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Error-L_2 String Stability of Passive Multi-Agent Systems depending on Information Flow Topology ⋆

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Abstract: This paper deals with the string stability problem for passive multi-agent systems. When multi-agent systems such as autonomous vehicles move forward direction, a small error of the vehicle at the front of a platoon can be amplified in the downstream direction even though the consensus of multi-agent systems is achieved. We analyze the string stability in view of L_2-gain of the error between the preceding and following agents' outputs, and present an analysis of the error string stability according to the communication network structure. It is shown that the error string stability is guaranteed when the following agent uses the information of the preceding agents, and the error gain decreases as the amount of information from the preceding agents increases. Furthermore, to verify the effectiveness of the proposed analysis, we apply this approach to the vehicle platooning problem, and a method for designing a proportional-derivative controller to ensure the passivity of the vehicle system is presented.

Keywords: Consensus, Multi-agent systems, Passivity-based control, Resilient networked control systems

1. INTRODUCTION

Distributed control techniques for networked multi-agent systems, compared to centralized control where a central manager gathers all information, offer significant robustness and scalability as control actions are based only on information exchange with neighboring agents (Jadbabaie et al. (2003)). Thus, cooperative control of multi-agent systems has been extensively studied (see, *e.g.*, Olfati-Saber et al. (2007); Qu (2009)) in various fields, such as formation control of mobile robots (Olfati-Saber (2006)), platooning of autonomous vehicles (Naus et al. (2010)), synchronization in sensor networks (Sundararaman et al. (2005)), and power grid control (Dorfler and Bullo (2012)). Various distributed control laws and network design methods have been proposed to solve the consensus problem for homogeneous or heterogeneous multi-agent systems with first-, second-, or higher-order dynamics (Seo et al. (2009); Qu and Simaan (2014); Joo et al. (2016)).

However, when multi-agent systems form platoons, achieving consensus is insufficient to ensure system stability since a small disturbance from a preceding agent can be amplified as it propagates downstream, leading to the string stability problem (Li et al. (2015); Naus et al. (2010); Ploeg et al. (2013a)). It is well-known that conventional control strategies for vehicle platooning, such as proportional-derivative (PD) controllers based on position error, are often insufficient to guarantee this property (Rajamani (2011)). Another study has analyzed string stability in the time domain by requiring that the L_p-norm of the output error does not increase along the platoon (Ploeg et al. (2013b)). Recently, an energy-based approach using the concept of passivity has been proposed to analyze string stability (Mokogwu and Hashtrudi-Zaad (2022)). This framework addresses the issue that the vehicle model itself often has a degree of non-passivity or passivity shortage. To deal with such issues arising from the vehicle model, a feedforward control is employed to resolve structural non-passivity, while an output feedback control is applied to compensate for the shortage of passivity. Although a feedforward passivation technique may induce implementation issues, the passivity-based approach provides a powerful tool for dealing with the string stability problem.

Passivity has been widely used in various control problems due to its useful properties. For instance, systems formed by the parallel or negative feedback interconnection of two passive systems remain passive. Also, passivity is determined solely by the input-output relationship, regardless of the system's order or complexity. It simplifies system representation and controller design, even for complex, high-order nonlinear dynamics. In particular, a closed-loop system can be stabilized using a simple static output feedback, which has enabled its effective application to the consensus problem in multi-agent systems (Arcak (2007); Chopra (2012)). However, the string stability of

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passive multi-agent systems has not been actively studied. Although recent work addressed string stability in vehicle platooning by considering passivity, it was limited to simple communication topologies using information only from the immediate predecessor (Mokogwu and Hashtrudi-Zaad (2022)).

This paper addresses the string stability problem for multi-agent systems from a passivity perspective. Specifically, we analyze string stability in terms of the L_2 gain of the error between preceding and following agents, and investigate how error-L_2 string stability changes with the information flow topologies. We prove that topologies where followers use information from preceding agents guarantee error-L_2 string stability, and that the L_2 gain decreases as the amount of look-ahead information increases. Moreover, we apply a PD controller to a non-passive vehicle platooning system to render it passive, and then validate our analysis through simulations.

Notations: The notations used in this paper are defined as follows. The 2-norm of a vector $a \in \mathbb{R}^k$ is $||a|| := \sqrt{a^T a}$, and the induced 2-norm of a matrix $A \in \mathbb{R}^{k\times m}$ is $||A|| := \sqrt{\lambda_{\max}(A^T A)}$. For a measurable and locally integrable signal $y(t) : [0, \infty) \to \mathbb{R}^n$, the L_2 norm of the signal is

$$\|y(t)\|_{L_2} := \sqrt{\int_0^\infty y^T(t)y(t)dt} < \infty.$$

When no confusion arises from the context, the time argument t and other indices are omitted.

2. PROBLEM STATEMENT

Consider a multi-agent system consisting of n agents described as

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + Bu_i(t), \\ y_i(t) &= Cx_i(t), \qquad i = 1, \dots, n \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}^m$, $u_i \in \mathbb{R}$, and $y_i \in \mathbb{R}$ denote the state, control input, and output, and $A \in \mathbb{R}^{m\times m}$, $B \in \mathbb{R}^{m\times 1}$, and $C \in \mathbb{R}^{1\times m}$ are system, input, and output matrices, respectively. For simplicity, we assume that all agents are initially at rest (*i.e.*, $x_i(0) = 0$, $i = 1, \dots, n$), and that a virtual leader generates the output $y_0(t)$ as a reference signal.

Each system described by (1) is passive if there exist a symmetric positive definite matrix $P = P^T \succ 0$ and the storage function $V(x) = (1/2)x^T Px$ such that the following conditions hold:

$$A^T P + PA \preceq 0, \qquad PB = C^T. \tag{2}$$

It is well-known as the *Kalman-Yakubovich-Popov (KYP) lemma* (see, *e.g.*, Khalil (2002)).

When all agents of multi-agent systems are arranged in platoon formation, *string stability* is considered a significant requirement for disturbance attenuation along the agent string (Ploeg et al. (2013b)). String stability has been defined in various notations (Ploeg et al. (2013a); Ploeg et al. (2013b)). In the following, we present the definitions of error-L_2 string stability. We define the error for the agent i as the difference between the output of the agents i and $i-1$, *i.e.*,

$$e_i(t) := y_{i-1}(t) - y_i(t). \tag{3}$$

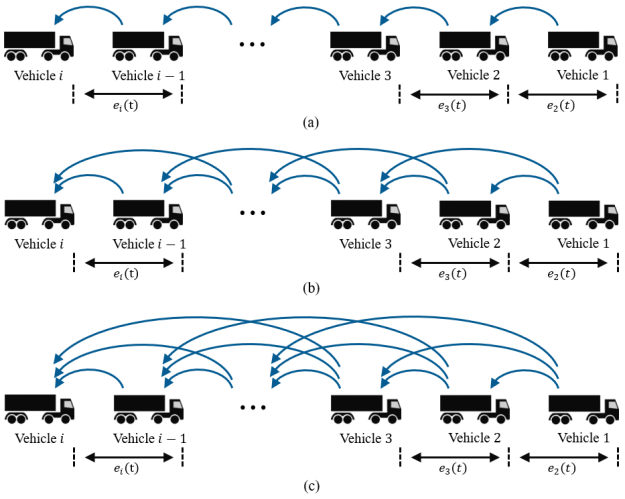


Fig. 1. Various types of information flow topology in vehicle platooning. (a) predecessor following (PF), (b) 2-predecessor following (2PF), (c) 3-predecessor following (3PF).

With (3), the definitions of error-L_2 string stability are provided as follows:

Definition 1. The multi-agent system is

- *error-L_2 string stable* if there exist positive constants $\gamma_i > 0$ such that
$$\|e_i\|_{L_2} \le \gamma_i \|e_1\|_{L_2}, \qquad i = 2, \dots, n,$$
- *semi-strictly error-L_2 string stable* if
$$\|e_i\|_{L_2} \le \|e_1\|_{L_2}, \qquad i = 2, \dots, n,$$
- *strictly error-L_2 string stable* if
$$\|e_i\|_{L_2} \le \|e_{i-1}\|_{L_2}, \qquad i = 2, \dots, n.$$

Definition 1 provides a clear framework for analyzing the propagation of the error within the platoon formation. The fundamental condition, error-L_2 string stability, ensures that the L_2 norm of the error signal for any agent i is bounded proportionally to the first agent's error e_1. The positive constant γ_i quantifies the maximum amplification factor relative to this initial error. Specifically, the condition $0 < \gamma_i < 1$ ensures that the error norm for any subsequent agent is strictly smaller than that of the first agent, guaranteeing error attenuation relative to the initial perturbation or disturbance.

To address the requirement for disturbance attenuation along the platoon, the notions of semi-strictly and strictly string stability are introduced. Semi-strict error-L_2 string stability imposes a more stringent condition where the error norm of any agent must not exceed that of the first agent. This ensures that e_1 represents the maximum error norm observed throughout the entire platoon, effectively preventing any amplification with respect to e_1. The most stringent condition is strict error-L_2 string stability, which requires the L_2 norm of the error to be non-increasing between any two consecutive agents. This property guarantees local error attenuation at every step along the string, ensuring that disturbances are effectively damped as they propagate downstream. This is often the desired behavior in practical platooning applications to ensure safety and ride comfort.

Each agent obtains information from its surrounding vehicles, where the communication structure is referred to as the information flow topology. Based on the given topology, a distributed controller is designed for each agent. Fig. 1 shows three types of topologies for a platoon formation of n agents.

(a) Predecessor following (PF) topology;
(b) 2-predecessor following (2PF) topology;
(c) 3-predecessor following (3PF) topology.

Besides these topologies, numerous other topologies can be analyzed using comparable methods (Zheng et al. (2015)).

3. ERROR-L_2 STRING STABILITY ANALYSIS

In this section, we demonstrate that the gain of error-L_2 stability, γ_i, is reduced by extending the information flow topology from PF to 2PF and 3PF.

Theorem 2. Assume that the system (1) is passive. Consider a static output feedback distributed controller under PF topology, defined as

$$u_i := k(y_{i-1} - y_i) = ke_i, \quad i = 1, \dots, n. \tag{4}$$

Then, the system (1) with (4) is *strictly error-L_2 string stable* for any positive gain $k > 0$, *i.e.*,

$$\|e_i\|_{L_2} \leq \|e_{i-1}\|_{L_2}, \quad i = 2, \dots, n. \qquad \square$$

Proof. We define the error state variables $\tilde{x}_i$ as the difference between the state of the predecessor and the follower, *i.e.,*

$$\tilde{x}_i := x_{i-1} - x_i, \quad i = 2, \dots, n. \tag{5}$$

Using (1), (3), and (4), the error dynamics is calculated as

$$\dot{\tilde{x}}_i = A\tilde{x}_i - kBe_i + kBe_{i-1}, \quad i = 2, \dots, n.$$

Let the storage function

$$V_i(\tilde{x}_i) := \frac{1}{2}\tilde{x}_i^T P \tilde{x}_i, \quad i = 2, \dots, n. \tag{6}$$

Since the system (1) is passive, we have

$$\begin{aligned}\dot{V}_i &= \frac{1}{2}\tilde{x}_i^T(A^TP + PA)\tilde{x}_i - k\tilde{x}_i^T PBe_i + k\tilde{x}_i^T PBe_{i-1} \\ &\leq -k\|e_i\|^2 + ke_i^T e_{i-1} \leq -\frac{1}{2}k\|e_i\|^2 + \frac{1}{2}k\|e_{i-1}\|^2.\end{aligned}$$

By integrating both sides over $[0, \tau]$, it follows that

$$\begin{aligned}&V_i(\tilde{x}_i(\tau)) - V_i(\tilde{x}_i(0)) \\ &\quad \leq -\frac{1}{2}k\int_0^\tau \|e_i(t)\|^2\,dt + \frac{1}{2}k\int_0^\tau \|e_{i-1}(t)\|^2\,dt.\end{aligned}$$

Since the storage function satisfies $V_i(\tilde{x}_i(\tau)) \geq 0$ and the initial condition $x_i(0) = 0$ implies $\tilde{x}_i(0) = 0$, it directly follows that

$$\int_0^\tau \|e_i(t)\|^2 dt \leq \int_0^\tau \|e_{i-1}(t)\|^2 dt.$$

From the definition of the L_2 norm of the signal, we conclude that the system (1) with (4) is strictly error-L_2 string stable, *i.e.*, $\|e_i\|_{L_2} \leq \|e_{i-1}\|_{L_2}$ for $i = 2, \dots, n$. ■

As shown in Theorem 2, PF topology ensures that the L_2 norm of the error is nonincreasing between any two consecutive agents. However, it does not guarantee that the error will decrease as the agent index i increases, that is, the error may simply be maintained without attenuation. Thus, we demonstrate that, by enabling each agent to use information from multiple preceding agents such as 2PF and 3PF topologies, error-L_2 string stability gain γ_i can be significantly reduced.

Theorem 3. Assume that the system (1) is passive. Consider a static output feedback distributed controller under 2PF topology, defined as

$$\begin{aligned}u_1 &:= k(y_0 - y_1) \\ u_i &:= k(y_{i-2} + y_{i-1} - 2y_i), \quad i = 2, \dots, n.\end{aligned} \tag{7}$$

Then, the system (1) with (7) is *error-L_2 string stable with gain* $\gamma_i = 1/2$ for any positive gain $k > 0$, *i.e.*,

$$\|e_i\|_{L_2} \leq \gamma_i \|e_1\|_{L_2}, \quad i = 2, \dots, n. \qquad \square$$

Proof. We define the error state variables $\tilde{x}_i$ as given in (5). In a similar manner to the proof of Theorem 2, the error dynamics is derived as

$$\begin{aligned}\dot{\tilde{x}}_2 &= A\tilde{x}_2 - 2kBe_2 \\ \dot{\tilde{x}}_i &= A\tilde{x}_i - 2kBe_i + kBe_{i-1} + kBe_{i-2}, \quad i = 3, \dots, n.\end{aligned}$$

We prove the theorem by using induction. Consider the storage function $V_i(\tilde{x}_i)$ as defined in (6). For $i = 2$, by differentiating $V_2(\tilde{x}_2)$ we have $\dot{V}_2(\tilde{x}_2) \leq -2k\|e_2\|^2$. Integrating both sides from 0 to τ gives

$$V_2(\tilde{x}_2(\tau)) - V_2(\tilde{x}_2(0)) \leq -2k\int_0^\tau \|e_2(t)\|^2\,dt.$$

With the initial rest condition ($x_i(0) = 0$), $V_2(\tilde{x}_2(0)) = 0$ and $V_2(\tilde{x}_2(\tau)) \geq 0$. Hence, we have

$$0 \leq V_2(\tilde{x}_2(\tau)) - V_2(\tilde{x}_2(0)) \leq -2k\int_0^\tau \|e_2(t)\|^2\,dt.$$

Since the gain k is positive, it can only be satisfied if $\|e_2(t)\|^2 = 0$ for all $t \geq 0$. Therefore, we conclude that

$$\|e_2\|_{L_2} = 0. \tag{8}$$

For $i = 3$, using Young's inequality, we get

$$\dot{V}_3(\tilde{x}_3) \leq -2k\|e_3\|^2 + ke_3^T e_1 \leq -k\|e_3\|^2 + \frac{k}{4}k\|e_1\|^2.$$

Then, by integrating both sides over $[0, \tau]$, it follows that

$$\int_0^\tau \|e_3(t)\|^2\,dt \leq \int_0^\tau \frac{1}{4}\|e_1(t)\|^2\,dt$$

and $\|e_3\|_{L_2} \leq (1/2)\|e_1\|_{L_2}$. Similarly, for $i = 4$,

$$\dot{V}_4(\tilde{x}_4) \leq -2k\|e_4\|^2 + ke_4^T e_3 \leq -k\|e_4\|^2 + \frac{k}{4}\|e_3\|^2.$$

Then, we have $\|e_4\|_{L_2} \leq (1/2)\|e_3\|_{L_2} \leq (1/4)\|e_1\|_{L_2}$. Meanwhile, for $i = 5, \dots, n$,

$$\dot{V}_i(\tilde{x}_i) \leq -k\|e_i\|^2 + \frac{1}{2}k\|e_{i-1}\|^2 + \frac{1}{2}k\|e_{i-2}\|^2.$$

Assume that $\|e_{i-2}\|_{L_2} \leq \gamma_{i-2}\|e_1\|_{L_2}$ and $\|e_{i-1}\|_{L_2} \leq \gamma_{i-1}\|e_1\|_{L_2}$, then we have

$$\begin{aligned}&\int_0^\tau \|e_i(t)\|^2 dt \\ &\quad \leq \frac{1}{2}\gamma_{i-1}^2\int_0^\tau \|e_1(t)\|^2 dt + \frac{1}{2}\gamma_{i-2}^2\int_0^\tau \|e_1(t)\|^2 dt \\ &\quad \leq \frac{\gamma_{i-1}^2 + \gamma_{i-2}^2}{2}\int_0^\tau \|e_1(t)\|^2 dt.\end{aligned}$$

Therefore,

$$\|e_i\|_{L_2} \leq \gamma_i\|e_1\|_{L_2}, \quad \gamma_i = \sqrt{(\gamma_{i-1}^2 + \gamma_{i-2}^2)/2}.$$

Here, if $\gamma_{i-1} \leq 1/2$ and $\gamma_{i-2} \leq 1/2$, then $\gamma_i \leq 1/2$. Since $\gamma_3 = 1/2$ and $\gamma_4 = 1/4$, we have $\gamma_i \leq 1/2$ for $i = 5, \dots, n$, and we conclude that $\gamma_i \leq 1/2$ for $i = 2, \dots, n$. ■

Next, we examine error-L_2 string stability of the multi-agent system with 3PF topology.

Theorem 4. Assume that the system (1) is passive. Consider a static output feedback distributed controller under 3PF topology, defined as

$$\begin{aligned} u_1 &:= k(y_0 - y_1), \quad u_2 := k(y_0 + y_1 - 2y_2) \\ u_i &:= k(y_{i-3} + y_{i-2} + y_{i-1} - 3y_i), \quad i = 3, \dots, n. \end{aligned} \tag{9}$$

Then, the system (1) with (9) is *error-L_2 string stable with gain* $\gamma_i = 1/3$ for any positive gain $k > 0$, *i.e.*,

$$\|e_i\|_{L_2} \le \gamma_i \|e_1\|_{L_2}, \quad i = 2, \dots, n. \qquad \square$$

Proof. With $\tilde{x}_i$ in (5), the error dynamics is derived as

$$\dot{\tilde{x}}_2 = A\tilde{x}_2 - 2kBe_2, \quad \dot{\tilde{x}}_3 = A\tilde{x}_3 - 3kBe_3,$$

$$\dot{\tilde{x}}_i = A\tilde{x}_i + kB(e_{i-1} + e_{i-2} + e_{i-3}) - 3kBe_i, i = 4, \dots, n.$$

Consider the storage function $V_i(\tilde{x}_i)$ as in (6). The proof follows the same methodology using induction on i.

For $i = 2$ and 3, calculating the derivative of $V_i(\tilde{x}_i)$ yields

$$\dot{V}_2 \le -2k\|e_2\|^2, \quad \dot{V}_3 \le -3k\|e_3\|^2.$$

By integrating these inequalities and following the same argument as in the proof of Theorem 3 (for the case $i = 2$), we conclude that

$$\|e_2\|_{L_2} = \|e_3\|_{L_2} = 0. \tag{10}$$

For $i = 4, \dots, 6$, the time derivatives of $V_i(\tilde{x}_i)$ are

$$\begin{aligned} \dot{V}_4 &\le -3k\|e_4\|^2 + ke_4^T e_1 \le -\frac{3}{2}k\|e_4\|^2 + \frac{1}{6}k\|e_1\|^2, \\ \dot{V}_5 &\le -3k\|e_5\|^2 + ke_5^T e_4 \le -\frac{3}{2}k\|e_5\|^2 + \frac{1}{6}k\|e_4\|^2, \\ \dot{V}_6 &\le -3k\|e_6\|^2 + ke_6^T e_4 + ke_6^T e_5 \\ &\le -2k\|e_6\|^2 + \frac{1}{2}k\|e_4\|^2 + \frac{1}{2}k\|e_5\|^2. \end{aligned}$$

So, we deduce that $\|e_4\|_{L_2} \le (1/3)\|e_1\|_{L_2}$, $\|e_5\|_{L_2} \le (1/9)\|e_1\|_{L_2}$, and $\|e_6\|_{L_2} \le (1/9)\|e_1\|_{L_2}$.

On the other hand, for $i = 7, \dots, n$, we obtain

$$\begin{aligned} \dot{V}_i &\le -3k\|e_i\|^2 + ke_i^T e_{i-3} + ke_i^T e_{i-2} + ke_i^T e_{i-1} \\ &\le -\frac{3}{2}k\|e_i\|^2 + \frac{1}{2}k\|e_{i-3}\|^2 + \frac{1}{2}k\|e_{i-2}\|^2 + \frac{1}{2}k\|e_{i-1}\|^2. \end{aligned}$$

Assume that $\|e_{i-3}\|_{L_2} \le \gamma_{i-3}\|e_1\|_{L_2}$, $\|e_{i-2}\|_{L_2} \le \gamma_{i-2}\|e_1\|_{L_2}$, and $\|e_{i-1}\|_{L_2} \le \gamma_{i-1}\|e_1\|_{L_2}$, then we have

$$\|e_i\|_{L_2} \le \gamma_i \|e_1\|_{L_2}, \quad \gamma_i = \sqrt{(\gamma_{i-1}^2 + \gamma_{i-2}^2 + \gamma_{i-3}^2)/3}.$$

In this case, if $\gamma_{i-3} \le 1/3$, $_{i-2} \le 1/3$, and $\gamma_{i-1} \le 1/3$, then $\gamma_i \le 1/3$. As calculated above, since $\gamma_4 = 1/3$, $\gamma_5 = 1/9$, and $\gamma_6 = 1/9$, we have $\gamma_i \le 1/3$ for $i = 7, \dots, n$. Therefore, $\gamma_i \le 1/3$ for $i = 2, \dots, n$. ■

Note that the multi-agent system is *semi-strictly error-L_2 string stable* under 2PF and 3PF topologies as well. Also, throughout Theorems 2–4, it is confirmed that error-L_2 string stability improves as an agent utilizes more information from its preceding agents.

4. APPLICATION TO VEHICLE PLATOON

In this section, we apply the results of Theorems 2–4 to vehicle platooning systems. Based on the vehicle dynamics, the relationships between position and velocity errors in the platoon are analyzed. Furthermore, a design method of the PD controller for the vehicle dynamics is proposed to ensure the passivity property of each vehicle. To validate the presented results, simulations are provided.

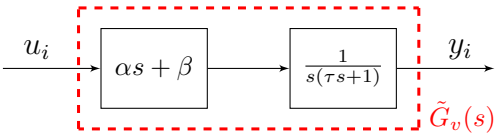


Fig. 2. Block diagram of the i-th vehicle system with PD controller

4.1 Longitudinal Vehicle Dynamics

Consider a multi-agent system (1) of n agents arranged in a platoon formation, as shown in Fig. 1, where each agent is modeled as a vehicle. The nonlinear longitudinal dynamics of each vehicle can be linearized using a feedback linearization technique (see, *e.g.*, Stankovic et al. (2000)). Then, the dynamics is represented as follows:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) \tag{11}$$

where

$$x_i(t) = \begin{bmatrix} p_i \\ v_i \\ a_i \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau} \end{bmatrix},$$

where p_i, v_i, a_i, and u_i denote the position, velocity, acceleration, and control input of the i-th vehicle, respectively, and τ is the inertial delay of the longitudinal dynamics.

According to Ploeg et al. (2013a), string stability analysis in the time domain using the L_2 norm is equivalent to that in the frequency domain. Thus, we derive the transfer functions corresponding to (11) to facilitate the subsequent analysis. Let $P_i(s)$, $V_i(s)$, and $U_i(s)$ denote the Laplace transforms of the position, velocity, and control input of the i-th vehicle, respectively. The transfer function $G_p(s)$ from $U_i(s)$ to $P_i(s)$ (*i.e.*, $C = [1\ 0\ 0]$) is obtained as

$$G_p(s) = \frac{P_i(s)}{U_i(s)} = \frac{1}{s^2(\tau s + 1)}.$$

Similarly, the model $G_v(s)$ from $U_i(s)$ to $V_i(s)$ (*i.e.*, $C = [0\ 1\ 0]$) is

$$G_v(s) = \frac{V_i(s)}{U_i(s)} = \frac{1}{s(\tau s + 1)}. \tag{12}$$

A key assumption in Theorems 2–4 is that each agent is passive. In the frequency domain, a system is passive if and only if its transfer function is *positive real* (Khalil (2002)). Note that the system $G(s)$ is passive only if the relative degree of $G(s)$ is zero or one and $G(s)$ is marginally stable. However, both transfer functions $G_p(s)$ and $G_v(s)$ are not passive and do not satisfy the requirements for Theorems 2–4. Therefore, we present a PD controller design to achieve passivity in the following subsection.

4.2 Velocity Model for Passivity Analysis

As mentioned in the previous subsection, due to the equivalence of the string stability analysis in the state space and frequency domains, we provide the following relation for the error ratio between the vehicles i and $i-1$.

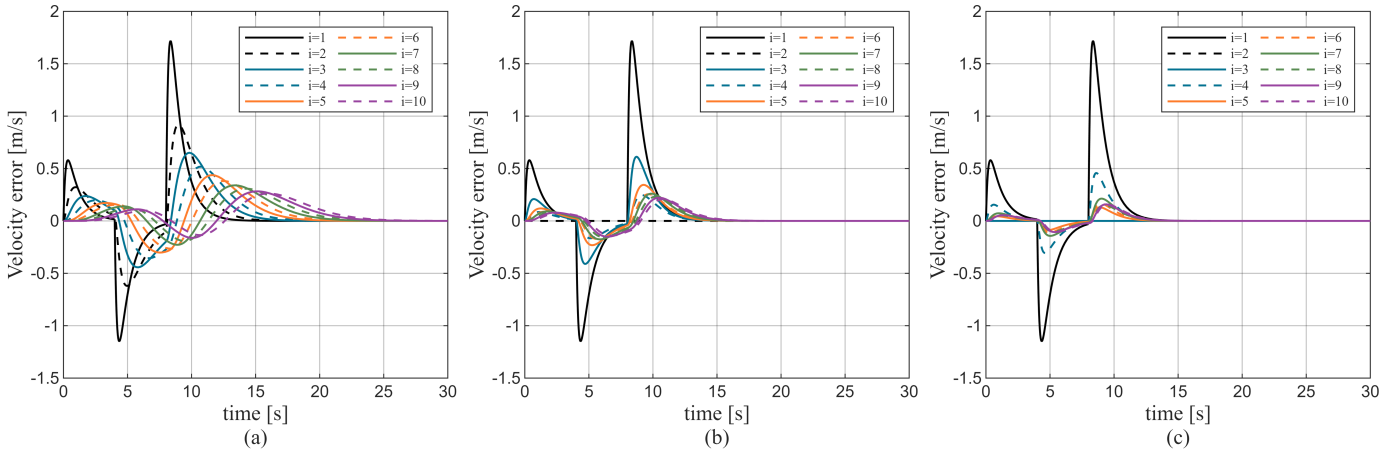


Fig. 3. Velocity error e_i of vehicle platoon. $\tau = 0.2$, $k = 0.5$, $\alpha = 0.8$, and $\beta = 2.6$. (a) PF, (b) 2PF, (c) 3PF.

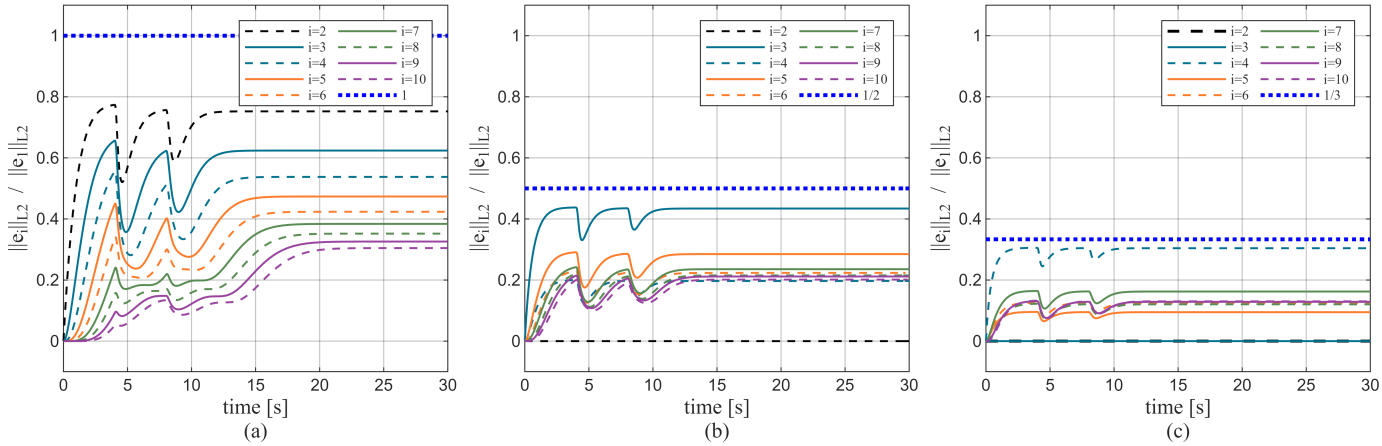


Fig. 4. Error L_2-norm ratio $\|e_i\|_{L_2}/\|e_1\|_{L_2}$ for the vehicle platoon. $\tau = 0.2$, $k = 0.5$, $\alpha = 0.8$, and $\beta = 2.6$. (a) PF, (b) 2PF, (c) 3PF.

$$\frac{E_{p,i}(s)}{E_{p,i-1}(s)} = \frac{P_{i-1}(s) - P_i(s)}{P_{i-2}(s) - P_{i-1}(s)} = \frac{V_{i-1}(s) - V_i(s)}{V_{i-2}(s) - V_{i-1}(s)} = \frac{E_{v,i}(s)}{E_{v,i-1}(s)}$$

where $E_{p,i}(s)$ and $E_{v,i}(s)$ denote position and velocity errors, respectively. It shows that the analysis of the velocity error propagation is sufficient for determining string stability with respect to the position error since the velocity error ratio is equivalent to the position error ratio. Therefore, we utilize $G_v(s)$ in (12) for the string stability analysis.

To achieve passivity, a PD controller $K(s) = \alpha s + \beta$ is added in the series connection with $G_v(s)$, as depicted in Fig. 2. It yields the compensated transfer function $\tilde{G}_v(s)$:

$$\tilde{G}_v(s) = K(s)G_v(s) = \frac{\alpha s + \beta}{s(\tau s + 1)}. \tag{13}$$

Since the relative degree of $\tilde{G}_v(s)$ is one, passivity can be achieved by properly selecting the control gains α and β. The condition for selecting these gains is presented in the following lemma.

Lemma 5. The system (13) is passive if and only if the non-negative control gains α and β satisfy $\alpha \geq \beta\tau$. □

Proof. The proof is omitted here since it can be directly proven via Corollary 11 in Chen and Smith (2009). ■

Therefore, by selecting α and β to satisfy the condition in Lemma 5, $\tilde{G}_v(s)$ is rendered positive real, and thus passive. The compensated model now serves as the basis for the subsequent error-L_2 string stability analysis.

4.3 Simulation

In this subsection, numerical simulations are carried out to validate the main results. We consider a vehicle platoon consisting of 10 identical vehicles under the three information flow topologies shown in Fig. 1. With $\tilde{G}_v(s)$ in (13), system and controller parameters are set as $\tau = 0.2$, $\alpha = 0.8$, and $\beta = 2.6$. The control gain is chosen as $k = 0.5$. Note that these parameters satisfy the passivity condition in Lemma 5 ($\alpha = 0.8 \geq 0.52 = \beta\tau$). The reference input $y_0(t)$ is generated by passing the signal $r(t)$ through a first-order low-pass filter with a time constant of 0.2 s as follows:

$$r(t) = \begin{cases} 0, & t < 0\,\text{s}, \\ 1, & 0\,\text{s} \leq t < 4\,\text{s}, \\ -1, & 4\,\text{s} \leq t < 8\,\text{s}, \\ 2, & t \geq 8\,\text{s}. \end{cases}$$

Fig. 3 illustrates the velocity errors, $e_i(t)$, for the different information flow topologies. Using these errors, Fig. 4 shows the error-L_2 norm ratio of the i-th vehicle's error relative to the first vehicle. Fig. 4(a) confirms that the system (13) is strictly error-L_2 string stable under PF topology, as proven in Theorem 2. This is clearly shown as all error-L_2 norm ratios relative to the first vehicle are bounded by one and monotonically decreasing. It means that e_i is not amplified compared to e_{i-1}. For 2PF topology, Fig. 4(b) confirms the result of Theorem 3. The plot shows that all error-L_2 norm amplification ratios with respect to the first vehicle remain below 1/2. In particular, it shows that $\|e_2\|_{L_2} = 0$, which precisely matches the analytical result (8) derived in the proof of Theorem 3. In 3PF topology, the amplification ratio is further bounded by 1/3, as depicted in Fig. 4(c). Moreover, it illustrates that both $\|e_2\|_{L_2} = 0$ and $\|e_3\|_{L_2} = 0$, which also matches the analytical result (10) derived in the proof of Theorem 4.

5. CONCLUSION

In this paper, we investigated error-L_2 string stability of passive multi-agent systems. The primary contribution is the development of an analytical framework that formally connects the extension of the information flow topology to a reduction in the error-L_2 stability gain γ_i, based on passivity principles. The analysis reveals that using information from the preceding agent guarantees error-L_2 string stability of the system. Furthermore, with the utilization of more information, the gain γ_i decreases. To validate these results, a numerical simulation was conducted on vehicle platoon systems. Nevertheless, the system considered in this paper is limited to minimum-phase homogeneous systems. Future work will focus on systems with higher relative degrees or passivity-short multi-agent systems relaxing passivity constraints instead of utilizing a PD controller. In addition, further analysis with regard to string stability of heterogeneous multi-agent systems will be explored.

DECLARATION OF GENERATIVE AI AND AI-ASSISTED TECHNOLOGIES IN THE WRITING PROCESS

During the preparation of this work the authors used *Google Gemini* in order to check grammar and improve phrasing. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

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