{w}))^T, 1]^T, \quad (10)$$

where $\mathbf{x}_k = [u_{k-1}, \dots, u_{k-n_u}, y_k, \dots, y_{k-n_y}]^T$ is an input vector of the NN with length ni, $\mathbf{c}$ and $\mathbf{w}$ are parameter vectors of the output layer and the hidden layer of the network with lengths $cf+1$ and $(ni+1)nf$, respectively. The scalar functions $\phi(\mathbf{x}_k, \mathbf{w})$ are sigmoidal activation functions of the neurons in the hidden layers.

Efficient exploitation for both control and model purposes of the system (1) based on the model (9) can be found, for example, in Zhu et al. [1999], Zhang [2003] or Zayed et al. [2006]. Note, that the model (9) has an affine structure similar to previous author works [Král and Šimandl, 2008, 2011b], but allows dual control design for a larger class of the nonlinear systems.

The coefficients of the polynomials $A(z^{-1}), B(z^{-1})$ and parameters of the neural network model are unknown. Before applying an estimation method to the parameter estimation, a suitable estimation model of the identified system has to be defined. For that, all unknown parameters of the model (9) will be included in one parameter vector

$$\boldsymbol{\Theta} = [b_0, \boldsymbol{\alpha}^T, \boldsymbol{\beta}^T]^T, \quad (11)$$

with its length denoted by n_Θ and where $\boldsymbol{\beta} = [\boldsymbol{w}^T, \boldsymbol{c}^T]^T$, $\boldsymbol{\alpha} = [b_1, \dots, b_{n_u}, a_1, \dots, a_{n_y}]^T$.

Now, attention can be focused on searching for the optimal parameter values of the model (9)-(11). The vector parameter $\boldsymbol{\Theta}$ of the model is considered to be a random variable with a time-variant characteristics, i.e.

$$\boldsymbol{\Theta}_{k+1} = \boldsymbol{\Theta}_k + \boldsymbol{v}_k \quad (12)$$

where $\{v_k\} \in \mathbb{R}$ is a white noise sequence which describes variation of the parameters and has zero mean and known covariance matrix $\boldsymbol{R}_k$. Further, it is assumed that the system (1) can be approximated with an arbitrary accuracy by the chosen nonlinear model (9)-(11). Then, it is possible to obtain the measurement equation from (9) by rewriting it as

$$y_{k+1} = h(\boldsymbol{\Theta}, \boldsymbol{y}_k, \boldsymbol{u}_k) + e_{k+1}, \quad (13)$$

where

$$h(\boldsymbol{\Theta}, \boldsymbol{y}_k, \boldsymbol{u}_k) = b_0 u_k + \boldsymbol{x}_k^T \boldsymbol{\alpha} + f^o(\boldsymbol{x}_k, \boldsymbol{\beta}). \quad (14)$$

Equations (12) and (13) define the estimation model of the system (1). Unfortunately, dependence of y_{k+1} on the parameters of model is nonlinear. Therefore, it is advisable to exploit a convenient method for finding the unknown vector parameter $\boldsymbol{\Theta}$.

It was shown [Singhal and Wu, 1989], that the well-known Extended Kalman filter may represent a convenient method for finding the unknown vector parameter $\boldsymbol{\Theta}$ of the model based on the MLP NN. From the paper point of view, its attractive features are namely practicality, computationally moderateness and tractability of the solution to estimation. Moreover, it will help in synthesis of the control law due to relatively easy calculation both the first and the second order moments of parameter estimates given by

$$p(\boldsymbol{\Theta}|\boldsymbol{I}^k) \approx \mathscr{N}\left\{\boldsymbol{\Theta} : \hat{\boldsymbol{\Theta}}_{k+1}, \boldsymbol{P}_{k+1}\right\}, \quad (15)$$

where the initial values for parameter estimation, i.e. $\boldsymbol{\Theta}_0$ is chosen by the designer.

Then, the parameters of the nonlinear model can sequentially be estimated by the following recursive relations

$$\hat{\boldsymbol{\Theta}}_{k+1} = \hat{\boldsymbol{\Theta}}_k + \mathbf{K}_k [y_k - \hat{y}_k], \quad (16)$$

$$\mathbf{P}_{k+1} = \mathbf{P}_k - \mathbf{K}_k \boldsymbol{\nabla}_k \mathbf{P}_k + \boldsymbol{R}_k, \quad (17)$$

$$\mathbf{K}_k = \frac{\mathbf{P}_k \boldsymbol{\nabla}_k}{(\boldsymbol{\nabla}_k)^T \mathbf{P}_k \boldsymbol{\nabla}_k + \sigma^2}. \quad (18)$$

where $\boldsymbol{\nabla}_k$ represents the Jacobian of the function $h(\cdot)$ with respect to the parameters $\boldsymbol{\Theta}$ define as

$$\boldsymbol{\nabla}_k = \left.\frac{dh}{d\boldsymbol{\theta}}\right|_{\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}_k} \quad (19)$$

Finally, with regards to the next steps of the control derivation, it it useful to rewrite $\boldsymbol{\nabla}_k$ and $\boldsymbol{P}_{k+1}$ as follows

$$\boldsymbol{\nabla}_k = \begin{bmatrix} \frac{dh}{db_0} \\ \frac{dh}{d\boldsymbol{\alpha}} \\ \frac{dh}{d\boldsymbol{\beta}} \end{bmatrix} = \begin{bmatrix} u_k \\ \boldsymbol{x}_k \\ \boldsymbol{\nabla}_k^{\beta} \end{bmatrix}, \quad (20)$$

$$\boldsymbol{P}_{k+1} = \begin{bmatrix} \boldsymbol{P}_{k+1}^{b} & \boldsymbol{P}_{k+1}^{b\alpha} & \boldsymbol{P}_{k+1}^{b\beta} \\ (\boldsymbol{P}_{k+1}^{b\alpha})^T & \boldsymbol{P}_{k+1}^{\alpha} & \boldsymbol{P}_{k+1}^{\alpha\beta} \\ (\boldsymbol{P}_{k+1}^{b\beta})^T & (\boldsymbol{P}_{k+1}^{\alpha\beta})^T & \boldsymbol{P}_{k+1}^{\beta} \end{bmatrix}, \quad (21)$$

where $\boldsymbol{\nabla}_k^b$, $\boldsymbol{\nabla}_k^{b\alpha}$ and $\boldsymbol{\nabla}_k^{b\beta}$ are subvectors of $\boldsymbol{\nabla}_k$ belonging to $\hat{b_0}$, $\hat{\boldsymbol{\alpha}}$, $\hat{\boldsymbol{\beta}}$ and have appropriate dimensions as well as the individual submatrices of $\boldsymbol{P}_{k+1}$ in (21).

Now, it is possible to obtain both the estimate $\hat{\boldsymbol{\Theta}}_{k+1}$ and the covariance matrix of the parameters $\mathbf{P}_{k+1}$ of the nonlinear model at each step of estimation algorithm which will be necessary for the following derivation of the control action u_k.

4. CONTROL DESIGN

In this section, the dual adaptive control law is derived. First, a non-dual control law based on generalised minimum variance (GMV) criterion is derived using the certainty equivalence principle. Subsequently, the final control is obtained by adding dual properties to the controller by the bicriterial approach utilization.

4.1 GMV controller based on CE assumption

To calculate an appropriate non-dual control law for system modelled by (9), the performance index, which defines the GMV control, can be chosen as:

$$J^{GMV} = E_{CE}\{(y^a_{k+1})^2\} = E_{CE}\{(P(z^{-1})y_{k+1} + Q(z^{-1})u_k - R(z^{-1})r_k - H(z^{-1})f_0)^2|\mathbf{I}^k\}, \quad (22)$$

where $P(z^{-1})$, $Q(z^{-1})$, $R(z^{-1})$ and $H(z^{-1})$ are weighting polynomials introduced to improve the system dynamics and steady-state characteristics of the overall control system, E_{CE} is the conditional expectation operator with enforcement of the CE principle and $\mathbf{I}^k$ is the information state containing all measurable inputs and outputs available up to time instant k.

Next, the following identity can be introduced

$$P(z^{-1}) = C(z^{-1})A(z^{-1}) + z^{-1}G(z^{-1}), \quad (23)$$

where $C(z^{-1}) = 1 + c_1 z^{-1} + \dots a_{nc} z^{nc}$ and $G(z^{-1}) = g_0 + g_1 z^{-1} + \dots g_{ng} z^{-ng}$ are polynomials of the orders $nc = d-1$ and $ng = n_y - 1$, respectively, with d representing delay of the system.

Multiplying (9) by $C(z^{-1})$ and substituting for $C(z^{-1})A(z^{-1})$ from (23) gives

$$P(z^{-1})y_{k+1} = G(z^{-1})y_k + B(z^{-1})C(z^{-1})u_k + C(z^{-1})f^o(\boldsymbol{x}_k, \boldsymbol{\beta}) + C(z^{-1})e_{k+1}. \quad (24)$$

Adding the term $Q(z^{-1})u_k - R(z^{-1})r_k - H(z^{-1})f^o(\boldsymbol{x}_k, \beta)$ to both sides of (24) and using (23) yields

$$y^a_{k+1} = \big[G(z^{-1})y_k + (B(z^{-1})C(z^{-1}) - Q(z^{-1}))u_k - R(z^{-1})r_k + (C(z^{-1}) - H(z^{-1}))f^o(\boldsymbol{x}_k, \beta)\big] + C(z^{-1})e_{k+1}. \quad (25)$$

The non-dual controller output u^{CE}_k which minimises the performance index (22) can be obtained by putting the first term in (25) equal to zero as

$$u^{HCE}_k = \frac{R(z^{-1})r_k - G(z^{-1})y_k - (C(z^{-1}) - H(z^{-1}))f^o(\boldsymbol{x}_k, \beta)}{B(z^{-1})C(z^{-1}) - Q(z^{-1})}. \quad (26)$$

Combining (9), (23) and (26), the closed-loop system behaviour can be expressed as

$$[B(z^{-1})P(z^{-1}) - Q(z^{-1})A(z^{-1})]y_{k+1} = B(z^{-1})R(z^{-1})r_k + (B(z^{-1})H(z^{-1}) - Q)f^o(\boldsymbol{x}_k, \beta) + (B(z^{-1})C(z^{-1}) - Q(z^{-1}))e_{k+1}. \quad (27)$$

To obtain the desired closed-loop dynamics, the following relation needs to be fulfilled

$$B(z^{-1})P(z^{-1}) - Q(z^{-1})A(z^{-1}) = T(z^{-1}), \quad (28)$$

with polynomials $P(z^{-1}) = 1 + p_1 z^{-1} + \dots p_{np} z^{-np}$ and $Q(z^{-1}) = q_0 + q_1 z^{-1} + \dots q_{nq} z^{-nq}$ of the orders $np = n_y - 1$ and $nq = n_u - 1$, respectively and $T(z^{-1}) = 1 + t_1 z^{-1} + \dots t_{nt} z^{-nt}$ is chosen by the designer, usually as a stable polynomial of the order 1 or 2.

Finally, to satisfy the static offset and eliminate the steady state of the nonlinear part, polynomials $R(z^{-1})$ and $H(z^{-1})$ can be tuned on-line, most simply, as

$$R(z^{-1}) = \frac{T(z^{-1})}{B(z^{-1})}\bigg|_{z=1} \qquad H(z^{-1}) = \frac{Q(z^{-1})}{B(z^{-1})}\bigg|_{z=1}. \quad (29)$$

The control action (26) is valid only for the situation, where parameters of the system are known a priori. In the opposite case, all unknown parameters are assumed to be equal to their mean values as a result of the CE principle application. So, based on the parameter estimation process described in Section 3, the coefficients of the polynomials $P(z^{-1})$, $Q(z^{-1})$ and $C(z^{-1})$, $G(z^{-1})$ can be found recursively at each control step from identities (28) and (23), respectively.

4.2 An enhancement of control to dual properties

Because of the CE principle utilization, the GMV controller output (26) does not allow to respect the uncertainty existing in the system. Therefore, the behaviour of the control action can be insufficient, especially at startup of the process. In this section, the disadvantage will be reduced by enhancing the HCE control from Section 4.1 by the idea of the dual control (in a sense of Fel'dbaum [1965]) based on the bicriterial approach described by the equations (3)–(5).

In the first step, the criterion J^c_k k will be minimised. Using (13)-(14) along with the CE assumptions, the nominal output $\hat{y}^{nom}_{k+1}$ is given

$$\hat{y}^{nom}_{k+1} = \hat{b}_{0,k+1}u^{HCE}_k + \boldsymbol{x}^T_k \hat{\boldsymbol{\alpha}}_{k+1} + f^o(\boldsymbol{x}_k, \hat{\boldsymbol{\beta}}_{k+1}), \quad (30)$$

where the values of the functions $\hat{b}_{0,k+1}$, $\hat{\boldsymbol{\alpha}}_{k+1}$, $\hat{\boldsymbol{\beta}}_{k+1}$ and y_{k+1} should be considered as random variables. By substituting (30) into (3); the criterion J^c_k can be rewritten as

$$J^c_k = E\{(\hat{b}_{0,k}u^{HCE}_k + \boldsymbol{x}^T_k \hat{\boldsymbol{\alpha}}_{k+1} + f^o(\boldsymbol{x}_k, \hat{\boldsymbol{\beta}}_{k+1}) - y_{k+1})^2|\mathbf{I}^k\}. \quad (31)$$

After multiplication and partial application of the mean operator over the information segment $\mathbf{I}^k$, it is possible to write

$$J^c_k = (\hat{b}_{0,k}u^{HCE}_k)^2 + (\boldsymbol{x}^T_k \hat{\boldsymbol{\alpha}}_k)^2 + (f^o(\boldsymbol{x}_k, \hat{\boldsymbol{\beta}}_{k+1}))^2 + 2\hat{b}_{0,k+1}u^{CE}_k \times \boldsymbol{x}^T_k \hat{\boldsymbol{\alpha}}_{k+1} + 2\boldsymbol{x}^T_k \hat{\boldsymbol{\alpha}}_{k+1} f^o(\boldsymbol{x}_k, \hat{\boldsymbol{\beta}}_{k+1}) + 2\hat{b}_{0,k+1}u^{CE}_k f^o(\boldsymbol{x}_k, \hat{\boldsymbol{\beta}}_{k+1}) - 2\hat{b}_{0,k+1}u^{HCE}_k E\{y_{k+1}\} - 2\boldsymbol{x}^T_k \hat{\boldsymbol{\alpha}}_{k+1} E\{y_{k+1}\} - 2f^o(\boldsymbol{x}_k, \hat{\boldsymbol{\beta}}_{k+1}) \times E\{y_{k+1}\} + E\{y^2_{k+1}\}. \quad (32)$$

The substitution of y_{k+1} into (32) results in

$$J^c_k = -2\hat{b}_{0,k+1}u^{HCE}_k E\{b_0\}u_k - 2\boldsymbol{x}^T_k \hat{\boldsymbol{\alpha}}_{k+1} E\{b_0\}u_k - 2f^o(\boldsymbol{x}_k, \hat{\boldsymbol{\beta}}_{k+1})E\{b_0\}u_k + E\{b^2_0\}u^2_k + 2E\{b_0 \boldsymbol{x}^T_k \boldsymbol{\alpha}\}u_k + 2E\{b_0 f^o(\boldsymbol{x}_k, \boldsymbol{\beta})\}u_k + c_1, \quad (33)$$

where c_1 represents all terms which need not be considered further because they are independent of u_k and thus have no influence on the value of the criterion J^c_k. Now, it is possible to determine the control action u^c_k as an extreme of criterion J^c_k along with utilization of the well-known relation $E\{ab\} = E\{a\}E\{b\} + \mathrm{cov}\{ab\}$

$$u^c_k = \frac{\hat{b}^2_{0,k+1}u^{HCE}_k - \boldsymbol{P}^{b\alpha}_{k+1}\boldsymbol{x}_k - \boldsymbol{P}^{b\beta}_{k+1}\nabla^{b\beta}_{k+1}}{\hat{b}^2_{0,k} + \boldsymbol{P}^b_{k+1}}, \quad (34)$$

where $\boldsymbol{P}^b_{k+1}$, $\boldsymbol{P}^{b\alpha}_{k+1}$, $\boldsymbol{P}^{b\beta}_{k+1}$ and $\boldsymbol{\nabla}^{b\beta}_{k+1}$ are defined by (21) and (20).

It should be pointed out, that the resulting control u^c_k in (34) respects the uncertainties in knowledge of the unknown model parameters $\hat{\boldsymbol{\Theta}}_{k+1}$ due to the elements of covariance matrix $\boldsymbol{P}_{k+1}$ and is equal to the caution control.

In the second step, the criterion (5) will be minimised according to (6)–(8). The criterion J_k^a could be rewritten using (21) and (20) as

$$J_k^a(u_k) = -\left(\nabla_{k+1}\mathbf{P}_{k+1}(\nabla_{k+1})^T + \sigma^2\right) = -\Big(\boldsymbol{P}_{k+1}^b u_k^2 + 2\boldsymbol{x}_k^T\boldsymbol{P}_{k+1}^{b\alpha}u_k + 2(\nabla_{k+1}^{\beta})^T\boldsymbol{P}_{k+1}^{b\beta}u_k\Big) + c_2, \quad (35)$$

where c_2 contains the elements that are independent of the variable u_k and need not be considered further.

The criterion $J_k^a(u_k)$ is a convex function of the variable u_k. Hence, the extreme is inevitable to be found within boundary of the domain Ω_k defined by (7). Substituting the variable u_k with the boundary points from (7) into the criterion J_k^a and subsequently comparing the results it can be detected, which one of two suspected points of the cost function represents the minimum. Therefore, it is possible to state the following relation

$$u_k = u_k^c + \delta_k \operatorname{sgn}\{J_k^a(u_k^c - \delta_k) - J_k^a(u_k^c + \delta_k)\}. \quad (36)$$

Substituting $u_k^c \pm \delta_k$ to u_k in (35) and using (36) it is possible to obtain

$$J_k^a(u_k^c - \delta_k) - J^a(u_k^c + \delta_k) = 4\delta_k(\boldsymbol{P}_{k+1}^b u_k^c + \boldsymbol{x}_k^T\boldsymbol{P}_{k+1}^{b\alpha} + \nabla_{k+1}^{\beta}\boldsymbol{P}_{k+1}^{b\beta}). \quad (37)$$

The control law is given using (34), (36) and (37) as

$$u_k = u_k^c + \delta_k \operatorname{sgn}\left\{\boldsymbol{P}_{k+1}^b u_k^c + \boldsymbol{x}_k^T\boldsymbol{P}_{k+1}^{b\alpha} + \nabla_{k+1}^{\beta}\boldsymbol{P}_{k+1}^{b\beta}\right\}, \quad (38)$$

where $4\delta_k$ from (37) can be omitted because it does not change the result of the signum operator.

Equations (8), (26), (34) and (38) represent the final adaptive control law. It is clear that the computational demands of the adaptive controller are moderate and the execution time is almost equal compared to the non-dual controller (26). Nonetheless, the controller (38) has the dual control ability. Note that the uncertainty measure is taken into account and after finishing the adaptation, the control signal is equivalent to the HCE-controller based on minimization of the criterion (22).

5. NUMERICAL EXAMPLE

Properties of the designed predictive dual controller are illustrated in the following numerical example [Zhu et al., 1999]

$$y_{k+1} = \frac{1.5y_k\sin(y_k)}{1 + y_k^2 + y_{k-1}^2} + 1.1y_{k-1} + 1.2u_k + 2u_{k-1} + e_{k+1}, \quad (39)$$

where e_k is a white noise with zero mean and variance $\sigma^2 = 0.00015$. System (39) is modelled by (9), where $n_y = n_u = 2$ and nonlinear function $f^0(\boldsymbol{x}_k, \boldsymbol{\beta})$ is approximated by MLP NN with 8 neurons in a single hidden layer. The initial parameters $\mathbf{\Theta}_0$ of the nonlinear model are generated from normal distribution $\mathcal{N}\{0,\ 0.1\mathbf{I}_{n_\Theta}\}$, the covariance matrix should reflect the confidence in the initial guess and is chosen as $\mathbf{P}_0 = 0.1\mathbf{I}_{n_\Theta}$. The parameters of the BD controller are chosen as follows: $\eta = 0.1$, desired polynomial $T(z^{-1}) = 1 - 0.5z^{-1}$ and the reference signal r_k was chosen as a square wave with unit amplitude and period 500 time instants. It can be mentioned, that there it is not any off-line training of the neural network based model.

Proposed adaptive dual control is compared with couple of non-dual solutions; cautious control and HCE control. These two controllers have not properties of dual control and can be obtained as a special case of the BDC, the cautious control by setting parameter $\eta = 0$ and HCE control by setting covariance matrix $\boldsymbol{P}_{k+1} = \mathbf{0}$ at each time instant of the simulation.

Emphasis in the example is given to the situation when the main part of the system uncertainty is considered in the unknown nonlinear functions (i.e. low level noise). Then the probing signal generating by the designed dual controller is a key part of the control action, especially at the beginning of the simulation. It brings better control quality compared to non-dual controllers (CA, HCE) as shown by below results. In opposite case, the high level noise could help to excitation of the system. However, it would play in favour of non-dual controllers, and it is an undesirable effect in the paper.

The quality of control will be evaluated by M Monte Carlo simulations. The cost function $\mathscr{L}$ is chosen as

$$\mathscr{L} = \frac{1}{N}\sum_{k=1}^{N-1}(y_{k+1} - r_{k+1})^2.$$

The value of the cost $\mathscr{L}$ for the particular j^{th} Monte Carlo simulation is denoted by $\mathscr{L}_j$ and the value of the criterion J is estimated by

$$\hat{J} = \frac{1}{M}\sum_{j=1}^{M}\mathscr{L}_j.$$

Variability of Monte Carlo simulations is expressed by

$$\operatorname{var}\{\mathscr{L}\} = \frac{1}{M-1}\sum_{j=1}^{M}(\mathscr{L}_j - \hat{J})^2$$

and the quality of the criterion estimate $\hat{J}$ is expressed by $\operatorname{var}\{\hat{J}\}$ which can be computed using the bootstrap technique [Efron and Tibshirani, 1994].

The criterion value estimates ($\hat{J}$), the accuracies of these estimates ($\operatorname{var}\{\hat{J}\}$), the variability of Monte Carlo simulations ($\operatorname{var}\{\mathscr{L}\}$) and the average time per simulation that were computed using $M = 1000$ Monte Carlo simulations with $N = 2000$ steps per simulation, are given in Table 1.

TABLE I. A quality control performance of the bicriterial dual (BD), cautious (CA) and heuristic certainty equivalence (HCE) controller.

	$\hat{J}$	$\operatorname{var}\{\mathscr{L}\}$	$\operatorname{var}\{\hat{J}\}$
HCE	38.11	719.45	0.02
CA	2.66	14.02	0.0005
BD	1.3	3.26	0.0003

The utilization of the idea of the bicriterial approach has a positive influence on the control quality as is evident from summarised results in Table I. The table compares the value of the criterion estimate $\hat{J}$, variance of this estimate $\operatorname{var}(\hat{J})$ and variability of the particular Monte Carlo runs. Is can be clearly seen that these three statistical measures are smaller for BDC compared to the nondual controllers (HCE, CA). This reflects the general superior performance of the BD controller, where the supplemented probing signal of the controller improves the quality of the parameter estimates. Thus, the performance of the proposed dual control qualitatively yields the best transient performance.

6. CONCLUSION

The functional adaptive dual control for nonlinear stochastic systems with non-minimum phase behaviour was proposed. This generalises the work proposed earlier by the same authors. The design procedure relaxed previous minimum phase assumption of the system; in addition, it has the advantage of the dual control based on the bicriterial approach. The proposed dual controller was derived independently of the type of the HCE adaptive controller. Therefore, it was used as an additional unit, which made possible to transform the GMV controller into a dual control one. It was shown that the proposed dual adaptive controller can handle with minimum phase behaviour of the system, but also that the performance of the proposed dual adaptive controller is consistently better compared to the HCE controller and the cautious controller as well.

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