Preprints of the 23rd IFAC World Congress
Busan, Republic of Korea, August 23-28, 2026

GMM-based Pareto Optimal Alarm Design for Multivariate Process Monitoring ⋆

Nachuan Yang *, **Yifei Tao** *, **Fanlin Jia** * and **Tongwen Chen** *

* *Department of Electrical and Computer Engineering, University of Alberta, Edmonton, Canada T6G 1H9.*

Abstract: Univariate alarm systems are usually inadequate for multivariate industrial processes, where strong process correlations often lead to alarm flooding and ineffective fault detection. In this paper, we investigate a multi-objective design of multivariate alarms, which remains an open research problem. Historical process data are first modeled using a Gaussian mixture model (GMM) to capture representative fault patterns. Based on these patterns, the alarm design is further formulated as a multi-objective optimization problem, which is then solved through quadratic programming and bisection methods. The proposed method jointly minimizes the false alarm rate, missing alarm rate, and cross false alarm rate, achieving a Pareto optimal solution among multiple alarm objectives. Compared with heuristic methods and manual tuning, the proposed method provides explicit rate characterization and theoretical guarantees, which are essential for safety-critical applications. The effectiveness of our proposed new method is demonstrated through case studies.

Keywords: Industrial alarm systems, alarm management, process monitoring.

1. INTRODUCTION

Alarm systems are integral components of distributed control systems (DCSs), ensuring operational safety and efficiency in complex industrial processes (Wang et al., 2024a). With increasing automation and massive process data, alarm management has become critical for maintaining system integrity and mitigating operational risks. Strong interdependence among process variables often leads to nuisance alarms and alarm floods, which hinder operators from responding effectively to genuine faults (Goel et al., 2017; Hu et al., 2023; Jia et al., 2025). To address these issues, industrial standards such as EEMUA-191 and ISA-S18.02 recommend using systematic and data-driven methods for alarm setup to improve operators' awareness and reduce unnecessary alarms. However, conventional alarms typically adopt univariate designs, where each variable is monitored independently. Such methods are inefficient for monitoring multivariate processes with strong correlations (Kondaveeti et al., 2009), as interacting variables can trigger nuisance alarms and alarm floods that obscure critical information.

Significant progress has been made in univariate alarm design in the past decade, including optimal linear alarm filters to minimize false and missed alarm rates (Cheng et al., 2013) and subsequent performance sensitivity analyses (Xiong et al., 2017). Roohi et al. (Roohi et al., 2021) further refined filter coefficients by incorporating process and control knowledge. Delay-timer research has evolved from traditional n-sample designs (Xu et al., 2011) to advanced configurations such as n_1-out-of-n frameworks (Adnan et al., 2013) and multi-setpoint strategies (Su et al., 2018). For deadband design, both analytical and adaptive methods have been proposed to handle stochastic or nonstationary processes (Adnan et al., 2011). Extensions to rank-order and variance-based filters have been reported (Roohi and Chen, 2020).

The research on multivariate alarm systems remains limited due to modeling complexity and the difficulty of handling coupling variables. Early studies combined multivariate control charts with data-mining algorithms to reduce false alarms (Kang and Kim, 2015), and analyzed process–alarm correlations (Yang et al., 2012). Subsequent work optimized alarm thresholds considering pairwise false and missing alarms (Han et al., 2016) and adaptive time scales (Chen and Wang, 2017). More recent developments include multivariate designs based on evidential reasoning (Weng et al., 2024), Markov chains (Taheri-Kalani et al., 2024), and normal operating zones (Wang et al., 2024b). However, these methods still rely on heuristic or manual tuning to determine alarm parameters, lack interpretability to some extent, and do not account for correlated false alarms arising from interactions among alarms.

This paper investigates the joint optimization of alarm rates for multivariate process monitoring. The main contributions of this work are summarized as follows:

1) A multi-objective optimization setup is proposed for multivariate alarm design, introducing the cross false alarm rate as a new metric to capture inter-alarm correlations and improve overall performance.
2) Based on the GMM modeling of process data, the feasibility of the multi-objective alarm design is established via quadratic programming, which characterizes all feasible alarm designs in closed form.

⋆ This work was supported by Natural Sciences and Engineering Research Council of Canada (NSERC).

©2026 the authors. Accepted by IFAC for publication under a Creative Commons Licence CC-BY-NC-ND

3) A directional search procedure with bisection methods is developed to solve the multi-objective optimization problem, and the Pareto optimality of the resulting alarm design is theoretically guaranteed.

The remainder of this paper is organized as follows. Section 2 presents the proposed optimization framework for multi-alarm configuration based on GMM-modeled fault patterns and Pareto optimal alarm design. Section 3 provides case studies on an aircraft system and the Tennessee Eastman Process to demonstrate the effectiveness. Section 4 concludes the paper and outlines future work.

Notations: $[A]_{ij}$ represents the element in the i-th row and j-th column of matrix A; while A^{T} denotes its transpose. The notation $A \succ 0$ (or $A \succeq 0$) indicates that matrix A is positive definite (or positive semidefinite). Similarly, $A \succ B$ (or $A \succeq B$) implies that $A-B \succ 0$ (or $A-B \succeq 0$). For scalar comparisons, $A > 0$ (or $A \geq 0$) indicates that A is positive (or non-negative), and $A > B$ (or $A \geq B$) means that $A - B$ is positive (or non-negative). For a square matrix $A \in \mathbb{R}^{n\times n}$, we use $|A|$ to denote its determinant. Given a positive semidefinite matrix A, the induced matrix norm for a vector x is defined as $\|x\|_A = x^{\mathrm{T}}Ax$. For probabilistic notation, we write $X \sim \mathcal{N}(\mu, \Sigma)$ to indicate that random variable X follows a Gaussian distribution with mean μ and covariance matrix Σ.

2. GMM-BASED MULTI-ALARM SYSTEM

In this section, we first use the GMM to model a multivariate process, and then introduce the alarm configuration and performance metrics to be optimized, followed by the formulation of a Pareto optimal design problem.

2.1 Process Modeling with Gaussian Mixture Models

Consider a multivariate industrial process characterized by N process variables at sampling time k:

$$x(k) = [x_1(k) \ \dots \ x_N(k)]^{\mathrm{T}} \in \mathbb{R}^N. \tag{1}$$

We make the following assumptions:
1) The process may experience M distinct fault types, which occur independently over time.
2) Historical process data is available with labels indicating normal and abnormal operations.

Given the historical data $\{x(k)\}_{k=1}^K$ containing both normal and faulty samples, we employ a Gaussian mixture model to characterize the underlying operational modes. Assuming M potential faults, the process data follows a mixture distribution with $M+1$ Gaussian components, one for normal operation and M for different fault patterns:

$$p(x) = \sum_{i=0}^{M} \pi_i \mathcal{N}(x|\mu_i, \Sigma_i) \tag{2}$$

where π_i is the mixing coefficient, $\mu_i \in \mathbb{R}^N$ is the mean vector, $\Sigma_i \in \mathbb{R}^{N\times N}$ is the covariance matrix, and $\mathcal{N}(x|\mu_i, \Sigma_i)$ denotes the multivariate Gaussian distribution. The index $i = 0$ corresponds to the normal operation mode.

The GMM parameters are estimated in a semi-supervised manner. For the normal component, we use maximum likelihood estimation on labeled normal samples to obtain μ_0, Σ_0, and $\pi_0 = K_n/K$, where K_n is the number of normal samples. For the fault components $\{\pi_i, \mu_i, \Sigma_i\}_{i=1}^M$, we apply the Expectation-Maximization (EM) algorithm on abnormal samples. The EM algorithm iteratively computes posterior probabilities (E-step) and updates parameters (M-step) until convergence. When fault labels are partially available, they can be used to initialize the corresponding components, improving convergence and estimation quality. Detailed EM steps can be found in (Murphy, 2012).

This GMM modeling has two key advantages: (1) it identifies operational modes from historical data without requiring complete fault labels, and (2) it provides Gaussian representations that facilitate analytical alarm design.

2.2 Linear Alarm Filters and Performance Metrics

In this subsection, we introduce the configuration of multivariate alarms for detecting the M abnormal operational modes. When the process operates in abnormal mode i, the variable $x(k)$ is approximately modeled as $\mathcal{N}(\mu_i, \Sigma_i)$, with parameters estimated using the EM algorithm.

We adopt linear alarm filters due to their simplicity and computational efficiency [1]. Each alarm is defined as:

$$\mathcal{A}_i(k) = \begin{cases} 1 & \text{if } w_i^{\mathrm{T}} x(k) \geq \zeta_i, \\ 0 & \text{if } w_i^{\mathrm{T}} x(k) < \zeta_i \end{cases} \tag{3}$$

where $w_i \in \mathbb{R}^N$ is the weight vector and $\zeta_i > 0$ is the threshold for alarm i. This multivariate approach requires only M alarms compared to conventional univariate designs that would need at least $2N$ alarms, thereby reducing nuisance alarms. We consider the following metrics:

False Alarm Rate (FAR): The probability that alarm i triggers under normal operation:

$$\begin{aligned} \mathrm{FAR}_i &= p(\mathcal{A}_i(k) = 1 \mid x(k) \sim \mathcal{N}(\mu_0, \Sigma_0)) \\ &= 1 - \Phi\left(\frac{\zeta_i - w_i^{\mathrm{T}}\mu_0}{\sqrt{w_i^{\mathrm{T}}\Sigma_0 w_i}}\right) \end{aligned} \tag{4}$$

where $\Phi(\cdot)$ is the standard normal CDF.

Missing Alarm Rate (MAR): The probability that alarm i fails to trigger when fault i occurs:

$$\begin{aligned} \mathrm{MAR}_i &= p(\mathcal{A}_i(k) = 0 \mid x(k) \sim \mathcal{N}(\mu_i, \Sigma_i)) \\ &= \Phi\left(\frac{\zeta_i - w_i^{\mathrm{T}}\mu_i}{\sqrt{w_i^{\mathrm{T}}\Sigma_i w_i}}\right) \end{aligned} \tag{5}$$

Cross False Alarm Rate (CFAR): The probability that alarm i is erroneously triggered by fault $j \neq i$:

$$\begin{aligned} \mathrm{CFAR}_{ji} &= p(\mathcal{A}_i(k) = 1 \mid x(k) \sim \mathcal{N}(\mu_j, \Sigma_j)) \\ &= 1 - \Phi\left(\frac{\zeta_i - w_i^{\mathrm{T}}\mu_j}{\sqrt{w_i^{\mathrm{T}}\Sigma_j w_i}}\right) \end{aligned} \tag{6}$$

Based on the above metrics, we formulate the alarm design as a multi-objective optimization problem:

$$\begin{aligned} \min_{w_i\in\mathbb{R}^N, \zeta_i>0, i=1,\dots,M} \quad & (\theta_f, \theta_m, \theta_c) \\ \text{subject to} \quad & \forall i, j = 1, \dots, M, j \neq i, \\ & \mathrm{FAR}_i \leq \theta_f, \\ & \mathrm{MAR}_i \leq \theta_m, \\ & \mathrm{CFAR}_{ji} \leq \theta_c, \end{aligned} \tag{7}$$

[1] The linear alarm filter can be extended to multi-time steps by augmenting variables using the same setup (Xiong et al., 2017).

where θ_f, θ_m, and $\theta_c \in (0, 0.5)$ denote the thresholds on the false alarm, missing alarm, and cross false alarm rates, respectively. This formulation captures the trade-offs among the three conflicting objectives.

Definition 1. A feasible triplet $(\theta_f, \theta_m, \theta_c)$ is said to be *Pareto optimal* for problem (7) if there exists no other feasible triplet $(\theta'_f, \theta'_m, \theta'_c)$ such that $\theta'_f \le \theta_f$, $\theta'_m \le \theta_m$, $\theta'_c \le \theta_c$, with at least one strict inequality holding.

The above model can be extended to large-scale industrial processes by incorporating more advanced multi-agent system methods (Ren et al., 2022). In the remainder of this paper, we develop efficient computational methods to obtain Pareto optimal solutions to (7).

3. PARETO OPTIMAL ALARM DESIGN

In this section, we first show that the multi-objective optimization problem is feasible via convex programming, and then propose a bisection method for solution.

3.1 Feasibility of Multi-Objective Optimization

When the thresholds $\theta_f, \theta_m, \theta_c$ are all fixed, the multi-objective optimization problem in (7) reduces to

$$\begin{aligned} \text{Find} \quad & w_i \in \mathbb{R}^N, \zeta_i > 0, i = 1, \dots, M \\ \text{subject to} \quad & \forall i, j = 1, \dots, M, j \neq i, \\ & \text{FAR}_i \le \theta_f, \\ & \text{MAR}_i \le \theta_m, \\ & \text{CFAR}_{ji} \le \theta_c. \end{aligned} \tag{8}$$

To establish feasibility, we characterize all feasible solutions through an equivalent convex reformulation.

Theorem 1. The set of all alarm weights and thresholds satisfying the constraints in (8) is given by

$$\{(w_i, \zeta_i) \mid w_i = \zeta_i v_i^*,\ \zeta_i > 0,\ i = 1, \dots, M\}, \tag{9}$$

where v_i^*, $i = 1, \dots, M$, are solutions to the following convex quadratic programming problem:

$$\begin{aligned} \text{Find} \quad & v_i \in \mathbb{R}^N,\ i = 1, \dots, M \\ \text{subject to} \quad & \forall i, j = 1, \dots, M, j \neq i, \\ & \Phi^{-1}(1-\theta_f)\|v_i\|_{\Sigma_0} + v_i^{\mathrm{T}}\mu_0 \le 1, \\ & \Phi^{-1}(\theta_m)\|v_i\|_{\Sigma_i} + v_i^{\mathrm{T}}\mu_i \ge 1, \\ & \Phi^{-1}(1-\theta_c)\|v_i\|_{\Sigma_j} + v_i^{\mathrm{T}}\mu_j \le 1, \end{aligned} \tag{10}$$

where $\Phi^{-1}(\cdot)$ is the inverse of the normal CDF $\Phi(\cdot)$.

Proof. From the definitions of alarm rates in (4)–(6), the constraints in (8) can be rewritten as

$$\begin{aligned} & \Phi\left(\frac{\zeta_i - w_i^{\mathrm{T}}\mu_0}{\|w_i\|_{\Sigma_0}}\right) \le \theta_f, \\ & \Phi\left(\frac{w_i^{\mathrm{T}}\mu_i - \zeta_i}{\|w_i\|_{\Sigma_i}}\right) \le \theta_m, \\ & \Phi\left(\frac{\zeta_i - w_i^{\mathrm{T}}\mu_j}{\|w_i\|_{\Sigma_j}}\right) \le \theta_c. \end{aligned} \tag{11}$$

Applying the monotonically increasing inverse function $\Phi^{-1}(\cdot)$ and rearranging the above inequalities yields

$$\begin{aligned} & \Phi^{-1}(1-\theta_f)\|w_i\|_{\Sigma_0} \le \zeta_i - w_i^{\mathrm{T}}\mu_0, \\ & \Phi^{-1}(\theta_m)\|w_i\|_{\Sigma_i} \ge \zeta_i - w_i^{\mathrm{T}}\mu_i, \\ & \Phi^{-1}(1-\theta_c)\|w_i\|_{\Sigma_j} \le \zeta_i - w_i^{\mathrm{T}}\mu_j, \end{aligned} \tag{12}$$

where we use $\Phi^{-1}(1-\theta)$ instead of $-\Phi^{-1}(\theta)$ to ensure non-negative coefficients. Observing that w_i and ζ_i appear homogeneously in these inequalities, we can normalize by dividing through by $\zeta_i > 0$, i.e.,

$$\begin{aligned} & \Phi^{-1}(1-\theta_f)\left\|\frac{w_i}{\zeta_i}\right\|_{\Sigma_0} \le 1 - \frac{w_i^{\mathrm{T}}}{\zeta_i}\mu_0, \\ & \Phi^{-1}(\theta_m)\left\|\frac{w_i}{\zeta_i}\right\|_{\Sigma_i} \ge 1 - \frac{w_i^{\mathrm{T}}}{\zeta_i}\mu_i, \\ & \Phi^{-1}(1-\theta_c)\left\|\frac{w_i}{\zeta_i}\right\|_{\Sigma_j} \le 1 - \frac{w_i^{\mathrm{T}}}{\zeta_i}\mu_j. \end{aligned} \tag{13}$$

Introducing $v_i = w_i/\zeta_i$ yields the formulation in (10). Since $\|v_i\|_{\Sigma_k} = \sqrt{v_i^{\mathrm{T}}\Sigma_k v_i}$ is a convex function and $v_i^{\mathrm{T}}\mu_k$ is a linear term, problem (10) is a convex quadratic program. The correspondence $w_i = \zeta_i v_i^*$ with arbitrary $\zeta_i > 0$ characterizes all feasible solutions. □

The above theorem transforms the nonlinear feasibility problem in (8) into the convex quadratic program in (10), which can be efficiently solved using standard convex optimization solvers with polynomial-time complexity. The homogeneity property indicates that feasible alarm designs can be scaled by any positive factor and remain feasible. In practice, the Gaussian parameters (μ_j, Σ_j) are estimated from data and inevitably contain approximation errors. To ensure robust performance under parametric uncertainties, we propose the following robust version of Theorem 1.

Corollary 1. Consider the case where the Gaussian distribution parameters of the operational modes are subject to parametric uncertainty. Specifically, suppose the true parameters lie within known bounds:

$$\underline{\mu}_i \le \mu_i \le \overline{\mu}_i, \quad \underline{\Sigma}_i \preceq \Sigma_i \preceq \overline{\Sigma}_i, \quad i = 0, 1, \dots, M, \tag{14}$$

where $\underline{\mu}_i, \overline{\mu}_i \in \mathbb{R}^N$ represent known lower and upper bounds on the mean vectors, and $\underline{\Sigma}_i, \overline{\Sigma}_i \succeq 0$ denote known lower and upper bounds on the covariance matrices in the positive semidefinite ordering. Then, the set of all alarm weights and thresholds that robustly satisfy the constraints in (8) for all parameter realizations within the uncertainty bounds is given by

$$\{(w_i, \zeta_i) \mid w_i = \zeta_i v_i^{\dagger},\ \zeta_i \neq 0,\ i = 1, \dots, M\}, \tag{15}$$

where $v_i^{\dagger}$, $i = 1, \dots, M$, are solutions to the following robust quadratic programming problem:

$$\begin{aligned} \text{Find} \quad & v_i \in \mathbb{R}^N,\ i = 1, \dots, M \\ \text{subject to} \quad & \forall i, j = 1, \dots, M, j \neq i, \\ & \Phi^{-1}(1-\theta_f)\|v_i\|_{\overline{\Sigma}_0} + v_i^{\mathrm{T}}\overline{\mu}_0 \le 1, \\ & \Phi^{-1}(1-\theta_f)\|v_i\|_{\overline{\Sigma}_0} + v_i^{\mathrm{T}}\underline{\mu}_0 \le 1, \\ & \Phi^{-1}(\theta_m)\|v_i\|_{\underline{\Sigma}_i} + v_i^{\mathrm{T}}\underline{\mu}_i \ge 1, \\ & \Phi^{-1}(\theta_m)\|v_i\|_{\underline{\Sigma}_i} + v_i^{\mathrm{T}}\overline{\mu}_i \ge 1, \\ & \Phi^{-1}(1-\theta_c)\|v_i\|_{\overline{\Sigma}_j} + v_i^{\mathrm{T}}\overline{\mu}_j \le 1, \\ & \Phi^{-1}(1-\theta_c)\|v_i\|_{\overline{\Sigma}_j} + v_i^{\mathrm{T}}\underline{\mu}_j \le 1. \end{aligned} \tag{16}$$

Proof. Follows from Theorem 1 and is omitted here. □

3.2 Directional Search for Optimal Thresholds

The remaining question is how to select the parameters $\theta_f, \theta_m, \theta_c$. We introduce a directional search approach that parameterizes the thresholds along a preference direction.

Define a weight direction vector $r = (r_f, r_m, r_c)$ with normalization $\|r\|_1 = 1$ and $r \geq 0$, where each component represents the relative importance of the corresponding metric. We parameterize the thresholds as:

$$(\theta_f, \theta_m, \theta_c) = s \cdot (r_f, r_m, r_c), \quad s > 0 \tag{17}$$

where s controls the overall magnitude of the thresholds. The alarm design problem becomes finding the smallest s such that (8) remains feasible:

$$\begin{aligned} \min_{s>0} \quad & s \\ \text{s.t.} \quad & (\theta_f, \theta_m, \theta_c) = s \cdot r, \\ & (8) \text{ is feasible.} \end{aligned} \tag{18}$$

Theorem 2. Problem (18) has a quasi-convex structure, i.e, if s_0 is feasible, then all $s' > s_0$ are feasible; if s_0 is infeasible, then all $s' < s_0$ are infeasible.

Proof. For any $s > 0$, the feasibility of (8) requires:

$$\begin{aligned} &\Phi^{-1}(1 - sr_f)\|v_i\|_{\Sigma_0} + v_i^{\mathrm{T}}\mu_0 \leq 1, \\ &\Phi^{-1}(sr_m)\|v_i\|_{\Sigma_i} + v_i^{\mathrm{T}}\mu_i \geq 1, \\ &\Phi^{-1}(1 - sr_c)\|v_i\|_{\Sigma_j} + v_i^{\mathrm{T}}\mu_j \leq 1, \quad \forall j \neq i. \end{aligned} \tag{19}$$

Since $\Phi^{-1}(\cdot)$ is monotonically increasing, and $\Phi^{-1}(1-sr_f)$ is decreasing in s while $\Phi^{-1}(sr_m)$ is increasing in s, the feasible region expands as s increases. Specifically, if v_i satisfies the constraints for $s = s_0$, it also satisfies them for any $s' > s_0$, establishing the monotonicity. □

The monotonic structure enables efficient solution via bisection search, as outlined in Algorithm 1.

Algorithm 1 Directional Bisection for Alarm Design

Require: Direction $r = (r_f, r_m, r_c)$, tolerance $\epsilon > 0$
Initialize: $s^{\text{low}} = 0$, $s^{\text{high}} = 1$ (or sufficiently large)
while $s^{\text{high}} - s^{\text{low}} > \epsilon$ **do**
  $s = (s^{\text{low}} + s^{\text{high}})/2$
  Set $(\theta_f, \theta_m, \theta_c) = s \cdot r$
  Solve QP (8) for v_i
  if QP is feasible **then**
    $s^{\text{high}} = s$, store $v_i^* = v_i$
  else
    $s^{\text{low}} = s$
  end if
end while
$s^* = s^{\text{high}}$
$(\theta_f^*, \theta_m^*, \theta_c^*) = s^* \cdot r$
Choose $\zeta_i^* > 0$ (e.g., $\zeta_i^* = 1$), set $w_i^* = \zeta_i^* v_i^*$
return (w_i^*, ζ_i^*), $(\theta_f^*, \theta_m^*, \theta_c^*)$

The solution returned by Algorithm 1 is *directional optimal*: it attains the smallest feasible scalar s^* along the ray $\{\theta = sr : s > 0\}$. However, this solution need not be Pareto optimal in the full three-dimensional objective space $(\theta_f, \theta_m, \theta_c)$. To obtain a Pareto optimal solution, we perform local refinement by solving

$$\begin{aligned} \max_{s_i>0} \quad & s_i \\ \text{s.t.} \quad & (\theta_f, \theta_m, \theta_c) = (\theta_f^*, \theta_m^*, \theta_c^*) - s_i e_i, \\ & (8) \text{ is feasible,} \end{aligned} \tag{20}$$

for each elementary direction $e_i \in \{e_f, e_m, e_c\}$, where $e_f = (1, 0, 0)$, $e_m = (0, 1, 0)$, $e_c = (0, 0, 1)$. Here, $(\theta_f^*, \theta_m^*, \theta_c^*)$ is the directional optimal solution from Algorithm 1.

Algorithm 2 Pareto Refinement via Coordinate Descent

Require: Initial solution $(\theta_f^*, \theta_m^*, \theta_c^*)$ from Algorithm 1
Require: Tolerance $\epsilon > 0$
repeat
  for $i \in \{f, m, c\}$ **do**
    Solve (20) via bisection method
    $\theta_i^* \leftarrow \theta_i^* - s_i$
  end for
until convergence
return (w_i^*, ζ_i^*), $(\theta_f^*, \theta_m^*, \theta_c^*)$

Theorem 3. The solution $(\theta_f^*, \theta_m^*, \theta_c^*)$ returned by Algorithm 2 is Pareto optimal, i.e., there exists no feasible solution $(\theta_f', \theta_m', \theta_c')$ such that $\theta_i' \leq \theta_i^*$ for all $i \in \{f, m, c\}$ with at least one strict inequality.

Proof. Suppose for contradiction that there exists a feasible triplet $(\theta_f', \theta_m', \theta_c')$ with $\theta_i' \leq \theta_i^*$ for all i and $\theta_k' < \theta_k^*$ for some k. Setting $\theta_k = \theta_k'$ and $\theta_j = \theta_j^*$ for $j \neq k$ would yield a feasible triplet with $s_k = \theta_k^* - \theta_k' > \epsilon$, contradicting the termination condition. Hence, no such dominating triplet exists. By definition, $(\theta_f^*, \theta_m^*, \theta_c^*)$ is Pareto optimal. □

The directional search followed by Pareto refinement provides a practical two-stage approach: (1) Algorithm 1 quickly finds a feasible solution aligned with user preferences encoded in r; (2) Algorithm 2 improves this solution to achieve Pareto optimality. The combined approach balances computational efficiency with solution quality. The proposed scheme can also be integrated with more advanced data-driven Kalman filtering algorithms for real-time analysis and design (Duan et al., 2025).

4. CASE STUDIES

In this section, two case studies are provided to verify the effectiveness of our proposed alarm design method.

4.1 Case Study 1: Aircraft Flight Dynamics

In this numerical example, we consider a lateral axis model of an L-1011 aircraft in cruise condition (Leibfritz and Lipinski, 2003) described by

$$\begin{cases} \dot{\xi}(t) = A\xi(t) + B\mu(t) + w(t) \\ \eta(t) = C\xi(t) + v(t) \end{cases} \tag{21}$$

where $\xi(t) \in \mathbb{R}^5, \mu(t) \in \mathbb{R}^2$ and $\eta(t) \in \mathbb{R}^4$ denote the state, actuator input and sensor output, respectively. The process and measurement noises are assumed to be zero-mean i.i.d. Gaussian with covariance matrix $1e-3I$. The aircraft system is regulated by a static output-feedback controller $\mu(t) = K\eta(t)$ with the gain matrix

$$K = \begin{bmatrix} -1.8791 & -2.5000 & -20.2350 & 0.3535 \\ 0 & 0.8853 & 0.9906 & 0.9104 \end{bmatrix}$$

such that it is asymptotically stable. Please refer to (Leibfritz and Lipinski, 2003) for more details about the model. We suppose there exist 6 fault modes within the actuators and sensors, as summarized in Table 1.

Table 1. Summary of Potential Faults in the L-1011 Aircraft System

Index	Location	Fault Description	Mathematical Model
F1	Actuator	Aileron effectiveness loss	$\mu_1 + \mathcal{N}(2.5, 0.01)$
F2	Actuator	Rudder effectiveness loss	$\mu_2 + \mathcal{N}(-1.0, 0.01)$
F3	Sensor	Sideslip sensor failure	$\eta_1 + \mathcal{N}(0.7, 0.01)$
F4	Sensor	Roll rate sensor bias	$\eta_2 + \mathcal{N}(-0.8, 0.01)$
F5	Sensor	Roll angle sensor offset	$\begin{bmatrix}\eta_3 + \mathcal{N}(1.0, 0.01)\\ \eta_4 + \mathcal{N}(0.7, 0.01)\end{bmatrix}$
F6	Sensor	Yaw rate sensor drift	$\eta_3 + \mathcal{N}(1.0, 0.01)$

Table 2. GMM Values for the Faults in L-1011 Aircraft System

Index	GMM Mean	Covariance (only diagonal)
F1	[-0.0079,-0.0041,0.1030,-0.5583]	[0.1112,0.0767,0.0171,0.0246]
F2	[-0.0006,0.0040,0.0160,1.0104]	[0.0865,0.0656,0.0157,0.0191]
F3	[0.6916,-0.0036,0.0009,0.0004]	[0.1234,0.0775,0.0175,0.0224]
F4	[-0.0041,-0.8011,0.0008,0.0086]	[0.0678,0.0586,0.0144,0.0165]
F5	[-0.0076,-0.0018,1.0002,0.7038]	[0.0665,0.0460,0.0238,0.0248]
F6	[-0.0097,-0.0056,1.0012,0.0102]	[0.1293,0.0901,0.0289,0.0270]

Table 3. Alarm Weights and Thresholds Designed by Quadratic Programming

Index	Normalized Alarm Weight (w_i)	Alarm Threshold (ζ_i)
F1	[-0.1631,0.0255,-0.1628,-0.9727]	0.2439
F2	[-0.0493,0.0860,-0.7308,0.6753]	0.2829
F3	[0.7518,0.6445,-0.1393,0.0091]	0.2313
F4	[-0.4062,-0.6332,-0.6534,-0.0845]	0.2180
F5	[0.2578,0.3064,0.6597,0.6360]	0.8756
F6	[-0.0309,0.0702,0.8051,-0.5882]	0.5833

Table 4. False, Missing and Cross False Alarm Rates of the Designed Multivariate Alarms

Index	FAR	MAR	CFAR
F1	0.0264	0.0365	$\{0, 0.0101, 0.0185, 2.7408e-12, 0.0063\}$
F2	0.0076	9.2827e-04	$\{3.2236e-08, 0.0088, 0.0019, 2.0947e-04, 1.6449e-10\}$
F3	0.0465	0.0672	$\{0.0752, 0.0740, 1.2898e-06, 0.0054, 0.0226\}$
F4	0.0204	0.0095	$\{0.0218, 0.0013, 1.4743e-05, 3.1086e-14, 4.4173e-11\}$
F5	4.6629e-15	0.0563	$\{0, 0.0300, 3.1715e-08, 0, 0.0752\}$
F6	2.1321e-08	0.0665	$\{0.0707, 0, 7.3692e-08, 1.3274e-09, 0.0923\}$

In this example, the sensor measurement data is available for us to use as monitoring variables, i.e.,

$$x(k) = \eta(k\Delta t) \tag{22}$$

where $\Delta t = 0.01$ min denotes the sampling period. We will design alarm parameters $(w_i, \zeta_i), i = 1, \ldots, 6$ for faults F1, F2, ..., F6, respectively. The Gaussian mixture components are computed based on historical trajectories, and partial results are summarized in Table 2. To design Pareto optimal alarms, we first initialize

$$\theta_f = 0.05,\ \theta_m = 0.07,\ \theta_c = 0.10. \tag{23}$$

By Theorem 1, we can compute the alarm weights and thresholds by solving 6 quadratic programming problems, and the solutions are obtained and summarized in Table 3. Based on the designed alarms, we can compute the false, missing and cross false alarm rates, as summarized in Table 4. It is easy to verify the feasibility, i.e.,

$$\begin{cases}\text{FAR}_i \le \theta_f = 0.05,\ i = 1, \ldots, 6,\\ \text{MAR}_i \le \theta_m = 0.07,\ i = 1, \ldots, 6,\\ \text{CFAR}_{ji} \le \theta_c = 0.10,\ i, j = 1, \ldots, 6, j \ne i.\end{cases}$$

Table 5. Alarm Rates Before and After Multi-Objective Optimization

Metric	Theoretical $(\theta_f, \theta_m, \theta_c)$	Theoretical $(\theta_f^*, \theta_m^*, \theta_c^*)$	Empirical $(\theta_f, \theta_m, \theta_c)$	Empirical $(\theta_f^*, \theta_m^*, \theta_c^*)$
mean FAR	0.0168	0.0111	0.0134	0.0069
max FAR	0.0465	0.0355	0.0376	0.0218
mean MAR	0.0395	0.0401	0.0236	0.0226
max MAR	0.0672	0.0650	0.0550	0.0509
mean CFAR	0.0172	0.0212	0.0110	0.0132
max CFAR	0.0923	0.0896	0.0957	0.0888

Then we implement the bisection methods in Algorithm 1 and Algorithm 2 with $\epsilon = 1e-8$, and obtain that

$$\theta_f^* = 0.0355 \le \theta_f,\ \theta_m^* = 0.0650 \le \theta_m,\ \theta_c^* = 0.0896 \le \theta_c.$$

Table 5 compares the two alarm systems. The results are obtained by Monte Carlo simulation with 1000 runs of 10,000 steps and a sampling time of $\Delta t = 0.01$ min. The designed Pareto optimal alarm shows better performance, especially lower worst-case alarm rates.

4.2 Case Study 2: Tennessee Eastman Process

We use the Tennessee Eastman Process dataset (Bathelt et al., 2015) to test the proposed alarm design method. The TEP models a realistic chemical process with five main units: reactor, condenser, separator, stripper, and compressor, involving two gas–liquid reactions that produce two liquid products. The dataset includes 53 variables: 12 manipulated (XMVs) and 41 measured (XMEASs). Eight fault scenarios are considered, covering feed composition changes, cooling system issues, and pressure losses, as listed in Tables 6 and 7. For each fault, 1000 samples are generated to estimate the GMM parameters.

We first initialize the reference thresholds as $\theta_f = \theta_m = \theta_c = 0.1$, for which the quadratic programming in Theorem

Table 6. Process Variables in TEP Dataset

Variables	Description	Units/Range
XMV(1-3)	Feed Flows (D,E,A)	Flow rate control
XMV(4)	A and C Feed Flow	Flow rate control
XMV(5)	Compressor Recycle Valve	Valve position
XMV(6)	Purge Valve	Valve position
XMV(7-8)	Separator & Stripper Flows	Flow rate control
XMV(9)	Stripper Steam Valve	Valve position
XMV(10-11)	Cooling Water Flows	Flow rate control
XMV(12)	Agitator Speed	Speed control
XMEAS(1-6)	Various Feed Flows	kscmh or kg/hr
XMEAS(7-9)	Reactor Conditions	kPa, %, °C
XMEAS(10-14)	Separator Operations	Various
XMEAS(15-19)	Stripper Operations	Various
XMEAS(20)	Compressor Work	kW
XMEAS(21-22)	Cooling Water Temps	°C
XMEAS(23-28)	Reactor Feed Analysis	Mole %
XMEAS(29-36)	Purge Gas Analysis	Mole %
XMEAS(37-41)	Product Analysis	Mole %

Table 7. TEP Fault Types and Descriptions

Fault ID	Description	Type
IDV(1)	A/C Feed Ratio, B Composition Constant	Step
IDV(2)	B Composition, A/C Ratio Constant	Step
IDV(4)	Reactor Cooling Water Inlet Temperature	Step
IDV(5)	Condenser Cooling Water Inlet Temperature	Step
IDV(6)	A Feed Loss	Step
IDV(7)	C Header Pressure Loss - Reduced Availability	Step
IDV(13)	Reaction Kinetics	Slow Drift
IDV(18)	Unknown Process Disturbance	Random

Table 8. Alarm Rates Before and After Multi-Objective Optimization

Metric	Theoretical $(\theta_f, \theta_m, \theta_c)$	Theoretical $(\theta_f^*, \theta_m^*, \theta_c^*)$	Empirical $(\theta_f, \theta_m, \theta_c)$	Empirical $(\theta_f^*, \theta_m^*, \theta_c^*)$
mean FAR	0.0055	0.0088	0.0060	0.0087
max FAR	0.0325	0.0317	0.0300	0.0280
mean MAR	0.0354	0.0205	0.0542	0.0424
max MAR	0.0906	0.0545	0.1833	0.1500
mean CFAR	0.0035	0.0044	0.0035	0.0036
max CFAR	0.0475	0.0351	0.0667	0.0479

1 is feasible. Then we utilize Algorithms 1 and 2 to design Pareto optimal alarms and obtain that

$$\theta_f^* = 0.0317 \leq \theta_f,\ \theta_m^* = 0.0545 \leq \theta_m,\ \theta_c^* = 0.0351 \leq \theta_c.$$

Table 8 compares the theoretical and empirical performance of the two alarm systems. The results show that the Pareto optimal design achieves superior performance in general, especially in the worst-case alarm rates.

5. CONCLUSION

This paper proposed a GMM-based framework for Pareto optimal design of multivariate alarm systems. By jointly minimizing the false alarm rate, missing alarm rate, and cross false alarm rate, the method systematically accounts for inter-alarm correlations and extends conventional univariate designs. The feasibility of the multi-objective formulation was established via quadratic programming, and a directional search with bisection was developed to obtain Pareto optimal solutions. Future work will focus on online implementation for large-scale industrial processes.

REFERENCES

Adnan, N.A., Cheng, Y., Izadi, I., and Chen, T. (2013). Study of generalized delay-timers in alarm configuration. *Journal of Process Control*, 23(3), 382–395.

Adnan, N.A., Izadi, I., and Chen, T. (2011). On expected detection delays for alarm systems with deadbands and delay-timers. *Journal of Process Control*, 21(9), 1318–1331.

Bathelt, A., Ricker, N.L., and Jelali, M. (2015). Revision of the Tennessee eastman process model. *IFAC-PapersOnLine*, 48(8), 309–314.

Chen, K. and Wang, J. (2017). Design of multivariate alarm systems based on online calculation of variational directions. *Chemical Engineering Research and Design*, 122, 11–21.

Cheng, Y., Izadi, I., and Chen, T. (2013). Optimal alarm signal processing: Filter design and performance analysis. *IEEE Transactions on Automation Science and Engineering*, 10(2), 446–451.

Duan, P., Liu, T., Xing, Y., and Johansson, K.H. (2025). Robust data-driven Kalman filtering for unknown linear systems using maximum likelihood optimization. *Automatica*, 180, 112474.

Goel, P., Datta, A., and Mannan, M.S. (2017). Industrial alarm systems: Challenges and opportunities. *Journal of Loss Prevention in the Process Industries*, 50, 23–36.

Han, L., Gao, H., Xu, Y., and Zhu, Q. (2016). Combining FAP, MAP and correlation analysis for multivariate alarm thresholds optimization in industrial process. *Journal of Loss Prevention in the Process Industries*, 40, 471–478.

Hu, W., Yang, G., Li, Y., Cao, W., and Wu, M. (2023). Root cause identification of industrial alarm floods using word embedding and few-shot learning. *IEEE Transactions on Industrial Informatics*, 20(2), 1465–1475.

Jia, F., Shang, J., and Chen, T. (2025). Early classification of industrial alarm floods with limited annotations. *IEEE Transactions on Industrial Electronics*, to appear.

Kang, J.H. and Kim, S.B. (2015). False alarm classification for multivariate manufacturing processes of thin film transistor–liquid crystal displays. *Journal of Process Control*, 35, 21–29.

Kondaveeti, S.R., Shah, S.L., and Izadi, I. (2009). Application of multivariate statistics for efficient alarm generation. *IFAC Proceedings Volumes*, 42(8), 657–662.

Leibfritz, F. and Lipinski, W. (2003). Description of the benchmark examples in COMPleib 1.0. *Tech. Report*.

Murphy, K.P. (2012). *Machine Learning: A Probabilistic Perspective*. MIT Press.

Ren, Y., Wang, Q., and Duan, Z. (2022). Optimal leader-following consensus control of multi-agent systems: A neural network based graphical game approach. *IEEE Transactions on Network Science and Engineering*, 9(5), 3590–3601.

Roohi, M.H. and Chen, T. (2020). Performance assessment and design of quadratic alarm filters. *IFAC-PapersOnLine*, 53(2), 494–499.

Roohi, M.H., Chen, T., Guan, Z., and Yamamoto, T. (2021). A new approach to design alarm filters using the plant and controller knowledge. *Industrial & Engineering Chemistry Research*, 60(9), 3648–3657.

Su, J., Guo, C., Zang, H., Yang, F., Huang, D., Gao, X., and Zhao, Y. (2018). A multi-setpoint delay-timer alarming strategy for industrial alarm monitoring. *Journal of Loss Prevention in the Process Industries*, 54, 1–9.

Taheri-Kalani, J., Latif-Shabgahi, G., and Shoorehdeli, M.A. (2024). Optimizing multivariate alarm systems: A study on joint false alarm rate, and joint missed alarm rate using linear programming technique. *Process Safety and Environmental Protection*, 191, 1775–1783.

Wang, J., Hu, W., and Chen, T. (2024a). *Intelligent Industrial Alarm Systems*. Springer.

Wang, Z., Wang, J., and Liu, J. (2024b). Multivariate process monitoring for safe operation of condensers in thermal power plants based on normal operating zones. *IEEE Transactions on Control Systems Technology*, 32(4), 1399–1409.

Weng, X., Xu, X., Feng, J., Shen, X., Meng, J., and Steyskal, F. (2024). A correlation analysis-based multivariate alarm method with maximum likelihood evidential reasoning. *IEEE Transactions on Automation Science and Engineering*, 21(4), 4974–4986.

Xiong, Y., Jing, Y., and Chen, T. (2017). Performance sensitivity analysis of linear alarm filters. In *2017 American Control Conference*, 4424–4429. IEEE.

Xu, J., Wang, J., Izadi, I., and Chen, T. (2011). Performance assessment and design for univariate alarm systems based on FAR, MAR, and AAD. *IEEE Transactions on Automation Science and Engineering*, 9(2), 296–307.

Yang, F., Shah, S.L., Xiao, D., and Chen, T. (2012). Improved correlation analysis and visualization of industrial alarm data. *ISA transactions*, 51(4), 499–506.