

## Hill-Climbing for Economic Plantwide Control

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**Abstract:** The application of hill-climbing control in the plantwide control context for driving the setpoint corresponding to an unconstrained degree of freedom to the economic optimum steady state is demonstrated for a reactor-separator-recycle process. Optimality via hill-climbing is sought for two operation modes. In Mode I, the throughput is fixed and hill-climbing maximizes the process energy efficiency. In Mode II, hill-climbing is used to maximize process throughput. Rigorous dynamic plantwide simulation results show that hill climbing control reduces steam consumption by ~3.7% and increases plant throughput by ~3.0%, compared to constant setpoint operation.

**Keywords:** Optimum operation, economic operation, hill-climbing control, plantwide control

### 1. INTRODUCTION

In the chemical process industry, the imperatives of fierce market competition and sustainability concerns are increasingly driving innovations for simple and practical plantwide process control system solutions that “seek” and drive the process operation towards the economic optimum steady state. The state-of-the-art consists of a hierarchical three-layered control structure with the layer below receiving setpoint adjustments from the layer above as shown in Fig.1. At the bottom is the regulatory layer with decentralized PI controllers, which closes the individual unit material, energy and component balances as well as the overall plantwide balances for safe and stable process operation. Conventionally, the regulatory layer throughput manipulator (TPM), which is the setpoint used to effect a production rate change, is located at a process fresh feed. Other regulatory layer setpoints that affect the plant steady state and hence the plant economics are adjusted periodically by a real-time optimizer (RTO) that optimizes these setpoints for an economic criterion using an adaptively fitted plant model. Since the optimum steady state solution always has multiple hard active constraints that must be controlled tightly to push the plant operation as close as possible to the optimum, an intermediate MPC supervisory layer adjusts appropriate setpoints in the regulatory layer to mitigate the transients in the constraint variables.

For a fixed active constraint set, recent literature reports have demonstrated that the regulatory layer control structure can and should be systematically altered via TPM relocation (Maity et. al. 2013) and input-output pairing choice (Jagtap et al., 2013) to propagate transients away from the economically dominant active constraints. The inventory control scheme of such a structure is then naturally aligned for tight control of the economically dominant hard active constraints and achieves significantly tighter constraint control compared to

supervisory MPC constraint control with a conventional regulatory structure (Kanodia and Kaistha, 2010).

With tight control of the active constraints, either using supervisory MPC control or by regulatory layer structure design, optimal operation boils down to proper management of the regulatory layer setpoints corresponding to the remaining unconstrained steady state degrees of freedom (dofs). These setpoints should be at their optimum steady state value, which is what the RTO approach attempts. An alternative to RTO is controlling an appropriate process variable, which when held constant provides near optimum process operation despite large process disturbances. In other words the economic penalty for constant setpoint operation with no re-optimization upon change in the process operating condition remains negligibly small. Such process variables are aptly referred to as self-optimizing (Skogestad, 2000).

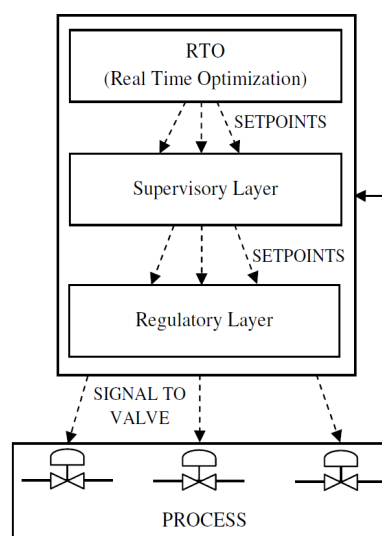


Fig.1. Hierarchical three-layered control

Another approach for managing the setpoints corresponding to the unconstrained steady state dofs is to track the Kuhn-Tucker (KKT) necessary conditions of optimality (NCO) using negative feedback. Realizing the importance of economic optimal operation, Shinsky (1967) proposed a hill-climbing feedback controller for optimizing a single unconstrained process variable, more than four decades ago. More recently, Bonvin and coworkers have demonstrated the application of NCO tracking for mid-course batch recipe corrections for minimizing batch time while ensuring on-target product quality. Despite the relevance of NCO tracking to the economic plantwide control problem, there are no literature reports that evaluate the same, at least to our knowledge. This is quite surprising and serves as the motivation towards exploring the approach in this work. We apply Shinsky's one dof hill-climber for economic optimum operation of a reactor-separator-recycle process. Optimality is sought for two operating modes. In Mode I, the plant steam (expensive utility) consumption is minimized while in Mode II, the plant throughput is maximized. In the following, we briefly describe the process along with optimization results for the two operating modes. The regulatory control structures for the two operating modes are then described. Dynamic hill-climbing control results for both modes are presented and the economic benefit is compared to constant set-point operation. The article ends with the conclusions that can be drawn from the work.

## 2. OPTIMUM PROCESS OPERATION

### 2.1 Process Description

The reactor-separator-recycle process flowsheet is shown in Fig. 2 along with the salient design and operating conditions. Fresh A ( $F_A$ ) and fresh B ( $F_B$ ) are mixed with the recycle stream and fed to a heated CSTR. The irreversible exothermic reaction  $A + B \rightarrow C$  occurs in the boiling reactor. The reactor effluent is sent to a simple distillation column which recovers 99 mol% pure C as the bottoms product and recycles the distillate with unreacted A and B with some C, back to the CSTR. The hypothetical component properties, reaction kinetics and thermodynamic fluid package used to simulate the process in Hysys are noted in Table 1.

Table 1. Modeling details of recycle process

Kinetics	A+B→C	$r = k \cdot x_A \cdot x_B$ $k = 2 \times 10^8 \cdot \exp(-70000/RT)$
Hypotheticals <sup>#</sup>	MW	NBP(°C)
A	50	70
B	80	100
C	130	120
VLE	Soave-Redlich-Kwong	

Reaction rate units:  $\text{kmol} \cdot \text{m}^3 \cdot \text{s}^{-1}$

<sup>#</sup>: Hydrocarbon estimation procedure used to estimate parameters for thermodynamic property calculations

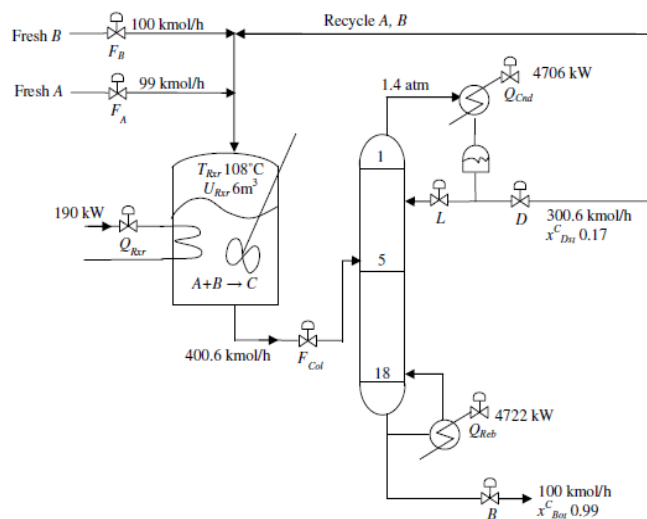


Fig.2. Recycle process flowsheet

### 2.2 Degrees of Freedom

The process has 9 independent control degrees of freedom (dofs) or valves. Of these 2 valves would be used for controlling the column reflux drum and bottom sump levels while another valve would be used for column operation at given design pressure. This leaves 6 remaining valves that may be adjusted to move the process to a particular steady state. The process steady state operating dof is then 6. These correspond to the two fresh feeds ( $F_A$  and  $F_B$ ), the reactor level and temperature ( $U_{Rrx}$  and  $T_{Rrx}$ ), the column reflux to feed ratio ( $L/F_{Col}$ ) and the product purity ( $x^C_{Bot}$ ). We use the following six specification variables to exhaust the steady state process dofs to solve the flowsheet:  $F_B$ ,  $U_{Rrx}$ ,  $T_{Rrx}$ ,  $L/F_{Col}$ ,  $x^C_{Bot}$  and  $x^B_{Rrx}$ .

### 2.3 Optimal Steady Operation

We consider optimum steady operation for two operating modes. In Mode I, the throughput ( $F_B$ ) is given, fixed e.g. by market demand-supply considerations, and the remaining steady state dofs are optimized to maximize the process energy efficiency. Since steam is the expensive utility here, optimal Mode I operation corresponds to minimizing column boilup,  $V_{Col}$ . In Mode II, all the six dofs are optimized to maximize the process throughput ( $F_B$ ). The Mode II solution is usually optimal in a seller's market, where the product demand far exceeds supply.

Table 2 summarizes the Mode I and Mode II steady state optimization, including the process constraints. For Mode I, results are presented for two process throughputs,  $F_B = 100$  kmol/h (design throughput) and  $F_B = 125$  kmol/h (increased throughput). In both modes, the minimum product quality ( $x^C_{Bot}^{MIN}$ ), the maximum reactor temperature ( $T_{Rrx}^{MAX}$ ) and level ( $U_{Rrx}^{MAX}$ ) constraints, are always active. These are soft active constraints with small short-term transient constraint

limit violations being acceptable. For a specified Mode I throughput, this leaves 2 unconstrained steady state dofs.

In Mode II, the maximum column boil-up ( $V_{Col}^{MAX}$ ) constraint goes active, in addition to the 3 Mode I active constraints. It signifies the initiation of flooding in the column and is a hard equipment capacity constraint. The unconstrained Mode II steady state dof remains 2 as  $F_B$  (throughput) is not specified and is an additional Mode II decision variable.

Table 2. Process Optimization Summary

Objective	Mode I Max ( $-V_{Col}$ )	Mode II Max ( $F_B$ )	
Constraints	$0 < \text{Material flow} < 2(\text{base-case})$ $0 < \text{Energy flow} < 2(\text{base-case})$ $0 < U_{Rxx} < 6\text{m}^3$ $0 < V_{Col} < 700\text{kmol/h}$ $90^\circ\text{C} < T_{Rxx} < 108^\circ\text{C}$ $x_{Bot}^C \geq 0.99$		
Optimized operating condition			
Variable			
$F_B(\text{kmol/h})$	100*	125*	133.7
$U_{Rxx}(\text{m}^3)$	6 <sub>max</sub>	6 <sub>max</sub>	6 <sub>max</sub>
$T_{Rxx}(^\circ\text{C})$	108 <sub>max</sub>	108 <sub>max</sub>	108 <sub>max</sub>
$x_{Rxx}^B$	0.172	0.212	0.218
$L/F_{Col}$	0.55	0.55	0.55
$x_{Bot}^C$	0.99 <sub>min</sub>	0.99 <sub>min</sub>	0.99 <sub>min</sub>
$V_{Col}^\#(\text{kmol/h})$	421.3	616.7	700 <sub>max</sub>

\*Specified

# calculated (not a decision variable)

The data in Table 2 suggests that the optimum  $L/F_{Col}$  values are the same. Further simulations showed that the economic objective function is quite insensitive to large changes in  $L/F_{Col}$ . It can therefore be treated as a self-optimizing controlled variable (CV) corresponding to one unconstrained steady state dof. For the second unconstrained dof, we consider  $x_{Rxx}^B$  as a candidate CV. As seen in Table 2, its optimum value changes significantly with throughput. Thus e.g., if the throughput is changed from 100 to 125 kmol/h and the  $x_{Rxx}^B$  specification is kept fixed at its optimum value at the former throughput, at the increased throughput, the column boilup is higher than the new optimum by ~3.7%. Since  $x_{Rxx}^B$  significantly affects  $V_{Col}$  and  $V_{Col}^{MAX}$  is the Mode II hard bottleneck constraint that limits the process throughput, a suboptimal  $x_{Rxx}^B$  value causes the throughput to be noticeably lower than the maximum achievable throughput of  $F_B = 133.7$  kmol/h. The CV,  $x_{Rxx}^B$  corresponding to the second Mode I/Mode II unconstrained dof, is then not self-optimizing. There then exists economic incentive for “seeking” its optimum, for which we will apply a one-dof hill-climbing controller.

### 3. ECONOMIC PLANTWIDE CONTROL SYSTEM

We now design the economic plantwide control system in light of the optimization results for optimal Mode I and Mode II operation.

#### 3.1. Control Structure for Mode I

In Mode I (given  $F_B$ ), we use a conventional regulatory control structure (CS1) with the TPM at  $F_B$ , a process fresh feed. All the downstream inventory controllers are then oriented in the direction of process flow, as shown in Fig. 3a. On the reactor, level ( $U_{Rxx}$ ) is controlled using the column feed ( $F_{Col}$ ) while temperature ( $T_{Rxx}$ ) is controlled using jacket heating duty ( $Q_{Rxx}$ ).  $F_A$  is maintained in ratio with  $F_B$  (TPM) and the ratio setpoint is adjusted to maintain the reactor composition,  $x_{Rxx}^B$ . The hill-climber adjusts the reactor composition setpoint to maximize  $-V_{Col}$ . On the column, the reflux drum and sump levels are controlled using the distillate and bottoms streams, respectively, while column pressure is controlled by manipulating condenser duty. The reflux ( $L$ ) is maintained in ratio with the  $F_{Col}$  and a sensitive stripping tray temperature ( $T_{Col}^S$ ) is controlled by manipulating the boilup,  $V_{Col}$ . A product purity controller adjusts the temperature setpoint to maintain  $x_{Bot}^C$ . Note that for economic optimality,  $U_{Rxx}^{SP} = U_{Rxx}^{MAX}$ ,  $T_{Rxx}^{SP} = T_{Rxx}^{MAX}$  and  $x_{Bot}^C^{SP} = x_{Bot}^C^{MIN}$  with no back-off as these are soft constraints. The remaining three dofs correspond to  $F_B^{SP}$  (desired throughput),  $L/F_{Col}^{SP}$  (self-optimizing) and  $x_{Rxx}^B^{SP}$ , the latter being adjusted by the hill climber for maximum energy efficiency.

#### 3.2. Control Structure for Mode II

For Mode II (maximum  $F_B$ ), in line with the recommendation of Jagtap and Kaistha (2012), the regulatory control structure is altered to CS2 with the TPM relocated to the hard bottleneck constraint, as shown in Fig. 3b. Accordingly,  $V_{Col}$  (bottleneck constraint variable) is tightly controlled by manipulating the column reboiler duty ( $Q_{Reb}$ ). For maximum throughput,  $V_{Col}^{SP}$  is set at  $V_{Col}^{MAX} - \delta$ ,  $\delta$  being the back-off necessary for ensuring the hard constraint limit is not violated during worst case transients. Note that the dynamically fast  $V_{Col}$ - $Q_{Reb}$  pairing gives the tightest possible  $V_{Col}$  control resulting in the back-off being negligible, and hence the maximum throughput being as high as possible. Since  $Q_{Reb}$  is used-up for tight  $V_{Col}$  control, it is unavailable for conventional column temperature control so that  $F_{Col}$  is manipulated instead, to maintain  $T_{Col}^S$ . The reactor level is then controlled using  $F_B$  with the remainder of the regulatory pairings being the same as in CS1. The setpoints  $U_{Rxx}^{SP}$ ,  $T_{Rxx}^{SP}$ ,  $x_{Bot}^C^{SP}$ ,  $V_{Col}^{SP}$ ,  $L/F_{Col}^{SP}$  and  $x_{Rxx}^B^{SP}$  correspond to the six steady state dofs. For maximum throughput, we set  $U_{Rxx}^{SP} = U_{Rxx}^{MAX}$ ,  $T_{Rxx}^{SP} = T_{Rxx}^{MAX}$ ,  $x_{Bot}^C^{SP} = x_{Bot}^C^{MIN}$  and  $V_{Col}^{SP} = V_{Col}^{MAX} - \delta$  ( $\delta = 0$  used here) with  $L/F_{Col}^{SP}$  at its self-optimizing value and  $x_{Rxx}^B^{SP}$  adjusted by a hill-climber for maximizing  $F_B$ .

Note that both CS1 and CS2 have the same regulatory layer CVs and differ only in the pairings used for controlling inventories upstream of  $V_{Col}$ . For clarity, economically significant setpoint choices and control loops are shown in maroon. We also highlight that in industrial practice, as  $F_B^{SP}$

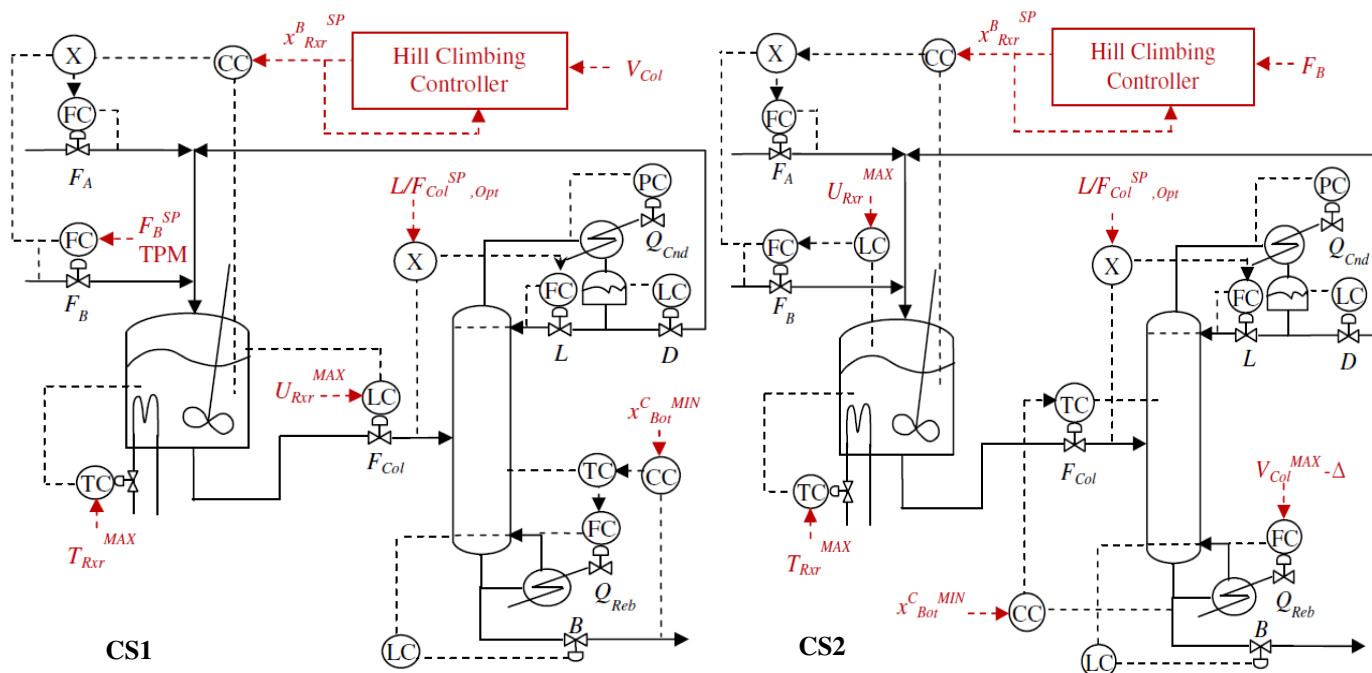


Fig. 3. Conventional regulatory control structures for CS1 and CS2 with hill-climbing controller

(TPM) is increased in CS1 to transition to maximum throughput and the  $V_{Col}^{MAX}$  constraint becomes active (column approaches flooding limit), overrides to handle the  $V_{Col}^{MAX}$  constraint would alter the pairings to reconfigure CS1 to CS2 (Jagtap et al., 2013).

### 3.3. Hill-Climber

The one-dof hill-climber using negative feedback was originally proposed by Shinsky (1967). An adapted version used here is shown in Fig. 4. We want to manipulate the regulatory layer setpoint  $u$  such that the economic objective  $J$  is driven to its maximum. In other words,  $u$  must be adjusted to drive the steady state slope  $y = dJ/du$  to zero. No direct measurement is available for  $y$  and it must be inferred from  $J$  and  $u$ , which are available. Process dynamics however get in the way of estimating  $y$ . We therefore use filters to smoothen the transients in  $J$  and  $u$  and obtain their long term variation post filtering,  $J_f$  and  $u_f$ . The long-term change  $\Delta J$  and  $\Delta u$  are then conveniently obtained by sampling  $J_f$  and  $u_f$ . Dividing  $\Delta J$  by  $\Delta u$  gives an estimate of the slope  $y$ . This estimate is driven to zero by using a feedback PI controller. The setpoint of the PI controller is zero, corresponding to the steady slope at the top of the hill. The output of the PI controller is sampled. Note that since we have a division operation, the output of the divider is trimmed to be between a maximum and minimum to guard against large slope estimates due to division by small numbers. Also, the estimation is not self-starting and needs a disturbance that gives a large enough derivative for division.

### 3.4. Dynamic Simulation and Controller Tuning

A rigorous dynamic simulation of CS1 and CS2 along with the respective hill-climbers as described above is built in

Hysys. The column drum and sump levels are set for 5 min liquid residence time at the design steady state at 50% level.

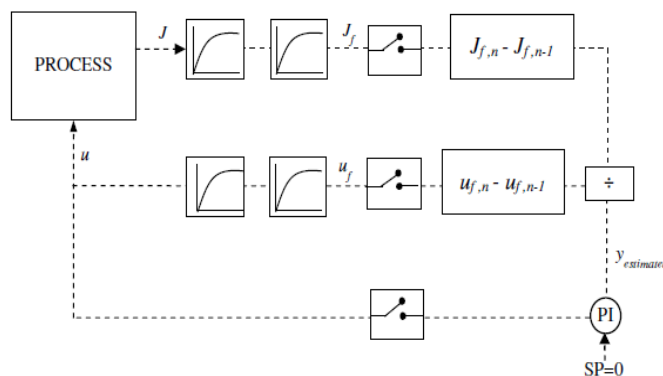


Fig. 4. block diagram the one-dof hill-climber

A consistent tuning procedure is followed for both CS1 and CS2. The column pressure controller is PI and tuned for tight pressure control. All flow controllers are PI and use a reset time of 0.5 mins and a controller gain of 0.3. All non-reactive liquid level controllers are P only and use a gain of 2. The reactor level controller is PI and is tuned for a slightly oscillatory servo response. The reactor temperature and column temperature controller gain is adjusted for a slightly oscillatory servo response with the reset time set to the time it takes for  $2/3^{rd}$  completion of the open loop step response. For realism, all temperature sensor readings are lagged by 2 mins. Also, a lag of 2 mins is applied to all 'direct Q' energy duty valves to account for heat transfer dynamics.

In the economic layer, the  $x_{Bot}^C$  PI controller is tuned by hit-and-trial for a slightly underdamped servo response. The composition sensor uses a sampling time of 5 mins and a dead-time of 5 mins. In our work, the one-dof PI hill-climber

is implemented in Matlab and linked with Hysys dynamics using object-oriented protocols. Both the Mode I and Mode II hill-climbers are tuned by hit-and-trial. Two 3 hr first order lags with are applied in series to  $J$  and  $x_{Rxx}^{B,SP}$  to filter out fast transients and obtain their long-term trends. We also apply a 1 hour sampling on the  $x_{Rxx}^{B,SP}$  adjustments by the hill-climber as well as to  $J_f$  and  $u_f$ . The salient parameters of the regulatory and economic loops used here in CS1 and CS2 simulations are noted in Table 3.

Table 3. Salient controller parameters for CS1/CS2<sup>\*,#,\$</sup>

CV	CS1/CS2		PV Range <sup>&amp;</sup>	MV Range <sup>&amp;</sup>
	K <sub>C</sub>	τ <sub>I</sub> (min)		
(A:B) <sub>Rxx</sub>	1.2/1.2	80/80	0.05-0.40	0.5-1.5
T <sub>Rxx</sub>	3/3	30/30	100-120°C	2x10 <sup>6</sup> kJ/h
U <sub>Rxx</sub>	2/2	20/20	10-100%	0-100%
T <sup>S</sup> <sub>Col</sub>	0.5/0.5	30/40	110-140°C	4x10 <sup>7</sup> kJ/h
x <sup>C</sup> <sub>Bot</sub>	0.35/0.35	40/40	0.98-0.995	110-140°C

\* All level loops use K<sub>C</sub> = 2 unless otherwise specified.

#Pressure/flow controllers tuned for tight control. &Minimum value is 0, unless specified otherwise. \$All compositions have a 5 min dead time and sampling time. All temperature measurements are lagged by 2 min.

the plantwide response completes in ~15 hrs, the hill-climber is switched on and it seeks the optimum value of  $x_{Rxx}^{B,SP}$  that maximizes  $-V_{Col}$  (minimizes  $V_{Col}$ ). The plantwide dynamic response of salient process variables is shown in Fig. 5. The product quality control is observed to be quite tight for the entire duration of the transient response. The hill-climber readjusts  $x_{Rxx}^{B,SP}$  towards its optimum value and in response,  $V_{Col}$  reduces towards its minimum value. The response shows that it takes only ~15 hrs for the minimum  $V_{Col}$  to be closely approached. For more aggressive hill-climber tuning, slight ringing around the optimum steady state is observed with the hill climber overshooting the peak and then stepping back. Further detuning of the hill-climber causes the time for the response to reach close to the optimum to increase significantly. For a +25 kmol/h throughput change, the action of the hill-climber causes  $V_{Col}$  to reduce from 639.5 kmol/h for constant  $x_{Rxx}^{B,SP}$  operation to 616.7 kmol/h, implying ~3.7% energy savings, which is not negligible. For a -25 kmol/h throughput change, the energy saving is somewhat lower at ~2.6%. These savings are significant enough to justify the additional cost of the hill-climber.

For Mode II (CS2) operation, the  $x_{Rxx}^{B,SP}$  is kept fixed at its optimum value at the design throughput, the CS2  $V_{Col}^{SP}$  is set at its constraint value of  $V_{Col}^{MAX}$  and the plant is allowed to settle to steady state. This initial steady state corresponds to

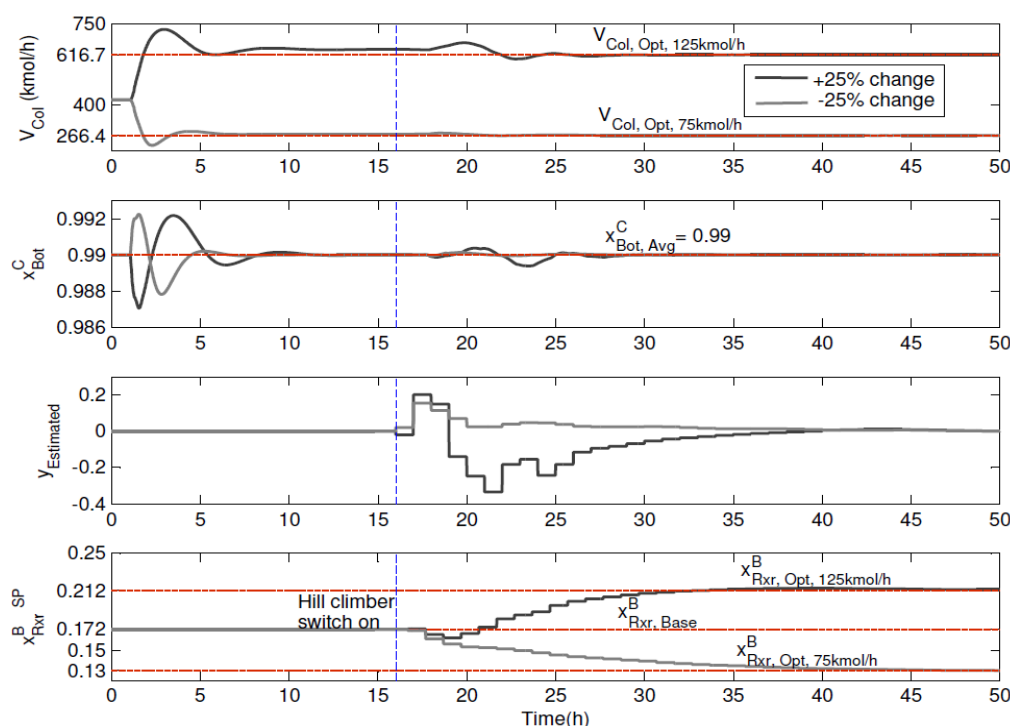


Fig. 5. Dynamic response for boilup minimization

#### 4. CLOSED LOOP RESULTS

The closed loop performance of the economic plantwide control system with the one-dof hill-climber for updating  $x_{Rxx}^{B,SP}$  is now obtained. In Mode I (CS1) operation, the throughput ( $F_B^{SP}$ ) is changed as a  $\pm 25$  kmol/h step around the design throughput ( $F_B = 100$  kmol/h). Initially, the  $x_{Rxx}^{B,SP}$  is kept constant at its design throughput optimum value. After

the maximum throughput with no re-optimization of  $x_{Rxx}^{B,SP}$ , which is held constant. At this steady state,  $F_B = 129.8$  kmol/h. The one-dof hill-climber is then switched on and it adjusts  $x_{Rxx}^{B,SP}$  to seek the value of  $x_{Rxx}^{B,SP}$  that maximizes  $F_B$ . The transient response of salient process variables in Fig. 6 shows that tight product quality control is achieved during the transient period. The  $x_{Rxx}^{B,SP}$  hill-climber causes  $F_B$  to increase towards the maximum achievable throughput value of 133.7



## REFERENCES

kmol/h. In about 15 hrs, this maximum value is approached quite closely. The hill-climber thus achieves a substantial ~3.0% increase in the maximum throughput. For a product sale price of \$30 per kmol, the increased throughput translates to ~\$1.0x10<sup>6</sup> additional yearly sales revenue. This clearly shows that a hill-climber to optimize an economically important unconstrained regulatory setpoint can lead to significant economic benefit compared to a constant setpoint operating policy. More importantly, the hill-climber ensures the unconstrained setpoint value, whose optimum is otherwise unknown, gets driven to the optimum so that the process operation remains near optimal.

## 5. CONCLUSIONS

To conclude, this work shows that hill-climbing to optimize CV setpoints corresponding to the process unconstrained steady state dofs can significantly improve economic process operation. For the disturbances considered in the example recycle process, the one-dof hill climber achieves more than 3% energy savings and maximum achievable throughput increase, compared to constant setpoint operation. Hill-climbing is thus a simple and effective alternative to and should be of increasing interest to industrial economic plantwide control applications.

- Bonvin, D. (1998). Optimal operation of batch reactors - A personal view. *Journal of Process Control* **8**, 355-368.
- Chachut, B., B. Srinivasan and D. Bonvin (2009). Adaptation strategies for real time optimization. *Computers and Chemical Engineering* **33**, 1557-1567.
- Jagtap, R. and N. Kaistha (2012). Throughput manipulator location selection for economic plantwide control. In: *Plantwide Control: Recent Developments and Applications* (G.P. Rangaiah and V. Kariwala), 121-145. John Wiley, New York.
- Jagtap, R., N. Kaistha and S. Skogestad (2013). Economic Plantwide Control Over a Wide Throughput Range: A Systematic Design Procedure. *American Institute of Chemical Engineers Journal* **59**, 2407-2426.
- Kanodia, R. and N. Kaistha (2010). Plantwide control for throughput maximization: A case study. *Industrial and Engineering Chemistry Research* **49**, 210-221.
- Maity, D., R. Jagtap and N. Kaistha (2013). Systematic top-down economic plantwide control of the cumene process. *Journal of Process Control* **23**, 1426-1440.
- Shinsky, F.G. (1996). *Process Control Systems: Application, Design, and Tuning*. McGrawHill, New York.
- Skogestad, S. (2000). Plantwide control: The search for the self-optimizing control structure. *Journal of Process Control* **10**, 487-507.

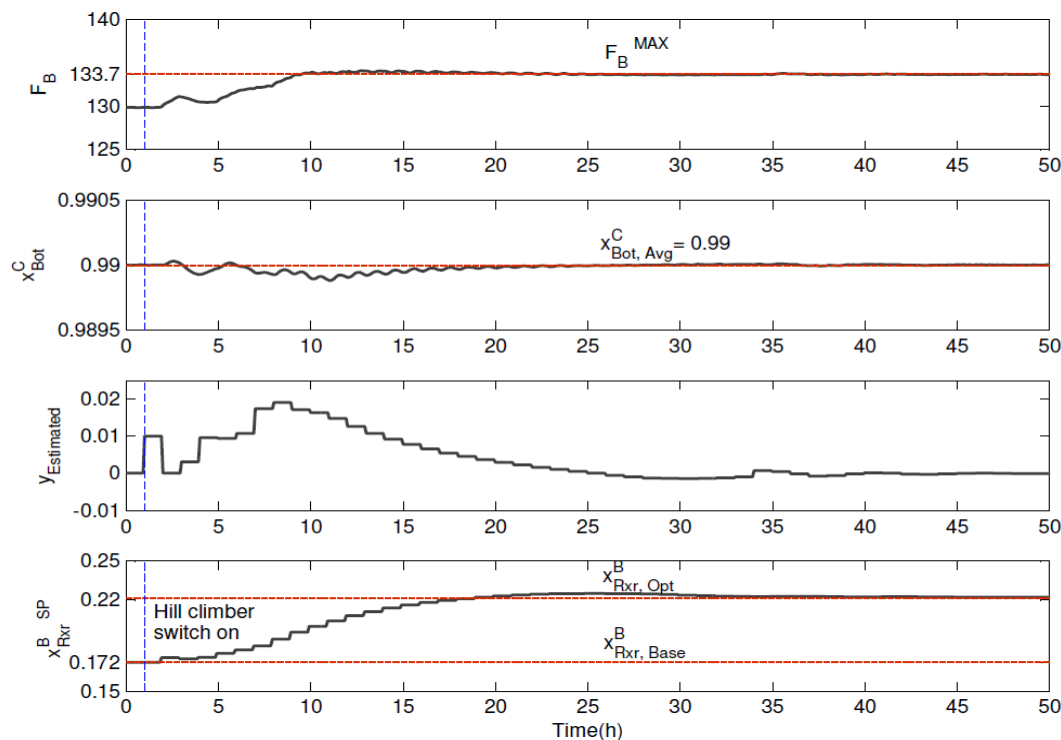


Fig. 6. Dynamic response for throughput maximization