Optimal Control of Digital Hydraulic Drives using Mixed-Integer Quadratic Programming

Mateusz Sniegucki *, **Markus Gottfried** *, **Uwe Klingauf** **

* *Bosch Rexroth AG, An den Kelterwiesen 14, 72160 Horb am Neckar, Germany (e-mail: mateusz.sniegucki@boschrexroth.de).*
** *Institut für Flugsysteme und Regelungstechnik, Technische Universität Darmstadt, Petersenstrasse 30, 64287 Darmstadt, Germany, (e-mail: klingauf@fsr.tu-darmstadt.de)*

Abstract: Control of dynamical systems gets considerably harder with an increasing number of control variables. Especially when the control variables are restricted to integer values, the solution is of combinatorial complexity. An example of such systems are Digital Hydraulic Drives, where several cylinders contribute to the output torque independently. In this work we present an optimal control approach for torque control of Digital Hydraulic Drives using Mixed-Integer Quadratic Programming in a Model Predictive Control framework. The nonlinear behavior and discrete valued inputs resulting from the use of on-off valves, are accommodated in the control model using a Mixed Logical Dynamical System representation. With the presented approach, optimal switching sequences for the electrical valves are computed that produce the desired torque trajectory with fast tracking and minimal ripple, while keeping switching events at a minimum and respecting physical system constraints.

Keywords: Digital Hydraulic Drives, Hybrid Systems, MIQP, MLD Systems, MPC

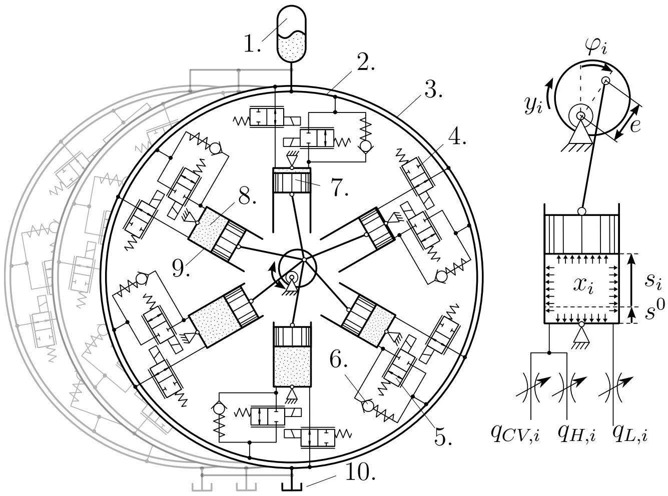
1. INTRODUCTION

Digital Hydraulics gain increasing attention in the fluid power community. As opposed to traditional analogue control components, such as proportional valves, which deliver a smooth output but are sensitive to external changes, digital hydraulics utilize discrete valued on-off valves. Similar to digital systems in electronics, digital hydraulic systems achieve robustness, repeatability and redundancy through the use of many yet simple components (Linjama and Vilenius [2011]).

A particular application of digital hydraulic systems are Digital Hydraulic Drives. They are comprised of cylinders, which independently generate a portion of the total torque output. In a radial design, the cylinders are radially arranged around an eccentric (Fig. 1). Each cylinder contains a piston, and electrically actuated on-off valves, that control the cylinder pressure. Due to the nature of on-off valves, the amplitude of the drive output cannot be altered continuously (i.e. either high- or low pressure is applied to the piston) but is rather quantized in portions according to the amount of cylinders present. This inherent feature creates ripple in the torque output, which can induce severe oscillations in the driveshaft, causing damage or inhibiting proper machine function. Especially in high-performance applications, such as hydraulic hybrid vehicles, this feature is very restricting. However, the amount of independent control variables holds potential for torque ripple mitigation, and due to the combinatorial complexity and nonlinear behavior, also makes control a challenging task. Control methods for Digital Hydraulic Drives that have been investigated in the past are limited and can be divided into two basic approaches:

Ehsan et al. [2000] consider torque generation as a succession of pulses in which the demanded torque output is modulated using delta-sigma ($\Delta\Sigma$) modulation. Similarly, other modulation techniques, such as Pulse-Width Modulation (PWM) can be considered as well. Because modulation yields an average value, the instantaneous torque output is afflicted with high errors in the form of torque ripple. Armstrong and Yuan [2006] propose setpoint tracking by scheduling all activated cylinders repeatedly at every sample step. Cylinder selection is based on the enumeration of switching possibilities, that yield the least deviation from the current setpoint. However, transient behavior and the number of switching events are not regarded, leaving room for further exploration.

In this work, we propose a nonlinear Model Predictive Control approach and employ a hybrid system model, which accommodates the nonlinear pressure dynamics and the discrete event behavior. By incorporating system knowledge, we show how torque ripple can be significantly reduced. Furthermore the presented methodology allows to shape the torque output according to given demands, such as frequency content and valve switching events. The work is structured as follows: In section 2 a short outline on related applications is given. Section 3 presents the physical system modeling and derives a mixed logical dynamical system representation. In section 4 the design of optimal control is shown. In section 5 we discuss the simulation results and conclude the work in section 6.



1. high pressure accumulator
2. high pressure line
3. low pressure line
4. low pressure valve (LPV)
5. high pressure valve (HPV)
6. check valve (CV)
7. piston
8. oil
9. cylinder
10. oil tank

Fig. 1. Digital Hydraulic Drive

2. HYBRID MODEL PREDICTIVE CONTROL

Model Predictive Control is an iterative control method, which optimizes an objective function on a finite horizon with respect to given demands (Maciejowski [2002]). As a model based control scheme, performance of MPC heavily relies on the employed system model. For the optimization to be manageable, low complexity models are demanded, while simultaneously maintaining a high prediction accuracy. Hence for the nonlinear behavior and the discrete nature of Digital Hydraulic Drives to be accounted for, using a hybrid system representation is favorable. In the context of speed control for Digital Hydraulic Drives, MPC describes the commutation of individual cylinders. In an outer loop, a conventional speed controller determines the demanded torque based on the current and desired shaft speed and feeds it into the MPC Controller (Fig. 2).

Related control tasks involving nonlinear, periodic system behavior and discrete inputs have been approached by employing MPC. Geyer et al. [2009] used MPC for the control of three phase AC electric drives. The motor torque and stator flux are kept within given hysteresis bounds choosing the optimal inverter switching combination to minimize switching events. Peyrl et al. [2009] examined MPC for torque control of switched reluctance motors (SRM). The optimization task for the nonlinear model is solved by enumeration of all possible switching possibilities for the DC-link power converter of a three phase SRM. Switching and continuous dynamics for optimal control of DC-DC Converters were considered by Asano et al. [2006] in utilizing a hybrid prediction model using a Mixed Logical Dynamical (MLD) system description. Modeling using MLD systems was introduced by Bemporad and Morari [1999] and an application of MPC using MLD system models was shown for a multi-tank system by Habibi et al. [2005].

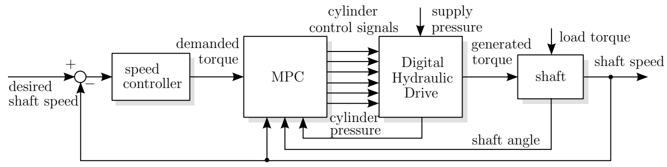


Fig. 2. MPC Control Strategy

3. SYSTEM MODELING

The basic element of the Digital Hydraulic Drive is a single cylinder with a moving piston, two actively controlled valves and a check valve (CV) that opens if the cylinder pressure exceeds the supply pressure. When the high pressure valve (HPV) is activated and the low pressure valve (LPV) is closed, fluid from the high pressure supply flows through an orifice into the cylinder, where pressure is built up. The resulting force on the piston generates torque and thus drives the shaft. When deactivated (open), the LPV permits fluid intake from the tank and inhibits decompression in the cylinder while the piston is in downstroke, whereas it permits fluid outtake and inhibits compression while in upstroke. When the LPV is activated (closed), it permits compression and decompression. The electrical valves determine the way pressure is built up in the cylinder, and thus shape the torque output accordingly. Each cylinder contributes to the overall torque output of the system by generating a periodic torque. Because all cylinders are phase shifted relative to each other around the eccentric, the instantaneous torque output of each cylinder is different.

3.1 Continuous Modeling

The continuity equation for compressible fluids

$$q_{in} - q_{out} = q = \frac{dV}{dt} + \frac{V}{\beta} \cdot \frac{dx}{dt} \tag{1}$$

describes the pressure change $\frac{dx}{dt}$ within the cylinder, when there is a flow q into the cylinder or a change in cylinder volume $\frac{dV}{dt}$. The pressure dynamics are characterized by the hydraulic compressibility coefficient β. Considering the cylinder geometry, (1) can be rearranged into

$$\dot{x}_i = (q_i - A \cdot \dot{s}_i(\varphi)) \cdot \frac{\beta}{A\,(s^0 + s_i(\varphi))}, \tag{2}$$

describing the evolution of pressure $x_i(t)$ over time t for cylinder $i = 1, 2, \ldots, N$. From the kinematic relation, the piston stroke s_i and thus the displaced volume $V_i = A \cdot s_i$ can be inferred from the shaft angle φ. The dead volume $A \cdot s^0$ is the part of the cylinder volume, that remains undisplaced after a full piston stroke. The kinematic relation between shaft angle and piston stroke can be described by

$$s_i(\varphi) = e \cdot (1 - \cos(\varphi + (i-1)\phi))\,, \quad \forall i = 1, 2, \ldots, N, \tag{3}$$

and accordingly the stroke velocity

$$\dot{s}_i(\varphi) = e\omega \sin(\varphi + (i-1)\phi), \tag{4}$$

where $\phi = \frac{2\pi}{N}$ is the phase shift, e the eccentricity and $\omega = \frac{d\varphi}{dt}$ the angular shaft velocity. The sum of flows into the cylinder is given by

$$q_i = q_{i,H} + q_{i,L} - q_{i,CV}. \tag{5}$$

Here the leakage behavior is accommodated in an adapted compressibility coefficient β. The valve hydraulics are described according to Akers et al. [2006] by

$$q_m = \alpha A_m \cdot \sqrt{\frac{2}{\rho}} \cdot \sqrt{|x_m - x|} \cdot \mathrm{sgn}(x_m - x) \cdot \nu_m \qquad (6)$$

as the flow through an orifice, given the orifice flow coefficient α, the oil density ρ, source pressure x_m, cylinder pressure x and the cross-sectional area of the orifice A_m. The valve stroke $\nu_m \in [0, 1]$ determines the orifice opening of the valve and ultimately its volume flow. With $m = \{H, L, CV\}$ the pressure sources for the respective valves are the supply pressure $x_H = x_{CV}$ for the HPV and CV and the tank pressure $x_L = 0$ for the LPV.

The resulting cylinder pressure dynamics in (2) consist of two parts, each with significantly different time constants:

$$\dot{x}_i = (\underbrace{q_i}_{\text{fast}} - \underbrace{A \cdot \dot{s}_i(\varphi)}_{\text{slow}}) \cdot \tilde{\beta}_i(\varphi), \qquad (7)$$

with $\tilde{\beta}_i(\varphi) = \frac{\beta}{A(s^0 + s_i(\varphi))}$. Therefore it is beneficial to decompose (2) into fast and slow dynamics. The fast dynamics are given by the volume flow through the opened valves, whereas the slow dynamics are governed by the piston stroke velocity. Decomposition yields the impulsive system with instantaneous state jumps:

$$\dot{x}_i = \begin{cases} 0,\ x_i(t+\tau) = x_H & \text{if } \nu_H = 1 \wedge \nu_L = 0 \text{ or } x_i(t) > x_H, \\ A\dot{s}_i(\varphi)\tilde{\beta}_i(\varphi) & \text{if } \nu_H = 0 \wedge \nu_L = 0, \\ 0,\ x_i(t+\tau) = 0 & \text{if } \nu_H = 0 \wedge \nu_L = 1 \text{ and } x_i(t) < 0, \end{cases} \qquad (8)$$

with τ representing a dead time due to present valve dynamics. The discrete time description is obtained by the forward Euler approximation

$$x_i(k+1) = \begin{cases} x_H & \text{if } \nu_H = 1 \wedge \nu_L = 0 \text{ or } x_i(k+1) > x_H, \\ x_i(k) + f_i(\varphi) & \text{if } \nu_H = 0 \wedge \nu_L = 0, \\ 0 & \text{if } \nu_H = 0 \wedge \nu_L = 1 \text{ and } x_i(k+1) < 0 \end{cases} \qquad (9)$$

where the change in pressure is governed by

$$f_i(\varphi) = \Delta T\ A\tilde{\beta}_i(\varphi)\dot{s}_i(\varphi). \qquad (10)$$

Because system commands are synchronized and triggered by the measured shaft angle, it is desirable to transform the system evolution from the time domain into the domain of the shaft angle φ. Given the relation $d\varphi = \omega(t)dt$, and assuming that $\Delta\omega \approx 0$, the sample time can be stated as $\Delta T = \frac{\Delta\varphi}{\omega(t)}$. Thus the discrete system evolution is described along the shaft angle $\varphi = k\Delta\varphi$, where k is the sample number. Equation (10) can be restated to

$$f_i(k) = \frac{\Delta\varphi}{\omega(k)}\ A\tilde{\beta}_i(k)\dot{s}_i(k). \qquad (11)$$

Note that from (4) $\dot{s}_i(k) \propto \omega(k)$, $f_i(k)$ becomes independent of the shaft speed and thus constant in the domain of the shaft angle. The torque output of a single cylinder is defined by the cylinder pressure $x_i(k)$ acting on the piston surface area A and the lever arm of the eccentric $l_i(k) = e\sin(k\Delta\varphi + (i-1)\phi)$, resulting in the output

$$y_i(k) = Ax_i(k)l_i(k). \qquad (12)$$

The overall torque output of the system is given by the superposition of all cylinder outputs $y(k) = \sum_{i=1}^{N} y_i(k)$.

3.2 MLD System Representation

The hybrid system model (9) is ruled by externally induced switching events through valve actuation and system state dependent switching. For the use in MPC and the related solution of optimal control problems, an integrated system model is required. This can be provided using a Mixed Logical Dynamical system formulation. In the framework of Bemporad and Morari [1999], systems described by physical laws, logic rules, and operating constraints, are described by linear dynamic equations subject to mixed-integer inequalities. Upon this description, the case dependent equation (9) can be expressed in the integral form

$$x_i(k+1) = x_i(k)\delta_i(k) + f_i(k)\delta_i(k) + x_H u_i(k), \qquad (13)$$

by introducing the logical variables $\delta_i(k), u_i(k) \in \{0, 1\}$. The nonlinear term $x_i(k)\delta_i(k)$ can be overcome by defining an auxiliary variable $z(k) = x_i(k)\delta_i(k)$, which together with the logical variables represents the input variables of the system. The value of $z_i(k)$ needs to be imposed by $\delta_i(k)$ such that

$$\begin{aligned} [\delta_i(k) = 1] &\rightarrow [z_i(k) = x_i(k)] \\ [\delta_i(k) = 0] &\rightarrow [z_i(k) = 0], \end{aligned} \qquad (14)$$

which can be expressed by the inequalities

$$\begin{aligned} z_i(k) &\leq M\delta_i(k) \\ z_i(k) &\geq m\delta_i(k) \\ z_i(k) &\leq x_i(k) - m(1 - \delta_i(k)) \\ z_i(k) &\geq x_i(k) - M(1 - \delta_i(k)), \end{aligned} \qquad (15)$$

with $m = 0$ and $M = x_H$ representing the range limits of the system state $x_i(k) \in [m, M]$. Now the system is described by the linear equation

$$x_i(k+1) = z_i(k) + f_i(k)\delta_i(k) + x_H u_i(k). \qquad (16)$$

The desired system behavior given by (9) is achieved by defining the logical relations

$$\begin{aligned} [x_i(k+1) \geq x_H] &\leftrightarrow [u_i(k) = 1] \wedge [\delta_i(k) = 0], \\ [0 < x_i(k) < x_H] &\leftrightarrow [u_i(k) = 0] \wedge [\delta_i(k) = 1], \\ [x_i(k+1) \leq 0] &\leftrightarrow [u_i(k) = 0] \wedge [\delta_i(k) = 0], \\ [\delta_i(k) = 1] &\rightarrow [u_i(k) = 0], \\ [u_i(k) = 1] &\rightarrow [\delta_i(k) = 0], \end{aligned} \qquad (17)$$

expressing the state saturation $x_i(k) \in [m, M]$, compression/decompression behavior, and restricting concurrent opening of high and low pressure valves to prevent hydraulic short-circuiting. Herein the logical variable $u_i(k)$ directly corresponds to the HPV state ν_H, whereas $\delta_i(k)$ corresponds to $\neg(\nu_H \wedge \nu_L)$. The resulting inequalities are

$$\begin{aligned} x_i(k+1) &\leq M \\ x_i(k+1) &\geq m \\ x_i(k) + f_i(k)\delta_i(k) &\leq M(\delta_i(k) + u_i(k)) \\ \mathrm{sgn}(s_i(k))y_i(k) + Al_i(k)f_i(k) &\leq M(\delta_i(k) + u_i(k)) \\ 1 &\geq \delta_i(k) + u_i(k). \end{aligned} \qquad (18)$$

Finally the entire system comprising N cylinders is expressed in the time variant MLD system form

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{z}(k) + \boldsymbol{B}_2(k)\boldsymbol{\delta}(k) + \boldsymbol{B}_1\boldsymbol{u}(k), \\ y(k) &= \boldsymbol{c}^T(k)\boldsymbol{x}(k), \\ \boldsymbol{E}_2(k)\boldsymbol{\delta}(k) + \boldsymbol{E}_3\boldsymbol{z}(k) &\leq \boldsymbol{E}_1\boldsymbol{u}(k) + \boldsymbol{E}_4(k)\boldsymbol{x}(k) + \boldsymbol{E}_5(k), \end{aligned} \qquad (19)$$

where $y(k) \in \mathbb{R}$ is the torque sum, $\boldsymbol{x}(k) \in \mathbb{R}^N$ the cylinder pressure vector, $\boldsymbol{z}(k) \in \mathbb{R}^N$ the auxiliary variable vector and $\boldsymbol{\delta}(k), \boldsymbol{u}(k) \in \{0, 1\}^N$ are the logical variable vectors. The system matrices are $\boldsymbol{B}_2(k) = \mathrm{diag}(f_1(k), f_2(k), \dots, f_N(k))$, $\boldsymbol{B}_1 = x_H\boldsymbol{I}_N$, the $N \times N$ identity matrix $\boldsymbol{I}_N$ and the output matrix $\boldsymbol{c}^T(k) = A\,[l_1(k)\ l_2(k)\ \dots\ l_N(k)]$. The inequalities (15) and (18) are contained in (19) with $\boldsymbol{E}_1 =$

$[\mathbf{0}_N, \mathbf{0}_N, \mathbf{0}_N, \mathbf{0}_N, -\boldsymbol{B}_1, \boldsymbol{B}_1, \boldsymbol{B}_1, \boldsymbol{B}_1, -\boldsymbol{I}_N]^T$ and $\boldsymbol{E}_2(k)$, $\boldsymbol{E}_3$, $\boldsymbol{E}_4(k)$, $\boldsymbol{E}_5(k)$ are established accordingly, where $\mathbf{0}_N$ is the $N \times N$ zero matrix.

4. OPTIMAL CONTROL OF A DIGITAL HYDRAULIC DRIVE

The goal of the optimization is to compute the input variables necessary to achieve a minimum deviation between the desired- and produced torque. The computation is carried out for a specified finite horizon of samples, yielding an optimal switching sequence for each electrical valve. The number of switching events is also considered in the optimization task to be minimized.

4.1 Design Objectives

The optimization task to be solved is described by the objective function

$$\min_{\boldsymbol{u},\boldsymbol{\delta},\boldsymbol{z}} J = \sum_{j=1}^{N_p} \underbrace{\|y(k) - w(k)\|_Q^2}_{\text{setpoint deviation}} + \underbrace{\|\Delta \boldsymbol{u}(k)\|_{\boldsymbol{R}_1}^2 + \|\Delta \boldsymbol{\delta}(k)\|_{\boldsymbol{R}_2}^2}_{\text{switching events}}, \tag{20}$$

where $w(k)$ is the setpoint and $\|\boldsymbol{x}\|_{\boldsymbol{P}}^2 = \boldsymbol{x}^T \boldsymbol{P} \boldsymbol{x}$. Equation (20) consists of conflicting objectives. Naturally, minimizing switching events $\Delta \boldsymbol{u}(k) = \boldsymbol{u}(k) - \boldsymbol{u}(k-1)$ and $\Delta \boldsymbol{\delta}(k) = \boldsymbol{\delta}(k) - \boldsymbol{\delta}(k-1)$ will result in a greater setpoint deviation, because less combinatorial possibilities arise for achieving the desired torque. Hence there is a trade-off based on the control objective. Priorities can be set by an appropriate choice of the weighting values $Q \geq 0, \boldsymbol{R}_1 \succeq 0, \boldsymbol{R}_2 \succeq 0$.

4.2 Control Constraints

Because highest priority and motivation of Digital Hydraulic Drives is a high efficiency operation, activating cylinders contributing with opposing torques is prohibited. Thus an additional constraint is formulated, where

$$y_i(k+1) \cdot \operatorname{sgn}(w(k+1)) \geq 0, \tag{21}$$

which also greatly reduces the solution space, as only half of the cylinders become available for control.

4.3 Algebraic Reformulation

The system equations (19) are rewritten as prediction equations for every step $j = k$ to $j = k + N_p$ in the horizon,

$$\boldsymbol{\mathcal{X}}(k) = \boldsymbol{\mathcal{Z}}(k) + \boldsymbol{\mathcal{B}}_2(k)\boldsymbol{\mathcal{D}}(k) + \boldsymbol{\mathcal{B}}_1\boldsymbol{\mathcal{U}}(k), \tag{22}$$

$$\boldsymbol{\mathcal{Y}}(k) = \boldsymbol{\mathcal{C}}(k) \cdot \boldsymbol{\mathcal{X}}(k) \tag{23}$$

$$\boldsymbol{\mathcal{E}}_2(k)\boldsymbol{\mathcal{D}}(k) + \boldsymbol{\mathcal{E}}_3\boldsymbol{\mathcal{Z}}(k) \leq \boldsymbol{\mathcal{E}}_1\boldsymbol{\mathcal{U}}(k) + \boldsymbol{\mathcal{E}}_4(k)\boldsymbol{\mathcal{X}}(k) + \boldsymbol{\mathcal{E}}_5(k), \tag{24}$$

so that $\boldsymbol{\mathcal{X}}(k) = [\boldsymbol{x}(k+1)\ \boldsymbol{x}(k+2)\ \ldots\ \boldsymbol{x}(k+N_p)]^T$ contains the state values over time for a given input sequence $\boldsymbol{\mathcal{Z}}(k) \in \mathbb{R}^{N \cdot N_p}, \boldsymbol{\mathcal{D}}(k) \in \{0,1\}^{N \cdot N_p}, \boldsymbol{\mathcal{U}}(k) \in \{0,1\}^{N \cdot N_p}$. Because $\boldsymbol{\mathcal{Z}}(k), \boldsymbol{\mathcal{D}}(k), \boldsymbol{\mathcal{U}}(k)$ are defined as system inputs, (22) alone represents a mere input-output relation. Only in conjunction with (24) the recursive character of dynamical systems becomes apparent. Considering the receding horizon policy of the MPC scheme, given the measured shaft angle velocity $\omega(k)$, the Matrices $\boldsymbol{\mathcal{B}}_2(k), \boldsymbol{\mathcal{C}}(k), \boldsymbol{\mathcal{E}}_2(k), \boldsymbol{\mathcal{E}}_4(k)$ and $\boldsymbol{\mathcal{E}}_5(k)$ need to be evaluated for every optimization step k, triggered at the respective shaft angle $\varphi = k\Delta\varphi$.

The objective function (20) can be restated to

$$\begin{aligned} \min_{\boldsymbol{u},\boldsymbol{\delta},\boldsymbol{z}} J = {} & (\boldsymbol{\mathcal{Y}}(k) - \boldsymbol{\mathcal{W}}(k))^T\, \boldsymbol{\mathcal{Q}}\, (\boldsymbol{\mathcal{Y}}(k) - \boldsymbol{\mathcal{W}}(k)) \\ & + \Delta\boldsymbol{\mathcal{U}}^T(k)\boldsymbol{\mathcal{R}}_1\Delta\boldsymbol{\mathcal{U}}(k) + \Delta\boldsymbol{\mathcal{D}}^T(k)\boldsymbol{\mathcal{R}}_2\Delta\boldsymbol{\mathcal{D}}(k), \end{aligned} \tag{25}$$

where $\Delta\boldsymbol{\mathcal{U}}(k) = \boldsymbol{\mathcal{K}}_{\boldsymbol{u}}\boldsymbol{\mathcal{U}}(k) + \boldsymbol{\mathcal{L}}_{\boldsymbol{u}}\boldsymbol{u}(k)$ and $\Delta\boldsymbol{\mathcal{D}}(k) = \boldsymbol{\mathcal{K}}_{\boldsymbol{\delta}}\boldsymbol{\mathcal{D}}(k) + \boldsymbol{\mathcal{L}}_{\boldsymbol{\delta}}\boldsymbol{\delta}(k)$, satisfying $\Delta\boldsymbol{u}(k) = \boldsymbol{u}(k) - \boldsymbol{u}(k-1)$ and $\Delta\boldsymbol{\delta}(k) = \boldsymbol{\delta}(k) - \boldsymbol{\delta}(k-1)$ respectively. By concatenating the inputs and prediction matrices, the optimization task can be formulated in standardized form

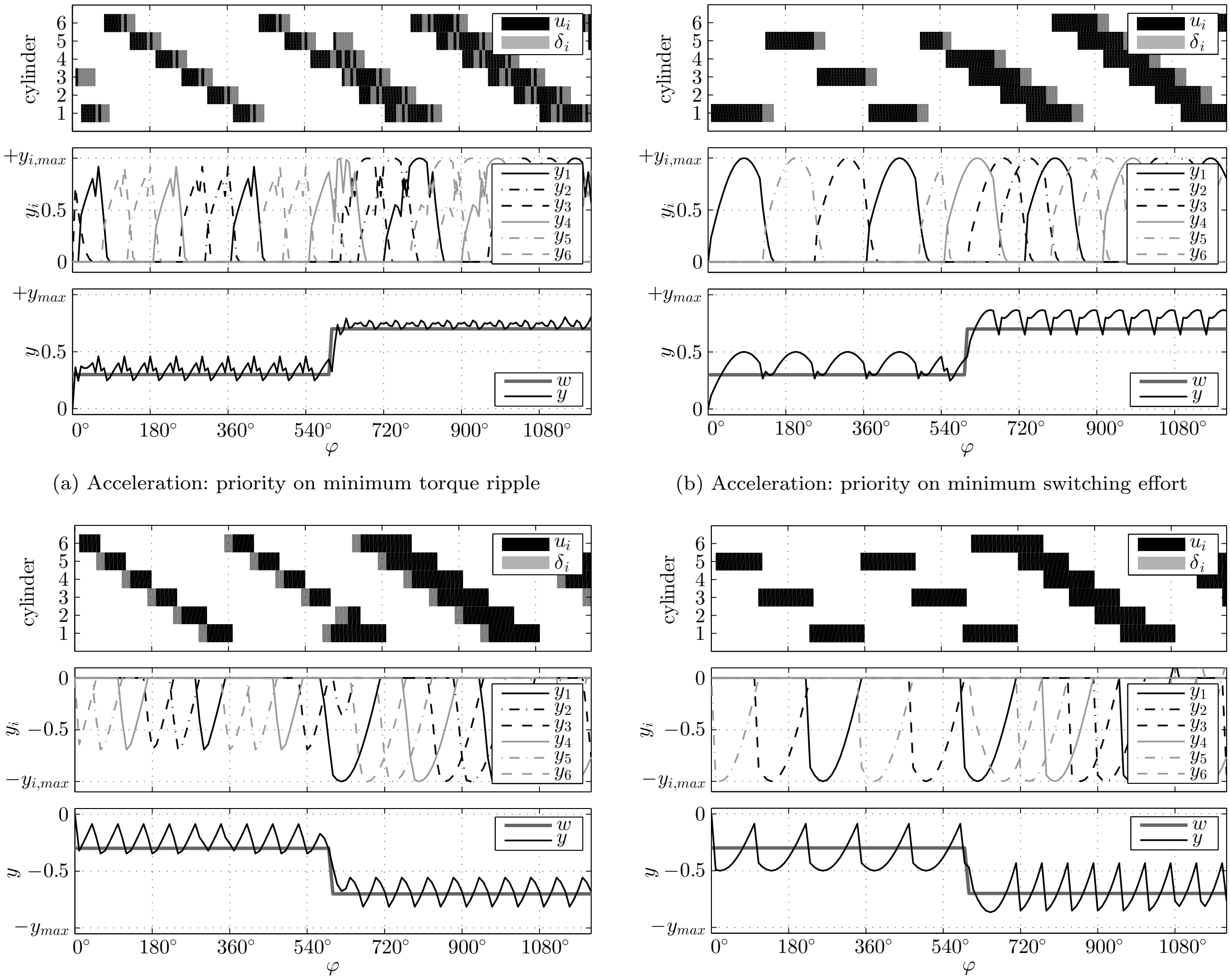
$$\begin{aligned} \min_{\boldsymbol{\mathcal{V}}(k)} \ & J = \boldsymbol{\mathcal{V}}^T(k)\boldsymbol{\mathcal{S}}_1(k)\boldsymbol{\mathcal{V}}(k) + \boldsymbol{\mathcal{S}}_2(k)\boldsymbol{\mathcal{V}}(k) \\ \text{s.t.} \ & \boldsymbol{\mathcal{F}}_1(k)\boldsymbol{\mathcal{V}}(k) \leq \boldsymbol{\mathcal{F}}_2(k) + \boldsymbol{\mathcal{F}}_3(k)\boldsymbol{x}(k), \end{aligned} \tag{26}$$

where $\boldsymbol{\mathcal{V}}(k) = [\boldsymbol{\mathcal{Z}}(k)\ \boldsymbol{\mathcal{D}}(k)\ \boldsymbol{\mathcal{U}}(k)]^T$ and the measured initial condition $\boldsymbol{x}(k)$. Solving the Mixed-Integer Quadratic Programming (MIQP) Problem (26) at every sample step k, yields the desired valve activation sequences.

5. SIMULATION RESULTS

The proposed control scheme was verified in a simulation for a Digital Hydraulic Drive with $N = 6$ cylinders, a normalized supply pressure $x_H = 1$, and a constant shaft speed $\omega(k) = 1000\frac{1}{\min}$. During operation, a setpoint change from $w = 0.35 \cdot y_{max}$ to $w = 0.7 \cdot y_{max}$ is to be tracked, with minimum deviation in the transient- and steady state. In a second approach, system deceleration is simulated, by a setpoint change from $w = -0.35 \cdot y_{max}$ to $w = -0.7 \cdot y_{max}$. The control performance substantially depends on the choice of the weighting matrices $\boldsymbol{\mathcal{Q}}, \boldsymbol{\mathcal{R}}_1, \boldsymbol{\mathcal{R}}_2$, the sample size $\Delta\varphi$ and the prediction horizon N_p. First the weighting is chosen to yield a minimum setpoint deviation, neglecting the amount of valve switching events. Subsequently, the weighting is shifted in favor of a lower switching effort. In both cases the prediction horizon needs to be sufficiently high to allow prediction of possible constraint violations. For instance, the prediction must always ensure, that any activated cylinder will not produce negative torque, for any given positive setpoint. Figure 4 shows the solution of the optimization depicted in Fig. 3(a) in detail. Here to respect constraints, the cylinder must be deactivated, so that the subsequent decompression is completed ($x_1 = 0$) prior to reaching the bottom dead center (nominal torque $\overline{y}_1 = 0$). Herefrom it can be concluded, that the prediction horizon has to be at least as large as the largest decompression time or angle. The sample size determines the resolution of the computed states and the complexity of the optimization task. While small sample sizes increase resolution, they also increase the problem complexity and computation time. Therefore an appropriate size is chosen. For the simulation, a prediction horizon of $N_p = 10$ samples and a sample size of $\Delta\varphi = 6°$ is chosen, which for $\omega = 1000\frac{1}{\min}$ equals a sampling time of 1 ms. Computation is performed using MATLAB R2011b running on an Intel Core i7-2820QM CPU at 2.3 GHz with 8 GB RAM in Win 7. For solving the MIQP problem, which is $\mathcal{NP}$-complete and in general hard [1] to solve (Papadimitriou and Steiglitz [1998]), CPLEX 12.3 is used with YALMIP version 3 (Löfberg [2004]) as a MATLAB interface. With an average of 60 active binary decision variables per iteration, the average solving time per iteration is 2 s.

[1] It is widely considered, that no polynomial time algorithm exists



(a) Acceleration: priority on minimum torque ripple

(b) Acceleration: priority on minimum switching effort

(c) Deceleration: priority on minimum torque ripple

(d) Deceleration: priority on minimum switching effort

Fig. 3. MPC results for acceleration, deceleration and different weighting values

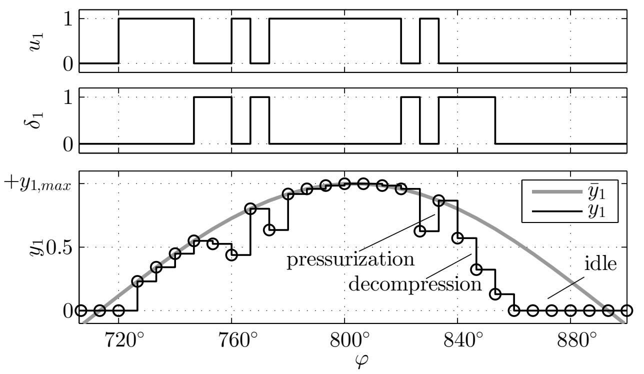
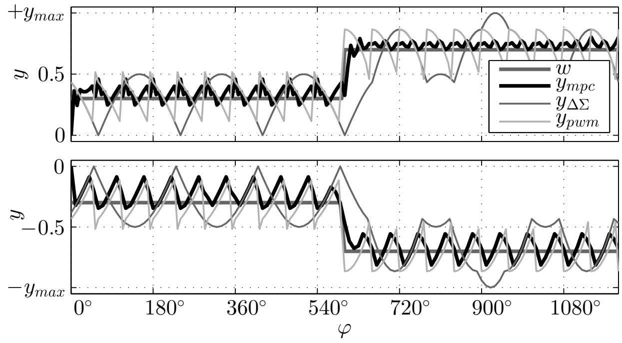


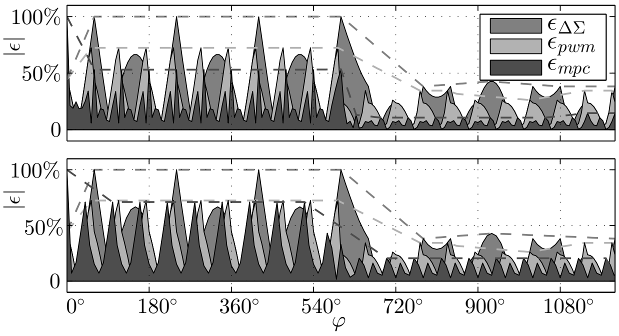
Fig. 4. System behaviour while accelerating

Figure 3(a) shows simulation results for a minimum setpoint deviation priority, set by the diagonal weighting matrices $\boldsymbol{\mathcal{Q}} = \boldsymbol{I}_{N_p}$ and $\boldsymbol{\mathcal{R}}_1 = \boldsymbol{\mathcal{R}}_2 = \boldsymbol{0}_{N \cdot N_p}$. The upper plot shows the switching values u_i and δ_i $\forall i = 1, \ldots, N$ for the respective cylinders (black $\equiv [u_i = 1]$, gray $\equiv [\delta_i = 1]$). The switching pattern yields the single torque values y_i (middle plot), which in sum create the overall output torque y (bottom plot). Figure 3(c) shows an optimal switching solution for shaft deceleration. As opposed to accelerating, in deceleration each cylinder acts as a pump where decompression is not available. Hence disabling cylinders while being activated is not possible until pistons change directions at their top dead center. This results in the phase cutting characteristics of y_i (middle plots) where essentially fewer combination possibilities arise and therefore more torque ripple retains. While activating u yields high pressure transients and thus high torque ripple, activation of δ enables de- or compression (depending on the shaft angle) resulting in smooth transients that reduce pulsation effects (Fig. 4). Therefore reducing torque ripple and minimizing setpoint deviation demands a frequent alternation between instantaneous pressurization and decompression, which increases control effort in terms of valve switching frequency. In order to decrease the control effort, weighting matrices were exemplarily adjusted to $\boldsymbol{\mathcal{Q}} = 0.5 \cdot \boldsymbol{I}_{N_p}$, $\boldsymbol{\mathcal{R}}_1 = \boldsymbol{\mathcal{R}}_2 = 0.5 \cdot \boldsymbol{I}_{N \cdot N_p}$, setting equal penalty values on setpoint deviation and control effort. The simulation results given in Fig. 3(b) and Fig. 3(d) reflect this in an increase in setpoint deviation and a decrease in switching frequency.

The direct comparison with the state of the art modulation based control scheme in Fig. 5 shows that the presented approach significantly reduces torque ripple. In contrast to



(a) torque trajectories for different control approaches



(b) relative setpoint deviation $|\epsilon| = |y - w|/w$ in %

Fig. 5. MPC vs. Modulation, top: acceleration; bottom: deceleration

$\Delta\Sigma$ and PWM modulation, the MPC approach instantly follows the setpoint change and has greater accuracy, which can be improved further by using more cylinders. Furthermore the weighting values implicitly determine the frequency content of the output torque. Higher switching frequencies yielding lower torque ripple, also correspond to higher frequencies in the output. This consequence can be exploited to shape the system behavior according to system requirements. In applications, induction of critical resonance frequencies can be avoided (e.g. juddering associated with driveline oscillations) by either producing low pulsation amplitudes or shifting excitation frequencies. Here weighting matrices are the tuning parameters within the MPC strategy to shape system behavior. Alternatively, admissible frequency content could possibly be formulated within the objective function or as constraints.

6. CONCLUSION AND OUTLOOK

In this work, a model predictive control approach for Digital Hydraulic Drives is presented. The nonlinear and discontinuous system behavior is accounted for by utilizing a MLD system formulation. Thereby a linear model with mixed-integer variables subject to mixed-integer linear inequalities is achieved. Employing the MPC scheme in conjunction with the developed model, simulations demonstrate that the instantaneous torque output can be kept close to a reference, thus diminishing critical pulsations. Valve switching events are explicitly considered within the control scheme. Therefore, with the appropriate choice of weights, switching sequences for an optimal trade-off between best setpoint tracking and minimum switching can be determined. In comparison with the state of the art methods to control Digital Hydraulic Drives, the presented hybrid MPC approach demonstrates superior performance by optimally exploiting the system's capability. Compared to the steady state accuracy of state of the art modulation strategies of about 60%, the presented MPC approach yields up to 90 % accuracy, significantly reducing torque ripple. The amount of decision variables present, results in a complex and computationally demanding control task. For online optimization to be tractable and compatible with the application, ways to improve computation time need to be explored. Here various approaches, from offline computation of affine control laws, up to individually tailored optimization solvers, offer promising solution possibilities. Furthermore, the system performance beyond the ideal model should be investigated, e.g. when dealing with uncertainties and disturbances.

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