Real-time Identification of different types of non-holonomic mobile robots

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Abstract: This paper presents the real-time identification of different types of non-holonomic mobile robot systems. Since the robot type is a priori unknown, the robot systems are formulated as a switched singular nonlinear system, and the problem becomes the real-time identification of the switching signal, and then the existence of the input-output functions and the distinguishability of the system are studied. We show in the simulations that the proposed technique is implemented easily and effectively, and it is robust to the noises as well.

1. INTRODUCTION

Wheeled mobile robots have been widely studied and attracted interests of many researches because of their wide applications in industries and theoretical challenges Kolmanovsky and McClamroch [1995]. More recently, wheeled mobile robots have been proposed for using in rescue missions Murphy et al. [2009], explorations Rooker and Birk [2007], tour guide Han et al. [2010], and even entertainment such as robot soccer games Camacho et al. [2006]. Indeed, the mobile robot navigation problem is of great importance, and there are considerable researches effort into solving the robot navigation problems in different applications Salichs and Moreno [2000]. The path planning and motion control of mobile robots are two main aspects in the navigation problem. Path planning consists in generating a collision-free trajectory from the known initial position to the desired final position, and some path planning algorithms have been proposed for wheeled mobile robots Defoort et al. [2009], Kokosy et al. [2008]. As for the motion control, which is the determination of the physical control inputs to the robot motion components, no matter what control approach is applied, like PID controllers Ardiyanto [2010], nonlinear feedback control approach Wan and Chen [2008], and sliding mode control Defoort et al. [2008], those controllers are designed based on the robot model. As a result, the robot models can not be avoided in the robot navigation problems, and the controller to be designed is going to be different according to different robot kinematic models. Thus, given an unknown model of wheeled mobile robots, the first task is to identify the robot kinematic model, and then we can design the model-based controller for it. Since the kinematic model of mobile robot depends on the construction manners and wheel configurations, thus seems difficult to be identified. Fortunately, by introducing the concepts of *degree of mobility* and of *degree of steerability*, the set of kinematic models of wheeled mobile robots can always be partitioned in five classes (see de Wit et al. [1996] for precise definition and classification of mobile robot types).

For those different types of non-holonomic mobile robot, this paper treats the identification problem of these kinematic models as a detection of active mode of a special switched system. A switched system is a dynamical system that consists of a family of subsystems (linear or nonlinear) and a logical rule, called the switching law, that orchestrates switching between these subsystems, and here only one value is possible. In recent years, there has been increasing interest in switched systems due to their significance from both a theoretical and practical point of view, and several important results for such systems have been achieved, for example, stability Vu and Liberzon [2005], stabilization Moulay et al. [2007], controllability results Xie et al. [2002], Sun et al. [2002], and tracking Bourdais et al. [2007]. Since switched system consists of different subsystems, if we model the subsystem as one of possible kinematic model of non-holonomic robots, then the robot model identification problem becomes the identification of the subsystems of this switched system.

The paper is structured as follows. Section 2 presents the description of different non-holonomic mobile robots. Section 3 presents the deductions of input-output functions of subsystems. Section 4 analyzes the distinguishability of subsystems. Simulation results are detailed in Section 5.

2. ROBOT DESCRIPTION

As stated in the introduction, by introducing the concepts of *degree of mobility* and *degree of steerability*, the set of kinematic models of wheeled mobile robots can be partitioned in five classes. This paper considers the first four classes, since the input-output relationship of the last class is not yet obtained. Let us take the simple unicycle model as an example, which is depicted in Fig. 1, with an arbitrary inertial base frame b being fixed in the plane of motion and a frame m being attached to the robot. For a general kinematic model, the state is given by $q = [q_1, q_2, q_3, q_4]^T$, where (q_1, q_2) is the coordinate of its origin O_m, q_3 is the orientation angle with respect to x-axis $\overrightarrow{X}_b$, q_4 is the angle of the plane of the steering wheel

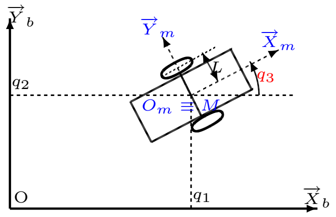

Fig. 1. Unicycle-type mobile robot

with respect to the robot frame Y_m when it exists. In the following, the robot types are distinguished as $(a.b)$, where a represents the *degree of mobility* and b represents the *degree of steerability* (de Wit et al. [1996]). Without loss of generality, the kinematic models under non-holonomic constraints of pure rolling and no slipping exists, thus they can be described as follows:

Type (2.0)

$$\Sigma_1 \begin{cases} \dot{q}_1 = \nu_1 \cos q_3 \\ \dot{q}_2 = \nu_1 \sin q_3 \\ \dot{q}_3 = \nu_2 \end{cases} \quad (1)$$

where the control input is $\nu = [\nu_1, \nu_2]^T$ with ν_1 and ν_2 being linear and angular velocity respectively.

Type (3.0)

$$\Sigma_2 \begin{cases} \dot{q}_1 = \nu_1 \cos q_3 - \nu_2 \sin q_3 \\ \dot{q}_2 = \nu_1 \sin q_3 + \nu_2 \cos q_3 \\ \dot{q}_3 = \nu_3 \end{cases} \quad (2)$$

where the control is $\nu = [\nu_1, \nu_2, \nu_3]^T$ with ν_1 and ν_2 being the robot velocity components along X_m and Y_m respectively, and ν_3 is the angular velocity.

Type (2.1)

$$\Sigma_3 \begin{cases} \dot{q}_1 = -\nu_1 \sin(q_3 + q_4) \\ \dot{q}_2 = \nu_1 \cos(q_3 + q_4) \\ \dot{q}_3 = \nu_2 \\ \dot{q}_4 = \nu_3 \end{cases} \quad (3)$$

where ν_3 is the angular velocity of the steering wheel, ν_1, ν_2 are defined as those of type (2.0) robot. The control input of this system is defined as that of type (3.0) robot.

Type (1.1)

$$\Sigma_4 \begin{cases} \dot{q}_1 = -L\nu_1 \sin q_3 \sin q_4 \\ \dot{q}_2 = L\nu_1 \cos q_3 \sin q_4 \\ \dot{q}_3 = \nu_1 \cos q_4 \\ \dot{q}_4 = \nu_2 \end{cases} \quad (4)$$

where L is half of the distance between the two fixed wheels, and the input is $\nu = [\nu_1, \nu_2]^T$ with ν_1 being the linear velocity and ν_2 being the angular velocity of the steering wheel.

Let us note that a last class exists (*Type* (1.2)), for which, until now no input-output relationship has been found.

3. INPUT-OUTPUT FUNCTIONS

3.1 Coordinate transformation

As stated above, the robot kinematic models are sets of ordinary differential equations (ODE), our objective is to identify which ODE is active. For each pair of the ODEs, they are distinguishable if for any non trivial input these two systems produce different outputs. In this paper, it is assumed that one can only measure the position of the robot, i.e. the outputs of the studied system are q_1 and q_2. Then it is necessary to study input-output functions of the systems to study the distinguishability of the subsystems. In order to facilitate the analysis, let us consider the following change of coordinates

$$\begin{cases} Z = q_1 + jq_2 \\ \Theta = e^{jq_3} \end{cases} \quad (5)$$

where j represents the imaginary unit. Applying the change of coordinates to systems (1) - (4), one can obtain a switched singular system of the following general form:

$$\begin{cases} E_{\sigma(t)}\dot{x} = G_{\sigma(t)}(x)u \\ Y \quad\quad = Cx \end{cases} \quad (6)$$

where $x = [Z, \Theta, q_4]^T$ is the system state, $u = [\nu_1, \nu_2, \nu_3]^T$ is the input, Y is the output with $C = [1, 0, 0]$. The switching function is defined as

$$\sigma(t) : \mathbb{R}^+ \to \mathcal{I},\ \mathcal{I} \triangleq \{1, 2, 3, 4\}$$

and for different subsystems, one has

$$E_1 = E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, E_3 = E_4 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$G_1(x) = \begin{pmatrix} \Theta & 0 & 0 \\ 0 & j\Theta & 0 \\ 0 & 0 & 0 \end{pmatrix}, G_3(x) = \begin{pmatrix} j\Theta e^{jq_4} & 0 & 0 \\ 0 & j\Theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$G_2(x) = \begin{pmatrix} \Theta & j\Theta & 0 \\ 0 & 0 & j\Theta \\ 0 & 0 & 0 \end{pmatrix}, G_4(x) = \begin{pmatrix} jL\Theta \sin q_4 & 0 & 0 \\ j\Theta \cos q_4 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

It is well known that for a singular system (switched or not), the output may be not differentiable due to the existence of the singular matrix ($E_{\sigma(t)}$ in system (6)). However, since system (6) possesses special structure, i.e. $C = CE_{\sigma(t)}$ and $G_{\sigma(t)}(x) = G_{\sigma(t)}(E_{\sigma(t)}x)$, thus the output of this system is successively differentiable, i.e. $Y \in C^\infty$.

3.2 Input-output functions

Now the problem formulated here becomes the real time computation of the switching signal $\sigma(t)$ to identify the subsystems of (6). Since one can identify the switching signal $\sigma(t)$ by using the input and the output of the system, it is clear that one needs to deduce some input-output representations of each subsystem. One can notice that system (6) is defined in complex domain, and we need to take complex transforms, thus let us firstly give some necessary definitions.

For a given scalar complex function of $x \in \mathbb{C}^n$, one can note it as $z(x) = a(x) + jb(x)$, where $z : \mathbb{C}^n \to \mathbb{C}$, $a : \mathbb{C}^n \to \mathbb{R}$, $b : \mathbb{C}^n \to \mathbb{R}$, the partial derivative of z with respect to x is defined as $\frac{\partial z}{\partial x} = \frac{\partial a}{\partial x} + j\frac{\partial b}{\partial x}$. If the matrix $\frac{\partial (z, \dot{z}, \ldots, z^{(n)})}{\partial x}$ has row rank r in complex domain, then we note as $rank_{\mathbb{C}} \frac{\partial (z, \dot{z}, \ldots, z^{(n)})}{\partial x} = r$. Define $\Pi_{\sigma(t)}(x) = (Z, \Theta)^T$ for $\sigma(t) = 1, 2$ and $\Pi_{\sigma(t)}(x) = (Z, \Theta, q_4)^T$ for $\sigma(t) = 3, 4$. Then we have the following theorem on the input-output functions.

Theorem 1. Given switched singular system of the form (6), where $x \in \mathbb{C}^n$, $u \in \mathbb{R}^m$, $E_{\sigma(t)} \in \mathbb{R}^{n \times n}$, $G_{\sigma(t)} \in \mathbb{C}^{m \times m}$,

$C \in \mathbb{C}^{1\times n}$ with $C = CE_{\sigma(t)}$ and $G_{\sigma(t)}(x) = G_{\sigma(t)}(E_{\sigma(t)}x)$, if $rank_{\mathbb{C}} \frac{\partial(Y,\dot{Y},\ldots,Y^{(l_{\sigma(t)}-1)})}{\partial\Pi_{\sigma(t)}(x)} = rank_{\mathbb{C}} \frac{\partial(Y,\dot{Y},\ldots,Y^{(l_{\sigma(t)})})}{\partial\Pi_{\sigma(t)}(x)}$, then there exists an input-output representation of each subsystem of (6), and this input-output function can be obtained by taking $l^{th}_{\sigma(t)}$ order derivative of the output Y.

Proof. Proof is omitted for sake of brevity, it is similar to Conte et al. [1999], which proves the existence of the input-output functions of the regular nonlinear systems.

From Theorem 1 one can conclude that there always exist input-output functions for each subsystem of (6), and the input-output functions can be obtained by taking 2^{nd} order derivative of the output Y. Taking the subsystem $\sigma(t) = 1$ as an example, since $Y = Z$, then one has

$$\dot{Y} = \dot{Z} = \nu_1\Theta \tag{7}$$

and

$$\ddot{Y} = \dot{\nu}_1\Theta + \nu_1\dot{\Theta} = (\dot{\nu}_1 + j\nu_1\nu_2)\Theta \tag{8}$$

which leads to

$$\frac{\partial(Y,\dot{Y})}{\partial\Pi_{\sigma(t)}(x)} = \begin{pmatrix} 1 & 0 \\ 0 & \nu_1 \end{pmatrix}$$

and

$$\frac{\partial(Y,\dot{Y},\ddot{Y})}{\partial\Pi_{\sigma(t)}(x)} = \begin{pmatrix} 1 & 0 \\ 0 & \nu_1 \\ 0 & \dot{\nu}_1 + j\nu_1\nu_2 \end{pmatrix}$$

One can see that, in the real field, $rank_{\mathbb{R}} \frac{\partial(Y,\dot{Y})}{\partial\Pi_1(x)} = 2$ and $rank_{\mathbb{R}} \frac{\partial(Y,\dot{Y},\ddot{Y})}{\partial\Pi_1(x)} = 3$. However, in the complex field, one has $rank_{\mathbb{C}} \frac{\partial(Y,\dot{Y})}{\partial\Pi_1(x)} = rank_{\mathbb{C}} \frac{\partial(Y,\dot{Y},\ddot{Y})}{\partial\Pi_1(x)} = 2$, thus the condition of Theorem 1 is satisfied, and the input-output function can be calculated with the 2^{nd} order derivative of the output Y. A easy calculation via equation (7) and (8) yields the following input-output equation:

$$\ddot{Y} = \frac{\dot{\nu}_1}{\nu_1}\dot{Y} + j\nu_2\dot{Y} \tag{9}$$

Analogously, one can obtain the input-output functions for other subsystems. When $\sigma(t) = 2$, one obtains

$$\ddot{Y} = \frac{\dot{\nu}_1 + j\dot{\nu}_2}{\nu_1 + j\nu_2}\dot{Y} + j\nu_3\dot{Y} \tag{10}$$

For $\sigma(t) = 3$, one has

$$\ddot{Y} = \frac{\dot{\nu}_1}{\nu_1}\dot{Y} + j(\nu_2 + \nu_3)\dot{Y} \tag{11}$$

and if $\sigma(t) = 4$, the input-output equation is of the following form

$$\ddot{Y} = \frac{\dot{\nu}_1}{\nu_1}\dot{Y} - \frac{\nu_2 L\cos(\arg\dot{Y})\frac{d(\arg\dot{Y})}{dt}}{Re(\dot{Y})}\dot{Y} + j\frac{d(\arg\dot{Y})}{dt}\dot{Y} \tag{12}$$

where $Re(\dot{Y})$ and $\arg\dot{Y}$ are the real part and the argument of the complex number $\dot{Y}$ respectively, and $\dot{F} = \frac{d(F)}{dt}$ represents the differentiation of function F with respect to t

4. DISTINGUISHABILITY

4.1 Distinguishability of input-output functions

Once the input-output functions are obtained, one can use them to analyze the distinguishability of the subsystems. Let us firstly recall the definition of the distinguishability.

Definition:Fliess et al. [2008a] The two subsystems are said to be strongly distinguishable if, and only if, the subsystems have the same input-output behavior only when $U = [\,0\ 0\ 0\,]$ and $Y = 0$. If not the two subsystems are said to be weekly distinguishable.

It is clear that the subsystems of (6) are distinguishable with non trivial inputs, thus the problem consists in seeking the peculiar inputs that produce the same output for the subsystems, in which cases the subsystems are not distinguishable.

Theorem 2. The subsystems of (6) can be distinguished, if and only if the input $\nu_1 \neq 0$, $\nu_2 \neq 0$ and $\nu_3 \neq 0$.

Proof. Firstly let us proof the sufficiency. One can see that if $\nu_1 \neq 0$, $\nu_2 \neq 0$ and $\nu_3 \neq 0$, there is no input-output function has both the same real part and imaginary part as another input-output function, thus the input-output functions are of the different form, and we can conclude that these subsystems are distanguishable if $\nu_1 \neq 0$, $\nu_2 \neq 0$ and $\nu_3 \neq 0$.

Then let us proof the necessity. Firstly one can notice that if the subsystems are distinguishable, we must have $\nu_1 \neq 0$, since the input-output functions can not be calculated when $\nu_1 \neq 0$. Now let us consider each pair of the subsystems.

If subsystems $\sigma(t) = 1$ and $\sigma(t) = 2$ are distinguishable, the functions (9) and (10) must be different, the two functions are of the same form if and only if when $\nu_2 = 0$ and $\nu_3 = 0$. Thus if the two subsystems are distinguishable, then we have $\nu_2 \neq 0$ and $\nu_3 \neq 0$.

For subsystems $\sigma(t) = 1$ and $\sigma(t) = 3$, (9) and (11) are of the same form if and only if when $\nu_3 \neq 0$. Thus we have $\nu_3 \neq 0$, if subsystems $\sigma(t) = 1$ and $\sigma(t) = 3$ are distinguishable.

Analogously for subsystems $\sigma(t) = 2$ and $\sigma(t) = 3$, we have $\nu_2 \neq 0$, if function (10) and (11) are dinstinguishable.

For subsystems $\sigma(t) = 1$ and $\sigma(t) = 4$, the functions are of the same form if and only if when $\nu_2 = \frac{d(\arg\dot{Y})}{dt}$ and $\nu_2 = 0$ or $\cos(\arg\dot{Y}) = 0$ or $\frac{d(\arg\dot{Y})}{dt} = 0$. However $\cos(\arg\dot{Y})$ is determined by the inputs and varies during the control process, thus we can conclude that the two functions are of the same form if and only if when $\nu_2 = \frac{d(\arg\dot{Y})}{dt} = 0$. Consequently, we have $\nu_2 \neq 0$, if subsystems $\sigma(t) = 1$ and $\sigma(t) = 4$ are distinguishable.

As for subsystems $\sigma(t) = 2$ and $\sigma(t) = 4$, functions (10) and (12) are of the same form if and only if when $\nu_2 = \nu_3 = \frac{d(\arg\dot{Y})}{dt} = 0$, thus we have $\nu_2 \neq 0$ or $\nu_3 \neq 0$ if subsystems $\sigma(t) = 2$ and $\sigma(t) = 4$ are distinguishable.

Analogously for subsystems $\sigma(t) = 3$ and $\sigma(t) = 4$, functions (11) and (12) are of the same form if and only if when $\nu_2 = \nu_3 = \frac{d(\arg\dot{Y})}{dt} = 0$, thus we have $\nu_2 \neq 0$ or $\nu_3 \neq 0$ if subsystems $\sigma(t) = 3$ and $\sigma(t) = 4$ are distinguishable.

In summary, we have $\nu_1 \neq 0$, $\nu_2 \neq 0$ and $\nu_3 \neq 0$, if the subsystems are distinguishable.

Since the input-output functions are distinguishable, one can use those equations to identify the switching signal, which will be detailed in the following.

4.2 Calculation of residuals

After having obtained the distinguishable input-output equation for each subsystem of (6), let us define the residual associated to the subsystem as follows:

$$R_i(t) = \begin{cases} \ddot{Y} - \frac{\dot{\nu}_1}{\nu_1}\dot{Y} - j\nu_2\dot{Y}, & i=1 \\ \ddot{Y} - \frac{\dot{\nu}_1 + j\dot{\nu}_2}{\nu_1 + j\nu_2}\dot{Y} - j\nu_3\dot{Y}, & i=2 \\ \ddot{Y} - \frac{\dot{\nu}_1}{\nu_1}\dot{Y} - j(\nu_2+\nu_3)\dot{Y}, & i=3 \\ \ddot{Y} - \frac{\dot{\nu}_1}{\nu_1}\dot{Y} + \frac{\nu_2 L\cos(\arg \dot{Y})\frac{d(\arg \dot{Y})}{dt}}{Re(\dot{Y})}\dot{Y} - j\frac{d(\arg \dot{Y})}{dt}\dot{Y}, & i=4 \end{cases}$$

It is clear that the current i^{th} subsystem is active if $R_i(t) = 0$. Since the residuals are complex numbers, if both the real part and the imaginary part go to zero within a short time period, the corresponding $\sigma(t)$ can be identified, thus

$$\sigma(t) = i, \quad if \int_{T_R} |R_i(t)| dt = 0, i \in \mathcal{I} \tag{13}$$

where T_R is a freely chosen but very short residual judging window.

It should be noted that the judging rule (13) is valid only for the case where one can precisely measure the input, the output and its derivatives. However, for the case where the input or the output are corrupted with noises, or the derivatives of the input and the output are not known (in this case one need to calculate them by some additional techniques), the calculated residuals are not equal to 0 within the window T_R, then the judging rule (13) can be replace by the following one:

$$\sigma(t) = \arg\min_{i\in\mathcal{I}} \int_{T_R} |R_i(t)| dt \tag{14}$$

In this case, the problem is then reduced to a real-time computation of time derivative of the input and the output of the studied system despite of noises, which makes the calculation of the derivative become a crucial issue.

4.3 Numerical differentiation

The numerical differentiation technique presented here was proposed by Fliess et al in Sira-Ramirez and Fliess [2006], and more details can be found in Fliess et al. [2008b], Mboup et al. [2009] and the references therein.

Consider a signal $y(t) = \sum_{k=0}^{\infty} y^{(k)}(0)\frac{t^k}{k!}$ which is assumed to be analytic around $t = 0$ and its truncated Taylor expansion $y_N(t) = \sum_{k=0}^{N} y^{(k)}(0)\frac{t^k}{k!}$, where $t > 0$. Its Laplace transform is of the following form:

$$Y_N(s) = \sum_{k=0}^{N} \frac{y^{(k)}(0)}{s^{k+1}} \tag{15}$$

Introducing the *algebraic derivation* $\frac{d}{ds}$, and multiply both sides of equation (15) by $\frac{d^\alpha}{ds^\alpha}s^N$, $\alpha = 0, 1, ..., N$, one has a triangular system of linear equations and from which the derivatives can be obtained:

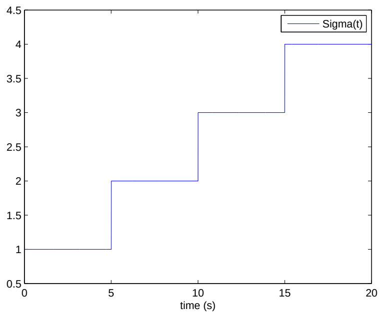

Fig. 2. Switching signal $\sigma(t)$

$$\frac{d^\alpha s^N Y_N}{ds^\alpha} = \frac{d^\alpha}{ds^\alpha}(\sum_{k=0}^{N} y^{(k)}(0)s^{N-k-1}) \tag{16}$$

which is independent of all the unknown initial conditions, and the coefficients $y(0), ..., y^{(k)}(0)$ are *linearly identifiable* Fliess and SiraRamrez [2003], then the $y^{(k)}(0)$ can be obtained by taking the inverse laplace transform of (16) over a time window T.

It is worth noting that the algebraic technique stated here is robust with respect to noises involved into the control inputs and outputs. Noises are viewed here as highly fluctuations around 0, therefore they can be attenuated by low-pass filters, as iterated integrals with respect to time. Moreover this algebraic technique has other advantages: it is of non-asymptotic nature, the desired estimation can be obtained instantaneously; it provides explicit formulae, which can be implemented directly; it does not require any assumption concerning the statistical distribution of the unstructured noise.

In practice, this algebraic technique is implemented with discrete measured data, thus it is necessary that the sampling time T_s should be small enough with respect to the duration time between two successive switchings [1] Mboup et al. [2009], Liu et al. [2011]. Moreover in Liu et al. [2011] some other analyses are discussed for several classes of noises.

5. SIMULATION RESULTS

Usually the model of the robot is not changing (except for some particular situations). However since we assume that the robot type is unknown, thus $\sigma(t)$ depends on different types of robots. In order to show the feasibility of the proposed method, it is assume that $\sigma(t)$ is a time-varying signal. In the previous section we have discussed the condition of the distinguishability, thus one can choose $v = [1.5, 1.3, 0.5]^T$ to avoid the indistinguishable cases. For the simulation setting, the sampling time of numerical differentiator is $Ts = 0.005s$, the sliding time window is $T = 0.5s$ and the residual judging window is $T_R = T = 0.5s$. The switching signal $\sigma(t)$ is shown in Fig. 2, and the output of the switching system is shown is Fig. 3. The first scenario supposes that the output and the differentiation of the output can be directly obtained without noises, and the simulation results are

[1] In practice it is at least 100 times smaller, thus the Zeno phenomenon are excluded

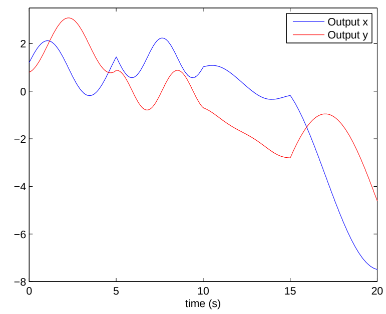

Fig. 3. Output

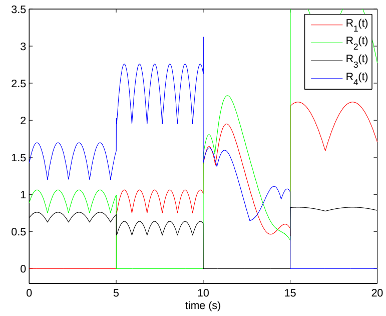

Fig. 4. Residuals when the differentiations can be obtained directly

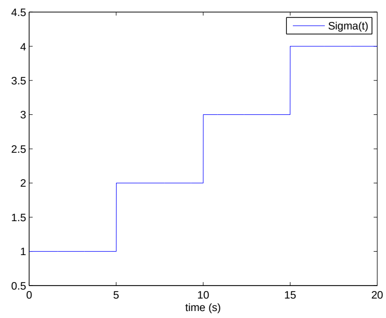

Fig. 5. Switching signal $\sigma(t)$ identified when the differentiations can be obtained directly

shown in Fig. 4 and Fig. 5. In this case, the switching signal is identified by $\int_{T_R} |R_i(t)|dt = 0$ and one can see that the identification of active mode is perfect. The second scenario assumes that the differentiation of the outputs is not directly known but without noises, and the 1st and 2nd order derivatives of the output are calculated by numerical differentiator presented in this paper. Simulation results are shown in Fig. 6 and Fig.7 with the same input and output as the former simulation. One can see that the residuals do not equal to 0 because of the calculation errors, and the switching signal is identified by $\sigma(t) = \arg\min_{i\in\mathcal{I}} \int_{T_R} |R_i(t)|dt$. One can notice as well that there exists a short time interval where $\sigma(t)$ can not be identified, and this is due to the fact that we use the numerical differentiator and the integral R_i over the residual judging window T_R, so this non identifiable interval is equal to T and can be reduced by reducing the time window T and residual judging window T_R. The final

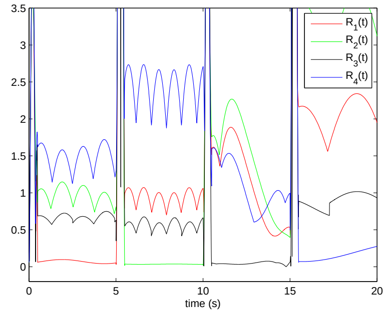

Fig. 6. Residuals when the differentiations are not known and calculated by the numerical differentiator

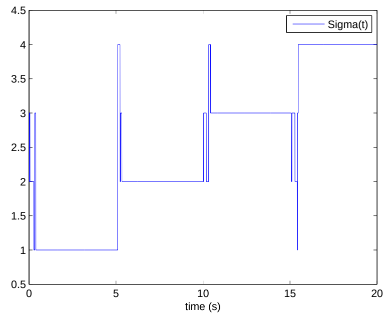

Fig. 7. Switching signal $\sigma(t)$ identified when the differentiations are not known and calculated by the numerical differentiator

scenario is similar to the second scenario, but supposes that there are noises adding to the output measurement, shown in Fig. 8. The simulation results are depicted in Fig. 9 and Fig. 10 with the white Gaussian noise of $SNR = 50dB$ (signal-to-noise ratio), and the identified switching signal $\sigma(t)$ is the same as previous simulations. One can conclude from the results that the proposed method is robust to noises, and the subsystems can be identified quickly in real time by using the judging rule: $\sigma(t) = \arg\min_{i\in\mathcal{I}} \int_{T_R} |R_i(t)|dt$.

6. CONCLUSION

The identification of mobile robot systems is discussed in this paper, and the problem is formulated as the identification of the switching signal of a switched singular nonlinear system. The distinguishability of the deduced switched singular system is studied. The proposed technique can be implemented in real-time and it is quite robust to the noises in the measurement. The good performance of the technique was validated by several simulations.

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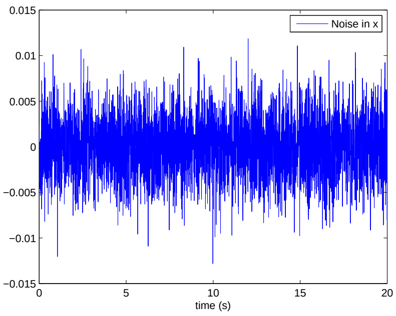
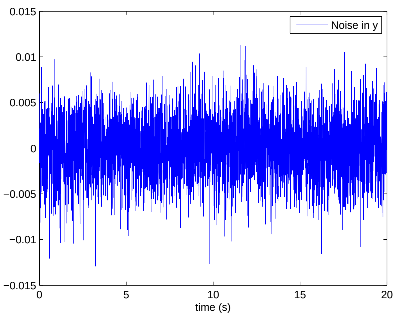


(a) Noise imposed in X



(b) Noise imposed in Y

Fig. 8. White Gaussian noise of $SNR = 50dB$

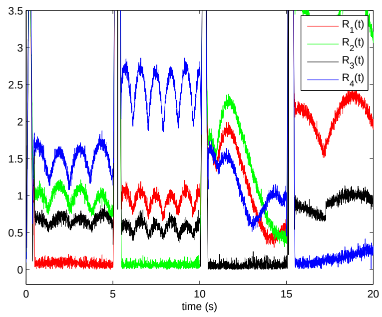


Fig. 9. Residuals with white noise SNR=50dB

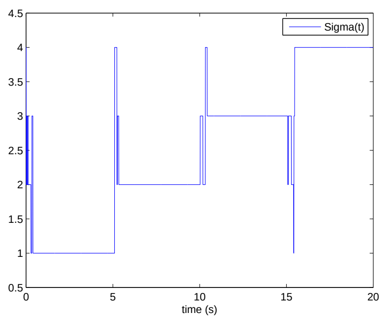


Fig. 10. Switching signal $\sigma(t)$ identified with white noise SNR=50dB

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