Discrete IDA-PBC design for 2D port-Hamiltonian systems ⋆

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Abstract: We address the discrete-time passivity-based control laws synthesis within port-Hamiltonian framework. We focus on IDA-PBC design for canonical port-Hamiltonian systems with separable energy being quadratic in momentum. For this class of systems, we define a discrete Hamiltonian dynamics that exactly satisfies a discrete energy balance. We then derive a discrete controller following the IDA-PBC procedure. The proposed methodology relies on an energy discretization scheme with suitable discrete conjugate port variables. The main result is illustrated on two examples: a nonlinear pendulum in order to compare with some simulation results of the literature, and the impact oscillator which requires robust discretization scheme.

1. INTRODUCTION

Discrete-time control design is of fundamental interest when considering control technics based on continuous-time model properties. One motivation would be stated as follows: as numerical integration scheme may not translate all model's properties, the resulting discrete dynamics may differ from the expected one. Therefore, information loss induced by the time-discretization step has to be controlled in order to frame the discrete behaviour. This is especially the case in the Hamiltonian framework considering passivity-based controllers.

Energy-based modeling, stemming from Lagrangian and Hamiltonian approaches in the field of mechanics, has provided a new mathematical framework for the analysis of dynamical systems. These approaches have been widely developed and extended to many engineering areas such as electronics, electromagnetics, mechatronics, etc. Subsequently, the concept of energy has been brought to control design issues.

In this paper, the port-Hamiltonian framework (introduced in [Maschke and van der Schaft, 1992]) is considered. It is known to handle systems representation and analysis in various physical domains. By nature, *Port-Hamiltonian systems* (denoted PHS) fit into the Dissipative systems theory [Willems, 1972]. This intrinsic property underlies the *interconnection damping assignment passivity-based control* method (denoted IDA-PBC) developed in [Ortega et al., 2002b]. In this framework, IDA-PBC appears crucial since it generates all asymptotically stabilizing controllers. However, since simulation results are obtained using a computer solver (regardless system properties), the discrete closed-loop behaviour may be not as efficient as expected for the reason mentioned above.

A common method used to design a discrete controller relies on sampling a continuous-time controller. Such a controller, called *emulation controller*, is easy to compute but its performances are inconstant. In practice, its implementation is done by evaluating the continuous-time controller at the computed discrete states. Its performance thus relies on the discretization scheme accuracy.

Otherwise, discrete controller synthesis can be tackled from a direct design viewpoint. In the Hamiltonian framework, this approach is based on the definition of a discrete port-Hamiltonian dynamics (*e.g.*, structure discretization [Talasila et al., 2006], time-discretisation [Laila and Astolfi, 2006b], sampled-data [Stramigioli et al., 2005, Monaco et al., 2009]). All these discrete Hamiltonian dynamics have been derived with respect to some initial intrinsic properties, such as passivity, (almost) energy balance. The direct discrete-time IDA-PBC design issues have been treated in the case of separable Hamiltonian [Laila and Astolfi, 2005] and for underactuated controlled Hamiltonian systems [Laila and Astolfi, 2006a, Gören-Sümer and Yalçin, 2011]. The drawback of these results resides on an almost energy balance equation (leading to numerical drift) that may be overcome to improve the performance. Anyway, all these references agree to the fact that emulation controller can be enhanced.

Our contributions concern discrete IDA-PBC design generating a closed-loop behaviour similar to the continuous one. To this end, we focus on discrete systems intrinsic properties: lossless characteristics and passivity. We first introduce a new definition of a discrete port-Hamiltonian dynamics that exactly satisfies a discrete energy balance. This definition is based on an energy-preserving integrator [Greenspan, 1974] and suitable conjugate port-output variables. The design is then derived for 2D fully actuated canonical port-Hamiltonian systems with separable energy being quadratic in momentum. Simulation results show the improvement of the numerical behaviours with respect to discrete controllers proposed in the literature.

The paper is organized as follows. In Section 2, we briefly introduce port-Hamiltonian systems and IDA-PBC design in this framework. Section 3 concerns our contributions: we define a discrete port-Hamiltonian dynamics which is conservative, and the discrete IDA-PBC design is then derived. Two examples have been implemented and the simulation results are discussed in Section 4. Section 5 concludes the paper.

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2. HAMILTONIAN FRAMEWORK & IDA-PBC DESIGN

In this section, we introduce the class of nonlinear systems and the associated control laws synthesis we are concerned with.

We first recall basics on the port-Hamiltonian framework [Maschke and van der Schaft, 1992, van der Schaft, 1999]. In particular, such systems satisfies a dissipative equality which we aim at preserving at the discrete level.
In this framework, it has been proven that all asymptotically stabilizing controllers can be generated by IDA-PBC design [Ortega et al., 2002b]. We recall here the fully actuated case that a discrete version will be derived in Section 3.

2.1 Port-Hamiltonian Systems (PHS)

We are interested in nonlinear systems with precise structure, the so-called *port-Hamiltonian systems* (also denoted PHS throughout the paper). More precisely, we consider canonical PHS which dynamics writes [van der Schaft, 1999]

$$\begin{cases} \begin{bmatrix} \dot q \\ \dot p \end{bmatrix} = \begin{bmatrix} 0 & \mathrm{Id} \\ -\mathrm{Id} & 0 \end{bmatrix} \begin{bmatrix} \nabla_q H \\ \nabla_p H \end{bmatrix} + \begin{bmatrix} 0 \\ B(q) \end{bmatrix} u(t) \\ y(t) = B(q)^T \nabla_p H \end{cases} \tag{1}$$

where $q \in \mathbb{R}^n$ is the generalized displacement, $p \in \mathbb{R}^n$ the generalized momenta, $\nabla_x H$ denotes the partial derivative of H with respect to x, $B(q) \in \mathbb{R}^{n\times m}$ is the input force matrix, with $B(q)u$ denoting the generalized forces resulting from the control inputs $u \in \mathbb{R}^m$, and $y \in \mathbb{R}^m$ is the conjugate port-output of the system.

The Hamiltonian (energy) function H is supposed to be separable and quadratic in p, that is, of the form

$$H(q,p) = \frac{1}{2} p^T M^{-1} p + V(q) \quad \text{with} \quad M^T = M > 0\,. \tag{2}$$

The time derivative of H along trajectories of (1) is

$$\frac{\mathrm{d}}{\mathrm{d}t} H(q(t),p(t)) = y^T(t) u(t) \tag{3}$$

which by integration leads to the dissipative equality

$$H(t) - H(t_0) = \int_{t_0}^{t} y^T(s) u(s) ds\,. \tag{4}$$

Equation (4) characterizes a fundamental property of PHS (1): the system is said to be conservative (*i.e.* lossless and passive). It states that the stored energy of such systems equals the energy supplied through the port variables.

Note that since the control laws synthesis we are considering is based on system passivity, we paid a particular attention to preserve this property while deriving a discrete Hamiltonian dynamics.

Furthermore, for control issues, two essentials cases have to be distinguished with respect to the range m of the input vector field. The PHS (1) is said to be fully actuated when $m = n$, and underactuated when $m < n$. Since we aim at illustrating our new discrete Hamiltonian dynamics for discrete controller design issues, we shall restrict our investigations on the simpler case of fully actuated systems. Thus, B is assumed to be invertible.

2.2 Continuous-time IDA-PBC design

We are concerned with control law synthesis following the IDA-PBC design presented in [Ortega et al., 2002b]. In the context of nonlinear systems given by (1), IDA-PBC technic aims at designing the desired closed-loop energy as

$$H_d(q,p) = \frac{1}{2} p^T M_d^{-1} p + V_d(q) \tag{5}$$

together with the closed-loop dynamics

$$\begin{bmatrix} \dot q \\ \dot p \end{bmatrix} = [J_d - R_d] \begin{bmatrix} \nabla_q H_d \\ \nabla_p H_d \end{bmatrix} \tag{6}$$

where

$$J_d = -J_d^T = \begin{bmatrix} 0 & M^{-1}M_d \\ -M_d M^{-1} & J_2(q,p) \end{bmatrix}$$

and

$$R_d = R_d^T = \begin{bmatrix} 0 & 0 \\ 0 & BK_{di}B^T \end{bmatrix}.$$

To summarize, IDA-PBC design consists of two steps.

The first one, called *energy shaping*, fixes the desired energy H_d which has a strict local minimum at the desired equilibrium. The associated control input u_{es} is obtained by solving the model matching (1) = (6) without dissipative matrix

$$\begin{bmatrix} 0 & \mathrm{Id} \\ -\mathrm{Id} & 0 \end{bmatrix} \begin{bmatrix} \nabla_q H \\ \nabla_p H \end{bmatrix} + \begin{bmatrix} 0 \\ B(q) \end{bmatrix} u_{es} = \begin{bmatrix} 0 & M^{-1}M_d \\ -M_d M^{-1} & J_2(q,p) \end{bmatrix} \begin{bmatrix} \nabla_q H_d \\ \nabla_p H_d \end{bmatrix}. \tag{7}$$

The energy shaping controller u_{es} has to satisfy

$$B(q)\, u_{es} = \left\{ \nabla_q H - M_d M^{-1} \nabla_q H_d + J_2 \nabla_p H_d \right\}. \tag{8}$$

Here comes the restriction we are concerned with, that is the system is fully actuated and the matrix B is assumed to be invertible. Hence the control law is easily computed by left multiplying both sides by $B(q)^{-1}$.

Remark 1. In the case of underactuated systems, a matching condition associated with the annihilator of B arises. A set of PDEs has to be solvable in order to derive the energy shaping controller using the Moore-Penrose pseudo-inverse. For details see [Ortega et al., 2002a,b].

The second step, called *damping injection*, consists of adding friction to the system in order to achieve asymptotic stabilization of the desired equilibrium. The damping injection controller u_{di} is constructed as

$$u_{di} = -K_{di} y = -K_{di} B(q)^T \nabla_p H_d\,, \quad K_{di} > 0\,. \tag{9}$$

The complete control law writes $u = u_{es} + u_{di}$.

3. MAIN RESULT

As pointed out in [Laila and Astolfi, 2005], discretizing Hamiltonian equations and evaluating the continuous-time IDA-PBC controller at discrete states yield a closed-loop behaviour with performance far from acceptable (compare the desired energy of the *continuous controller* with the *emulation controller* in Figure 1). In the literature, there are only few references that address discrete IDA-PBC design. Basically, those investigations focus on direct discrete-time IDA-PBC controller design based on a modified integration scheme that aims at satisfying an energy balance up to the second order [Laila and Astolfi, 2005, 2006a, Gören-Sümer and Yalçin, 2011].

We shall present here a discrete port-Hamiltonian dynamics which exactly satisfies an energy balance equation. This formulation is used to design a discrete IDA-PBC controller with closed-loop performance similar to the designed continuous performance. In particular, we shall illustrate energy conservation of the closed-loop system when only energy shaping is considered.

Our result combines an energy-preserving discretization scheme introduced in [Greenspan, 1974] with a suitable definition of discrete conjugate port variables. The IDA-PBC controller synthesis is then considered as a model matching issue in the restricted case of a fully actuated controlled Hamiltonian system.

Remind that we only deal with the 2-dimensional fully actuated case, with separable energy being quadratic in momentum.

3.1 Discrete port-Hamiltonian systems

The energy-preserving scheme proposed in [Greenspan, 1974] concerns canonical Hamiltonian systems with separable energy $H(q,p)$ quadratic in p, as given in equation (1) with $u=0$. Notice that this scheme preserves the structure of the equation (that is, the structure matrix J). It writes

$$\begin{bmatrix} \dfrac{q_{n+1}-q_n}{\Delta t} \\ \\ \dfrac{p_{n+1}-p_n}{\Delta t} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \dfrac{V(q_{n+1})-V(q_n)}{q_{n+1}-q_n} \\ \\ \dfrac{p_{n+1}+p_n}{2} \end{bmatrix}. \qquad (10)$$

This descretization scheme exactly preserves the Hamiltonian. Indeed, on a time mesh, the energy variation writes

$$\begin{aligned} \Delta H_n &\overset{\Delta}{=} H(q_{n+1},p_{n+1}) - H(q_n,p_n) \\ &= \left[\frac{1}{2}(p_{n+1})^2 + V(q_{n+1})\right] - \left[\frac{1}{2}(p_n)^2 + V(q_n)\right]. \end{aligned}$$

Rearranging the terms, it remains to see that

$$\begin{aligned} \frac{1}{2}\left[(p_{n+1})^2 - (p_n)^2\right] &= \left(\frac{p_{n+1}+p_n}{2}\right)(p_{n+1}-p_n) \\ &\overset{(10)}{=} \left(\frac{q_{n+1}-q_n}{\Delta t}\right)(p_{n+1}-p_n) \end{aligned}$$

and

$$V(q_{n+1}) - V(q_n) \overset{(10)}{=} -\left(\frac{p_{n+1}-p_n}{\Delta t}\right)(q_{n+1}-q_n)$$

to conclude that $\Delta H_n \equiv 0$ regardless the time step Δt.

We now introduce port variables. The control input u naturally arises in the $\dot{p}$ equation, whereas the conjugate port-output y can be defined by several manner. In the literature, following the continuous-time definition of a PHS, a classical definition of y_n relies on the gradient of H evaluated at (q_n,p_n). The slight difference here resides on the use of a discrete gradient.

Let us now introduce our definition of discrete PHS dynamics.

Definition 2. A (canonical) *discrete port-Hamiltonian dynamics* is defined by

$$\begin{cases} \dfrac{q_{n+1}-q_n}{\Delta t} = \dfrac{p_{n+1}+p_n}{2} \\ \dfrac{p_{n+1}-p_n}{\Delta t} = -\dfrac{V(q_{n+1})-V(q_n)}{q_{n+1}-q_n} + B(q_n)u_n \\ y_n = B(q_n)\dfrac{q_{n+1}-q_n}{\Delta t} \end{cases} \qquad (11)$$

Among the discrete Hamiltonian systems formulation that can be found in the literature (*e.g.* structure discretization [Talasila et al., 2006], time-discretisation [Laila and Astolfi, 2006b], sampled-data [Stramigioli et al., 2005, Monaco et al., 2009]), the discrete Hamiltonian dynamics (11) encodes two fundamental properties: energy conservation and passivity.

Proposition 3. The discrete PHS (11) is conservative w.r.t. the same storage function H.

Proof. The statement of the proposition is straightforward once noticed that on a time-mesh the energy balance writes

$$H(q_{n+1},p_{n+1}) - H(q_n,p_n) = \Delta t\, y_n u_n .$$

The discrete Hamiltonian dynamics (11) satisfies a dissipative equality (the system is lossless) with a storage function rate given as the input/output product (the system is passive), it is thus conservative. ■

3.2 Discrete IDA-PBC design

Based on the discrete Hamiltonian dynamics defined by (11), we design a discrete-time controller following IDA-PBC.

The open-loop discrete dynamics is given by (11), and the desired closed-loop discrete dynamics is considered as

$$\begin{bmatrix} \dfrac{q_{n+1}-q_n}{\Delta t} \\ \\ \dfrac{p_{n+1}-p_n}{\Delta t} \end{bmatrix} = [J-R_d] \begin{bmatrix} \dfrac{V_d(q_{n+1})-V_d(q_n)}{q_{n+1}-q_n} \\ \\ \dfrac{p_{n+1}+p_n}{2} \end{bmatrix}, \qquad (12)$$

which is the discrete version of (6) with desired energy H_d (5).

If we now translate the model matching equation (7) within discrete dynamics, the energy shaping controller writes

$$(u_{es})_n = B^{-1}(q_n)\left(\frac{V(q_{n+1})-V(q_n)}{q_{n+1}-q_n} - \frac{V_d(q_{n+1})-V_d(q_n)}{q_{n+1}-q_n}\right). \qquad (13)$$

The damping injection controller given by (9) is computed with the suitable discrete conjugate output defined in (11) as follows

$$(u_{di})_n = -K_{di}y_n = -K_{di}B(q_n)\frac{q_{n+1}-q_n}{\Delta t}. \qquad (14)$$

Finally, the discrete control law writes $u_n = (u_{es})_n + (u_{di})_n$. Let us now state the closed-loop property obtained with u_n.

Proposition 4. Consider the discrete dynamics (11) and the desired closed-loop dynamics (12) where V_d has an isolated minimum at q^*. Then

(i) $(q^*,0)$ is a (locally) stable equilibrium of the closed-loop system with $u_n = (u_{es})_n$ given by (13).
(ii) $(q^*,0)$ is an asymptotically stable equilibrium of the closed-loop system with $u_n = u_{es}(n) + u_{di}(n)$ given by (13) and (14).

Proof. Notice first that, by construction, the control law u_n applied to the system (11) leads to the closed-loop system (12). We shall use Lyapunov's second theorem to prove the statement of the proposition. Let $L(x) = H_d(x) - H_d(x^*)$ be the Lyapunov candidate, where $x^* = (q^*,0)$. Then L is positive definite in a neighborhood of x^* and $\Delta L_n = (\Delta H_d)_n$. It follows

(i) $\Delta L_n \equiv 0$ in the case $u_n = (u_{es})_n$, hence x^* is stable
(ii) $\Delta L_n < 0$ in the case $u_n = (u_{es})_n + (u_{di})_n$ since a straightforward calculation leads to

$$\begin{aligned} \Delta L_n &\overset{\Delta}{=} L(q_{n+1},p_{n+1}) - L(q_n,p_n) \\ &= -\frac{1}{\Delta t}(q_{n+1}-q_n)^T B K_{di} B^T (q_{n+1}-q_n) \end{aligned} \qquad (15)$$

Hence x^* is asymptotically stable for any $K_{di} > 0$. ■

Notice that, as in the continuous setting, the dissipative property of the discrete closed-loop system is an intrinsic property of the system.

4. EXAMPLES

We shall now illustrate the performances of the proposed discrete-time design method. Two classical examples have been implemented: the nonlinear pendulum which has been studied in the literature, and the impact oscillator which is considered to have a very rich dynamic (its momenta has to change very rapidly on a short time interval).

In order to compare numerical results, a reference curve is needed, namely the continuous-time solution. Generally speaking, no analytical solution is available, thus we shall consider our reference, the tagged *continuous* curves, as the numerical solution obtained with high order accuracy solver and small time step.

4.1 Nonlinear pendulum

Consider the nonlinear pendulum given in Hamiltonian form (1) with energy function

$$H(q,p) = \frac{1}{2}p^2 - \cos(q)\,. \tag{16}$$

The dynamics then explicitly writes

$$\begin{cases} \dot{q} = p \\ \dot{p} = -\sin(q) + u(t) \\ y = p \end{cases} . \tag{17}$$

The desired closed-loop energy is chosen as

$$H_d(q,p) = \frac{1}{2}p^2 - \cos(q) + \frac{K_{es}}{2}q^2 + 1\,. \tag{18}$$

The continuous-time IDA-PBC design, as recalled in equations (8) and (9), yields the control laws

$$u_{es}(t) = -K_{es}\,q(t) \quad \text{and} \quad u_{di}(t) = -K_{di}\,p(t) \tag{19}$$

whereas the discrete-time IDA-PBC derived in this paper yields the control laws $(u_{es})_n$ and $(u_{di})_n$ given equations (13) and (14) respectively. The simulation results are compared with the closed-loop behaviours obtained with: 1) the emulation controller, that is evaluating (19) at stage n, and 2) the controller presented in [Laila and Astolfi, 2005, 2006a] based on a modified Euler scheme.

All numerical results concern the same initial conditions $(q_0, p_0) = (0.5\pi - 0.2, 0.5)$. The tagged *continuous* curves have been obtained with MATLAB `ODE45` solver with fixed time step $\Delta t = 10^{-4}$(s), and the remaining curves have been implemented with a time step $\Delta t = 0.35$(s).

We first consider the energy shaping controllers u_{es}. The Figure 1 shows the closed-loop energies H_d computed with the different schemes. One notices that the emulation and the literature controllers are not energy preserving. In contrast, the design method proposed in this paper exactly preserves the energy of the closed-loop system. This feature relies on the conservative property of the discrete Hamiltonian dynamics (11) as stated in Proposition 4.

Consider now the whole IDA-PBC steering law $u_{es} + u_{di}$. The Figure 2 shows the orbits in the phase plane. Clearly, with

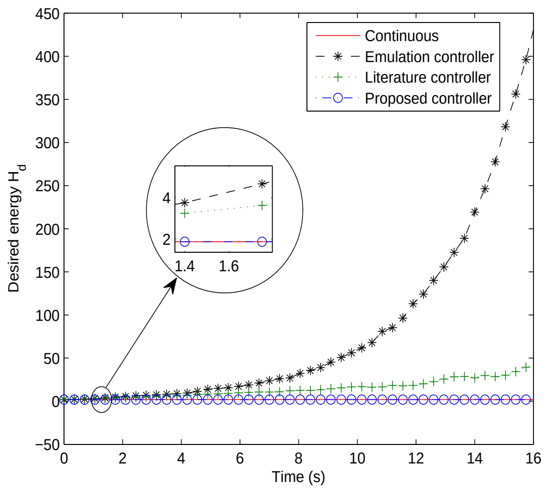

Fig. 1. Energy of the closed-loop with u_{es} controllers ($u_{di} = 0$).

the chosen energy shaping and damping gains, all trajectories converge to the equilibrium point (the origin). However the convergence rates differ. The closed-loop behaviour obtained with the discrete controller of proposition 4(ii) and the continuous one are similar, and they do have the same convergence rate. But the two remaining discrete approaches are less satisfactory. Indeed, as the latter dynamics do not fully satisfy the energy balance, the controller has to compensate the trajectory errors due to the energy drift at any time mesh, hence the convergence rate decrease.

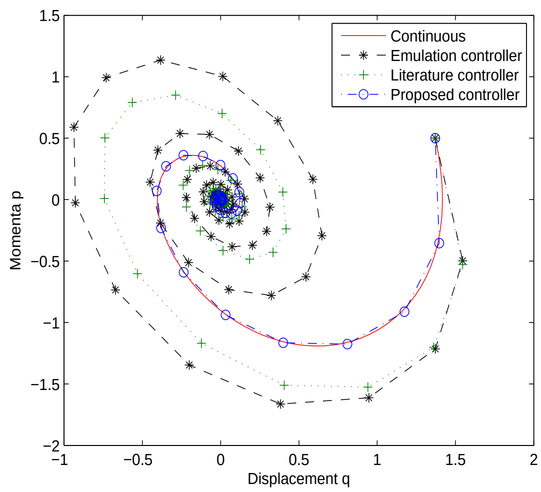

Fig. 2. Trajectories of the pendulum with control $u_{es} + u_{di}$.

Furthermore, it is worth noting that, if the discrete energy balance is not satisfied, the control law stabilizes the equilibrium only if the damping injection gain is able to compensate the energy drift at any time mesh. The explanation is the following. The continuous-time closed-loop energy balance involves a skew-symmetric product (associated with J_d) minus a symmetric non-negative product (associated with R_d). The first one is zero by skew-symmetry, and the energy variation is thus negative. However, in discrete settings, preserving the structure of the equation (that is the J_d matrix) is no longer sufficient to guarantee energy conservation. As an illustration, focus on the one mesh detail presented in the Figure 1. It is there obvious that if $(u_{di})_n$ is not able to compensate the numerical growth of energy, the desired energy will not converge (to zero in the present case) and the associate control law will not be stabilizing. So, if the energy is not conserved, there exists a critical damping gain depending on the choice of the discretization scheme and on the time step (as the scheme and the step size characterize the numerical growth of energy on a mesh). Hence dissipativity is no longer an intrinsic property of the discrete dynamics. Stabilizing neither. This will be illustrated in the following example.

4.2 Impact oscillator

Let us now consider the so-called *impact oscillator* which is considered to have rich dynamics [Hairer et al., 2002]. Its Hamiltonian function is given by

$$H(q,p) = \frac{1}{2}p^2 + \frac{0.15}{2}q^2 + \frac{1}{q^2}, \quad q \neq 0, \tag{20}$$

generating the dynamics

$$\begin{cases} \dot{q} = p \\ \dot{p} = -0.15\,q + \dfrac{2}{q^3} + u(t) \\ y = p \end{cases} . \tag{21}$$

The desired closed-loop energy is chosen as

$$H_d(q,p) = H(q,p) + 0.35\,\cos(q) - \frac{1}{2}\ln(q) - \frac{3}{2}q \tag{22}$$

As previously, one computes the continuous-time IDA-PBC control laws following the equations (8) and (9)

$$u_{es}(t) = 0.35\sin(q(t)) + \frac{1}{2q(t)} + \frac{3}{2}$$

and

$$u_{di}(t) = -K_{di}\,p(t) \tag{23}$$

The discrete energy shaping control design of Proposition 4 writes

$$\begin{aligned}(u_{es})_n = & -0.35\frac{\cos(q_{n+1}) - \cos(q_n)}{q_{n+1} - q_n} \\ & + \frac{\ln(q_{n+1}) - \ln(q_n)}{2(q_{n+1} - q_n)} + \frac{3}{2}\end{aligned} \tag{24}$$

which has been solved using a fixed point method. As in the previous example, we shall compare the simulation results with the discrete controllers proposed in the literature.

All numerical results concern the same initial conditions $(q_0, p_0) = (4, -0.75)$, damping gain $K_{di} = 0.05$, and simulation time $t_{end} = 150$(s). The tagged *continuous* curves have been obtained with MATLAB `ODE23T` solver (as specifically recommended in [Leimkuhler and Reich, 2005]) and the remaining curves have been implemented with a time step $\Delta t = 0.8$(s).

We first consider the energy shaping controllers u_{es}. All the computed orbits are presented in figure 3. As expected, one notices that the continuous-time controller and the discrete controller proposed here (Proposition 4(i)) have a similar behaviour. As the proposed discrete Hamiltonian dynamics is conservative, its orbit winds around the continuous one. As expected, the trajectories computed with the discrete controllers taken from the literature suffer from discretization and design errors, hence the divergent orbits.

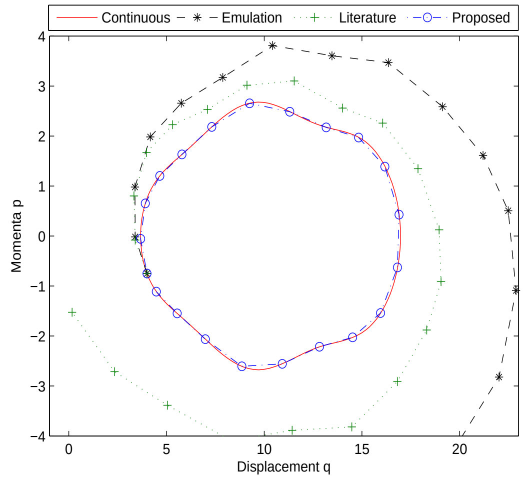

Fig. 3. Orbits of the impact oscillator with control u_{es} ($u_{di} = 0$).

Consider now the whole IDA-PBC steering law $u_{es} + u_{di}$. The trajectories are presented in the Figure 4. Once again, the continuous-time controller and that one of Proposition 4(ii) lead to similar closed-loop dynamics. For the remaining curves, representing the closed-loop behaviours obtained with the controllers taken from the literature, the orbits diverge. Despite of adding damping in the system, the amount of added dissipation is not enough to compensate the discretization and design errors. Dissipativity is thus no longer an intrinsic property of the discrete dynamics. The discretization step did not translate this genuine fundamental property which is essential to forecast discrete closed-loop behaviour.

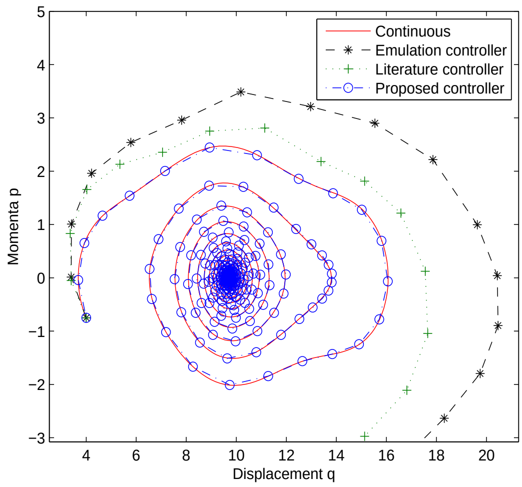

Fig. 4. Trajectories of the impact oscillator with control $u_{es} + u_{di}$.

To emphasize the role of the discrete system properties (resulting from the choice of the discretization scheme and the definition of suitable conjugate port output), let us apply the discrete controllers proposed in the literature to a discrete Hamiltonian dynamics derived by an energy-preserving integrator such as (10). The trajectories are presented in the Figure 5. The trajectories associated with the discrete controllers taken from the literature are no longer divergent (compare with Figure 4). They both converge to a limit cycle. Again, the control law proposed in [Laila and Astolfi, 2005] seems more efficient since it steers the system closer to the desired equilibrium. Notice that these results still depend on the step size. However, it is worth noting that each controller generates its own limit cycle: the trajectories actually converge to distinct limit cycles! The discrete closed-loop behaviour is thus highly connected to the controller settings (and the step size) although the design is originally based on intrinsic system properties. This illustrates a severe drawback of such discrete controller design. The closed-loop behaviour analysis becomes no more systematic and has to be studied case-by-case. Indeed, Greenspan integrator ensures a stable energetic behaviour (hence convergent trajectories compared to Figure 4), but design error remains. The observed limit cycles precisely correspond to the exact balance of the design error by the stable energetic behaviour. Therefore, one may translate a limit cycle as the numerical energy level associated with the design error.

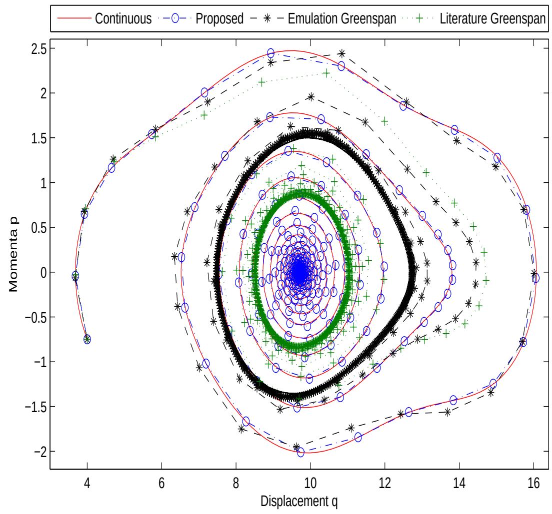

Fig. 5. Trajectories of the impact oscillator with control $u_{es}+u_{di}$ computed with Greenspan scheme.

5. CONCLUSION

In this paper, the discrete IDA-PBC design has been addressed. In continuous-time, this design relies on intrinsic system properties (such as energy conservation and passivity). The idea is thus to preserve these properties while deriving a discrete dynamics. We give here a definition of a discrete port-Hamiltonian dynamics which exactly satisfies a discrete energy balance. We then use this discrete Hamiltonian dynamics to design a discrete controller following the IDA-PBC procedure. It has been shown that the desired equilibrium is (asymptotically) stable with this discrete controller. Then, the efficiency of this design method has been discussed on two examples. The first one illustrates the efficiency improvement with respect to discrete controllers proposed in the literature, especially concerning its convergence rate. In the second one, we stress that discretization and/or design errors disrupt discrete dynamics properties and generate aleatory behaviours (in the sense that they have to be studied case-by-case). We point out the intrinsic system properties that have to be carried out to the discrete system in order to forecast its closed-loop behaviour.

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