Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Extended Binary Model Reference Adaptive Control overcomes limitations of L1 Adaptive Control ⋆

Hsu, L. * **Battistel, A.** * **Nunes, E. V. L.** *

* *COPPE/UFRJ, Department of Electrical Engineering; (e-mail: liu@coep.ufrj.br, battistel@ufrj.br, eduardo@coep.ufrj.br).*

Abstract:
The aim of this paper is to introduce a combination of two control techniques proposed in the 90's, namely the Smooth Sliding Control (SSC) and the Binary Model Reference Adaptive Control (BMRAC) that shares common features with the recently proposed L1 Adaptive Controller (L1-AC). The basic L1-AC structure is shown to be similar to that of the SSC, except that the SSC is based on sliding mode control instead of adaptive control. On the other hand, the B-MRAC provides a smooth transition between adaptive and sliding mode control. The natural combination of both control techniques, called extended BMRAC (eBMRAC), is shown to overcome important limitations of the L1-AC, such as poor tracking performance to time-varying reference signals, the need of excessively large gains and the difficulty to extend it to output feedback control. In contrast to the L1-AC designs, the new output feedback controller eBMRAC does not need to be structurally redesigned for different types of systems.

1. INTRODUCTION

A recently proposed control architecture has attracted notable attention in the past few years. The so-called L1 Adaptive Control (L1-AC), firstly published in [Cao and Hovakimyan, 2006a]-[Cao and Hovakimyan, 2006b] and later in the book [Hovakimyan and Cao, 2010] is claimed to provide fast adaptation with guaranteed transient properties. The controller basically consists of a modified Model Reference Adaptive Control (MRAC) using an input filtered control, a state prediction loop and high-gain adaptation law with parameter projection. From now on, the latter will be referred to as *projection adaptation*, for simplicity.

Despite the fact that several works reporting successful applications of L1 can be found in the literature, see [Hovakimyan et al., 2011] and references therein, some recent papers questioning the efficiency of this technique can also be found [Ortega and Panteley, 2014a,b], [Boskovic and Mehra, 2013].

Criticisms regarding L1-AC include the use of excessively high adaptation gains, the inability to track a time-varying reference and the coincidence of L1-AC control signal with a full state PI controller, which suggests that adaptation is unnecessary in such scheme [Ortega and Panteley, 2014b], [Ortega, 2013].

It should also be noted that the L1-AC scheme is different according to the application. For instance, if the plant input gain is unknown, the algorithm has to be modified to a more complex architecture [Hovakimyan and Cao, 2010] (pp.35). This is similar for output feedback.

⋆ This work was supported in part by the Brazilian Research Agencies CNPq, CAPES & FAPERJ.

On the other hand, the ideas of using high gain, input filtering and prediction loop is not new in tracking control of uncertain systems. The Smooth Sliding Control (SSC), proposed by Hsu [1997] as a solution to avoid chattering in sliding mode control (SMC) systems, also relies on input filtered control together with an output error prediction loop, similarly to the L1-AC. The high-gain naturally appears since the control signal is generated by an amplitude modulated relay function. The discontinuous control is filtered prior to being injected into the plant, providing a smooth control signal. Despite the similarity, however, an essential difference is apparent since the SSC explicitly employs a reference model - which L1-AC does not. It is then possible to track a time-varying reference with a small residual error with the SSC while such property is not guaranteed with the L1-AC.

The use of high gain projection adaptation laws was proposed by Hsu and Costa [1994], under the designation of Binary-MRAC (BMRAC), as a method to improve adaptation transient and to achieve the good performance and robustness properties of a sliding mode controller (SMC), while avoiding chattering. It was argued that the BMRAC tended to a sliding mode controller as the adaptation gain was increased. Therefore, it can be expected that the SMC can be replaced by a BMRAC loop.

In this context, this paper seeks to discuss how to overcome some of the flaws of L1-AC by combining the SSC and BMRAC schemes. The key idea is to use the BMRAC high gain projection adapdation law with the input filtering and prediction architecture of the SSC and compare the resulting scheme, named extended BMRAC (eBMRAC) with the L1-AC. Simulations with simple examples illustrate that the eBMRAC outperforms the L1-AC.

2. MRAC ERROR EQUATIONS

Consider an uncertain SISO linear time invariant plant

$$\dot{x}_p = A_p x_p + b_p u\,, \qquad y_p = h_p^T x_p\,, \tag{1}$$

where $x_p \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ is the input and $y_p \in \mathbb{R}$ is the output. The corresponding input-output model is

$$y_p = G_p(s)u, \qquad G_p(s) = k_p \frac{N_p(s)}{D_p(s)},$$

where $K_p \in \mathbb{R}$ is the high frequency gain, $N_p(s)$ and $D_p(s)$ are monic polynomials.

We assume that the plant parameters are uncertain and only known within finite bounds and consider the usual MRAC design assumptions:

A1) $G_p(s)$ is minimum phase.
A2) The plant is controllable and observable.
A3) The order of the plant (n) is known.
A4) The plant relative degree n^* is known.
A5) The sign of K_p is known and assumed positive without loss of generality.

Considering that a reference signal y_m is generated by the following reference model:

$$y_m = M(s)r \tag{2}$$

where $M(s)$ is stable with relative degree n^*. The main objective is to find a control law u such that the output error $e_0 := y_p - y_m$ tends asymptotically to zero, for arbitrary initial conditions and uniformly bounded arbitrary piecewise continuous reference signal $r(t)$.

When the plant is known a control law which achieves the matching between the closed-loop transfer function and $M(s)$ is given by $u^* = \theta^{*T}\omega$, where the parameter vector is written as $\theta^* = \left[\theta_1^{*T}\ \theta_2^{*T}\ \theta_3^*\ \theta_4^*\right]^T$, with $\theta_1^*, \theta_2^* \in \mathbb{R}^{n-1}, \theta_3^*, \theta_4^* \in \mathbb{R}$ and the regressor vector $\omega = [\omega_u^T\ \omega_y^T\ y_p\ r]^T \in \mathbb{R}^{2n}$ is obtained from I/O state variable filters given by:

$$\dot{\omega}_u = \Lambda\omega_u + gu, \qquad \dot{\omega}_y = \Lambda\omega_y + gy_p, \tag{3}$$

where $\Lambda \in \mathbb{R}^{n-1\times n-1}$ is Hurwitz and $g \in \mathbb{R}^{n-1}$ is chosen such that the pair (Λ, g) is controllable. The matching conditions require that $\theta_4^* = K_m/K_p$ [Ioannou and Sun, 1996]. The error equation is developed as usual for MRAC [Ioannou and Sun, 1996], [Hsu and Costa, 1994].

$$\dot{x}_e = A_c\ x_e + k^* b_c\ [u - u^*], \qquad e_0 = h_c^T\ x_e, \tag{4}$$

or in an input/output form

$$e_0 = k^* M(s)\ [u - u^*] \tag{5}$$

3. BINARY MODEL REFERENCE ADAPTIVE CONTROL (BMRAC)

The Binary-MRAC for arbitrary relative degree plants was proposed by Hsu and Costa [1994] and consists of a high gain projection adaptation based MRAC. The resulting system presents better transient performance and robustness to unmodeled dynamics than with conventional adaptive controllers.

Since the parameter vector is not known, the control input is designed using an estimate θ of the ideal parameter θ^*. The implementable control law is given by:

$$u(t) = \theta^T(t)\omega(t) \tag{6}$$

For the case $n^* = 1$, θ is obtained by a projection-type adaptation law

$$\dot{\theta}(t) = -\sigma\theta - \gamma e_0 \omega \tag{7}$$

with high gain γ and the projection factor defined as

$$\sigma = \begin{cases} 0\,, & \text{if } \|\theta\| < M_\theta \text{ or } \sigma_{eq} < 0 \\ \sigma_{eq}\,, & \text{if } \|\theta\| \geq M_\theta \text{ and } \sigma_{eq} \geq 0 \end{cases} \tag{8}$$

with a constant $M_\theta \geq \|\theta^*\|$ and

$$\sigma_{eq} = \frac{-\gamma e_0 \theta^T \omega}{\|\theta\|^2} \tag{9}$$

The BMRAC scheme is depicted in Fig. 1.

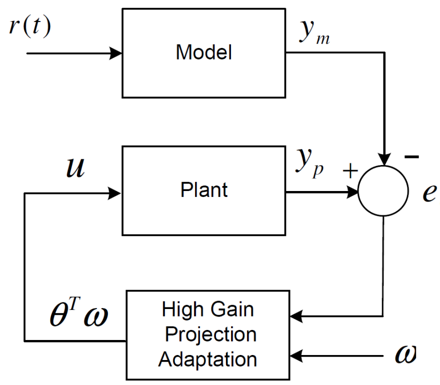

Fig. 1. BMRAC block diagram

4. SMOOTH SLIDING CONTROL (SSC)

The Smooth-Sliding Control technique was proposed by Hsu [1997] as a solution to avoid chattering in variable structure model reference control systems. The architecture of L1-AC resembles the SSC closely, but with a few key differences: i) the absence of an explicit reference model; ii) the use of projection based adaptation instead of relay switched control; iii) the need to structurally redesign the algorithm to deal with output feedback and unknown input gain; and iv) the inability to track a time-varying reference with acceptable error as will be seen in Section. 6. Here, we focus on the case $n^* = 1$ for the sake of simplicity, since only straightforward modifications are needed to deal with arbitrary relative degree [Hsu, 1997].

4.1 The case $n^* = 1$

The SSC is also based on the MRAC framework where the error equations are given by (4)-(5). The SSC employs a filtered input and a prediction loop, which is also used in the L1-AC. The smooth control law is obtained using an averaging filter with sufficiently small time constant τ such that the control u is replaced by u_0^{av}, an approximation of the equivalent control $(u_0)_{eq}$. The control law is

$$u = u^{nom} - u_0^{av}; \quad u_0^{av} = (1/F_{av}(\tau s))u_0 \tag{10}$$

$$u_0 = f(t) sign(\varepsilon_0) \tag{11}$$

where ε_0 is an output prediction error associated with the prediction loop

$$\varepsilon_0 = e_0 - \hat{e}_0; \quad \hat{e}_0 = k^{nom} M[u_0 - u_0^{av}]; \tag{12}$$

since $\hat{e}_0$ can be interpreted as a predicted output error by considering k^{nom} and u_0 as estimates of k^* and $u^{nom} - u^*$, respectively. With correct estimates the prediction would be exact, as seen in eq. (6). The modulation function $f(t)$ is chosen such that $f(t) \geq |u^*(t) - u^{nom}(t)|\,; \forall t$. The SSC scheme is presented in Fig 2.

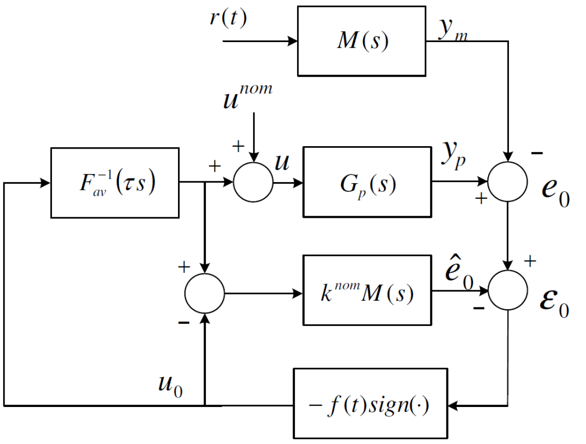

Fig. 2. Smooth sliding control block diagram

5. LIMITATIONS WITH L1-AC

The L1-AC formulation considers the plant as

$$\dot{x}(t) = Ax(t) + b(u(t) + \theta^T x(t)), \quad y(t) = c^T x(t); \quad (13)$$

where $A \in \mathbb{R}^{n \times n}$, $b, c \in \mathbb{R}^n$ are assumed as known [Hovakimyan and Cao, 2010] (pp.18). The following control structure is used

$$u(t) = u_m(t) + u_{ad}(t), \quad u_m(t) = -k_m^T x(t) \quad (14)$$

where k_m renders $A_m \triangleq A - bk_m^T$ Hurwitz, while $u_{ad}(t)$ is an adaptive component that will be defined shortly. Thus, the partially closed-loop system is given by:

$$\dot{x}(t) = A_m x(t) + b(\theta^T x(t) + u_{ad}(t)), \quad y = c^T x(t) \quad (15)$$

The following state-predictor is used

$$\dot{\hat{x}}(t) = A_m \hat{x}(t) + b(\hat{\theta}^T x(t) + u_{ad}(t)) \quad \hat{y} = c^T \hat{x}(t) \quad (16)$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the state of the predictor and $\hat{\theta}(t) \in \mathbb{R}^n$ is the estimate of the parameter θ, obtained by a projection based adaptive law

$$\dot{\hat{\theta}} = \gamma Proj(\hat{\theta}(t), -\tilde{x}^T(t)Pbx(t)), \quad \hat{\theta}(0) = \hat{\theta}_0 \in \Theta; \quad (17)$$

the prediction error is defined as $\tilde{x}(t) \triangleq \hat{x}(t) - x(t)$, $\gamma \in \mathbb{R}^+$ is the adaptation gain, $P = P^T$ is the solution of Lyapunov equation $A_m^T P + PA_m = -Q$ for arbitrary symmetric $Q = Q^T > 0$. The projection is confined to the set Θ. The adaptive control signal in the frequency domain is

$$u_{ad}(s) = -C(s)(\hat{\eta}(s) - k_g r(s)) \quad (18)$$

where $r(s)$ and $\hat{\eta}(s)$ are the Laplace transforms of $r(t)$ and $\hat{\eta}(t) = \hat{\theta}^T(t)x(t)$, respectively. The input gain $k_g \triangleq -1/(c^T A_m^{-1} b)$ is assumed known and $C(s)$ is a stable filter. The L1-AC architecture is presented in the block diagram of Fig. 3. It is interesting to note that the L1-AC needs to be adjusted to be applicable to different situations. For instance, if the states are not measurable, the scheme showed above is no longer suitable. Similarly, if the input gain is unknown, the structure has to be redesigned. Also,

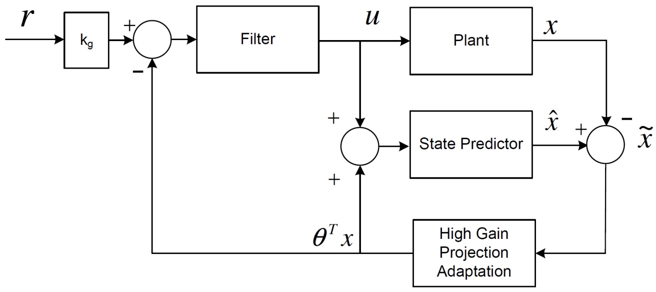

Fig. 3. L1-AC architecture

the L1-AC is not able to track time-varying references nor it presents parameter convergence.

In [Ortega and Panteley, 2014a] and [Ortega and Panteley, 2014b], the control signal generated by the L1-AC is shown to be equivalent to a full state PI controller, suggesting that adaptation would be unnecessary.

This can be illustrated using a first order stable LTI filter in L1-AC scheme, although it is also true for general filters according to Ortega and Panteley [2014b]. Note that the filtered input can be rewritten as

$$\dot{u} = -k(u - \hat{\theta}^T x) \quad (19)$$

the control signal generated by (19) coincides with the output of a perturbed LTI PI controller

$$\dot{v} = kb^\dagger A_m x - \mu k \tilde{\theta}^T x; \quad u = v - kb^\dagger x \quad (20)$$

where $b^\dagger$ is the pseudo-inverse of b, given by $b^\dagger = (b^T b)^{-1} b^T$. This means that if the parameter error converges to zero, the obtained controller converges to an LTI controller that could be obtained without adaptation.

It is also important to note that L1-AC analysis does not guarantee zero tracking error for time-varying reference signals, as shown in [Hovakimyan and Cao, 2010]. The same applies to parameter error, such that only the prediction error is assured to be uniformly bounded.

Note that this controller requires full state measurement, which as mentioned is quite restrictive and it also requires knowledge of input gain. Even though the L1-AC theory is able to extend the idea shown above to contour these limitations, it is important to note that this is achieved by changing the control architecture.

The proposed controller is able to overcome these difficulties, since it does not require knowledge of input gain and is able to track a time-varying reference with residual error using output feedback without the need to modify the control scheme.

6. COMBINING SSC AND B-MRAC

The core idea of this paper is to propose a controller that combines the SSC and BMRAC by using the SSC architecture with the BMRAC adaptation. The resulting scheme is named Extendend BMRAC (eBMRAC) To that end, the relay of the SSC is replaced by an output feedback projection adaptation law with standard MRAC parametrization. The scheme is seen in Fig. 4.

Consider the MRAC error equations (4)-(5). As well as the SSC, the eBMRAC uses an input filtered control signal

$$u = C(s)[u_0]; \quad u_0 = \theta^T \omega \tag{21}$$

where θ is an adaptive parameter. For the sake of simplicity, the plant high frequency gain K_p is assumed known. Nevertheless, the case where only $sign(K_p)$ is known can be addressed using a similar development as in [Hsu, 1997].

Consider the auxiliary errors

$$\hat{e}_0 = k^* M(s)[u - u_0], \quad \varepsilon_0 = e_0 - \hat{e}_0 \tag{22}$$

The adaptation law with parameter projection is

$$\dot{\theta}(t) = -\sigma\theta - \gamma\varepsilon_0\omega \tag{23}$$

$$\sigma = \begin{cases} 0\,, & \text{if } \|\theta\| < M_\theta \text{ or } \sigma_{eq} < 0 \\ \sigma_{eq}\,, & \text{if } \|\theta\| \geq M_\theta \text{ and } \sigma_{eq} \geq 0 \end{cases} \tag{24}$$

$$\sigma_{eq} = \frac{-\gamma\varepsilon_0\theta^T\omega}{\|\theta\|^2} \tag{25}$$

The eBMRAC equations are $e = y_p - y_m$, Eq. (21)–(25) with ω as defined in Section 2. The block diagram is shown in Fig. 4. The predicted error $\hat{e}_0$ state dynamics can

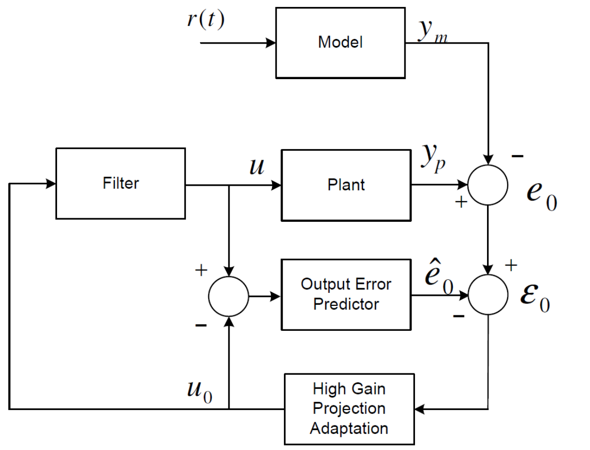

Fig. 4. Extended BMRAC (eBMRAC) block diagram

be written as

$$\dot{\hat{x}}_e = A_c\, \hat{x}_e + k^* b_c\, [u - u_0], \qquad \hat{e}_0 = h_c^T\, \hat{x}_e, \tag{26}$$

which allows to obtain the state dynamics for the prediction error

$$\dot{x}_\varepsilon = A_c\, x_\varepsilon + k^* b_c\, [u_0 - u^*], \qquad \varepsilon_0 = h_c^T\, x_\varepsilon, \tag{27}$$

$$\varepsilon_0 = k^* M(s)[u_0 - u^*]; \tag{28}$$

Note that the prediction error does not depend on the filtered input.

The following properties are guaranteed by the projection based adaptation law:

Theorem 1. Consider the error system described by (4)-(5) and the auxiliary errors (22),(27). The control signal is given by (6) with adaptation law (23)–(25). Assume that assumptions (A1)-(A4) hold, $\|\theta(0)\| \leq M_\theta$ and K_p is known. If τ is sufficiently small, then

i) $\|\theta\| \leq M_\theta, \forall t \geq 0$;
ii) $\|x_\varepsilon(t)\|^2 \leq c_1 e^{-\lambda_1 t}\|x_\varepsilon(0)\|^2 + \mathcal{O}(\gamma^{-1}), \forall t \geq 0$ for some positive constants c_1 and λ_1;
iii) The prediction error ε_0 tends asymptotically to zero;
iv) e_0 tends exponentially to some small residual interval of order τ.

The more general case, when only $sign(K_p)$ is known and unmodeled dynamics (including delays) are present, can also be considered following the same developments presented by Hsu [1997].

7. SIMULATION RESULTS

7.1 Example 1

In order to show the efficiency of the proposed controller in comparison with L1-AC, we use a simple first order example. Consider the following plant, state predictor and filter:

$$\dot{x} = 3x + u + \theta x; \quad \dot{\hat{x}} = -2\hat{x} + u; \quad C(s) = \frac{c}{s+c} \tag{29}$$

It is desired to track a sinusoidal reference signal, given by $r(t) = 10\sin(0.5t)$. High-gain is used as suggested by Hovakimyan and Cao [2010]. In this case, $\Gamma = 10^4$ and $c = 160$. The unknown parameter is assumed to be in the set $\theta = [-10,\ 10]$. For $\theta = -5$ the result is seen in Fig. 5, note that the system output does not track the reference input. The same plant, reference model and filter is used

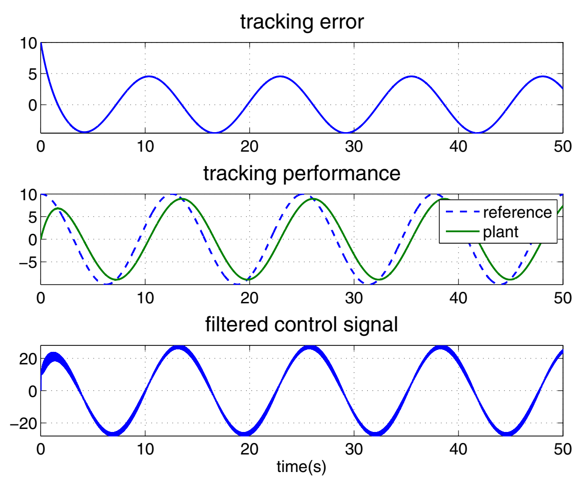

Fig. 5. L1-AC performance for Example 1

for the eBMRAC, that is

$$G(s) = \frac{1}{s-3-\theta}; \quad M(s) = \frac{1}{s+1}; \quad F_{av} = \frac{1}{\tau s+1} \tag{30}$$

Design parameters are chosen as $\gamma = 10$; $M_\theta = 10$ and $\tau = 0.02$. The result is shown in Fig. 6, where it is possible to note a good tracking performance.

7.2 Example 2

L1-AC: To further compare the two schemes, the simulation results in this section consider the second order plant used in [Cao and Hovakimyan, 2006a,b], already including the unknown parameter θ of (13), that is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1.4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u; \quad y = [1\ 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{31}$$

The reference signal is $r(t) = 100cos(0.2t)$ and the filter is designed as $C(s) = 160/(s+160)$. Adaptation gain is $\Gamma = 10^4$ and θ is assumed to be in the set $\theta_i = [-10,\ 10],\ i = 1, 2$.

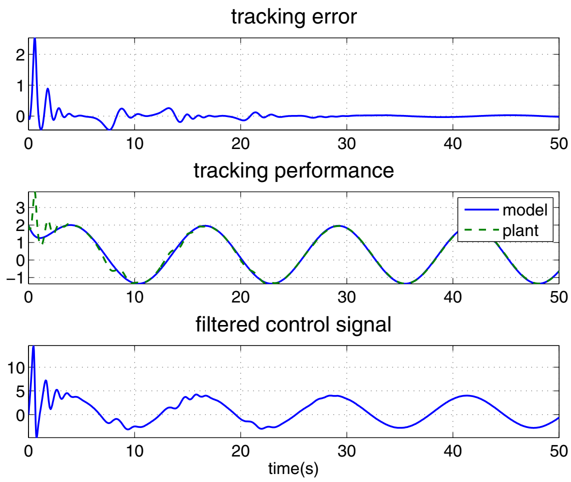

Fig. 6. eBMRAC performance for for Example 1

The plant transfer function has relative degree $n^* = 2$. However, when dealing with state feedback, one can obtain an output of relative degree $n^* = 1$. The results of Figs. 7 reproduce the results shown in [Cao and Hovakimyan, 2006b] and it is possible to note a poor tracking performance as well as oscillatory parameter values.

The limitations of L1-AC can be more clearly shown in two different scenarios: i) if the frequency is increased, which severely impairs the performance as seen in Fig. 8 when reference signal is $r(t) = 100cos(t)$; and ii) if the input gain is not known. In this case, the whole L1-AC scheme has to be redesigned, which is a restrictive constraint. Fig. 8 shows that it is not even possible to track a unit step input

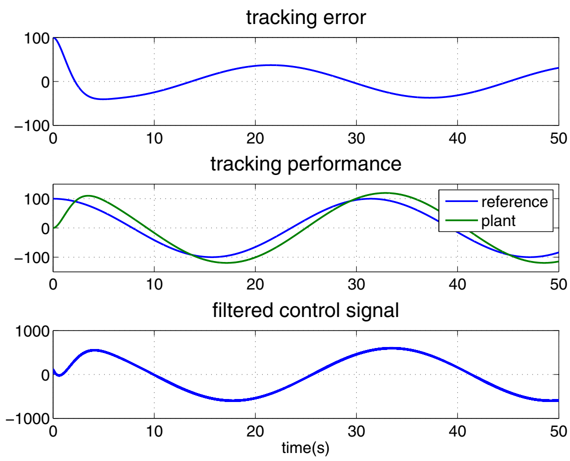

Fig. 7. L1-AC: Example 2 with $r(t)=100cos(0.2t)$

eBMRAC: The same plant of Eq. (31) is used assuming there is prior knowledge on the system states such that it is possible to obtain an output of relative degree $n^* = 1$. Considering both states are measurable, an output of relative degree one is obtained by combining the states such that the output is $\bar{y}_p = Pb_p x_p$.

It is important to note that this is done for the sole purpose of providing a fair comparison, since the L1-AC is designed as state feedback. The more general version

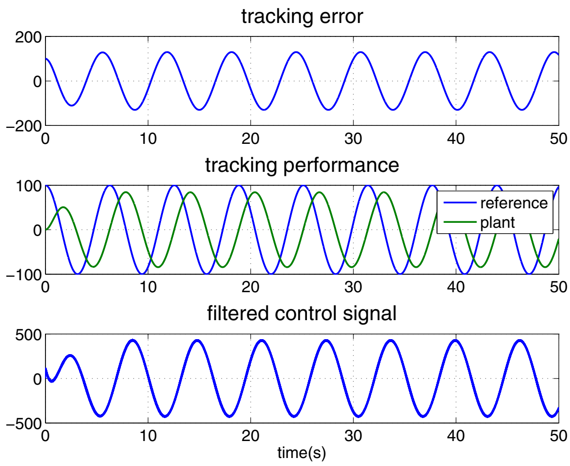

Fig. 8. L1-AC: Example 2 with $r(t) = 100cos(t)$

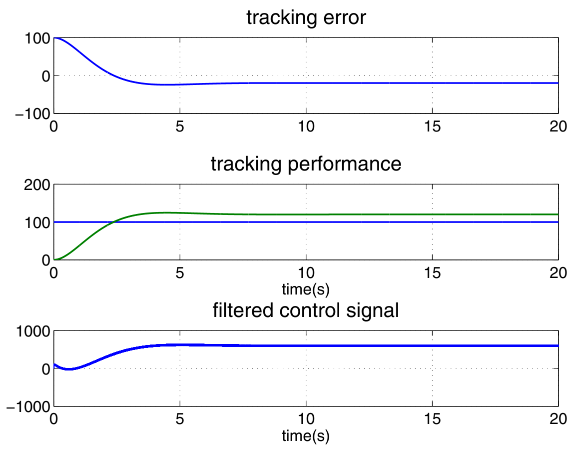

Fig. 9. L1-AC step response for unknown input gain in Example 2

of the eBMRAC would be able to deal with systems of higher relative degree, however for the sake of simplicity and for a more intuitive illustration of this comparison, it is chosen to deal with output feedback of a system with relative degree $n^* = 1$.

The averaging filter is designed with $\tau = 0.02$, and I/O state variable filters are $\Lambda(s) = 1/(s+1)$. Projection adaptation parameters are $\gamma = 10$ and $M_\theta = 10$. Results show good tracking performance with residual error, as seen in Figs. 10, and a similar result when the frequency is increased, showing in Fig. 11. Zero tracking error is still obtained if k_p is unknown. The result when $k^{nom} = 1.2$ is shown in Fig. 12 Note that an excessively large gain is not needed and that the error can be made smaller choosing a smaller τ. It is important to stress that the eBMRAC inherits the robustness of the SSC to unmodeled dynamics such as delays and nonminimum phase dynamics.

8. CONCLUSION

This paper shows that the combination of two early control strategies, the Smooth Sliding Control and the Binary-MRAC, provides an adaptive controller that overcomes fundamental limitations of the L1-AC, such as poor tracking performance to time varying reference and the use of

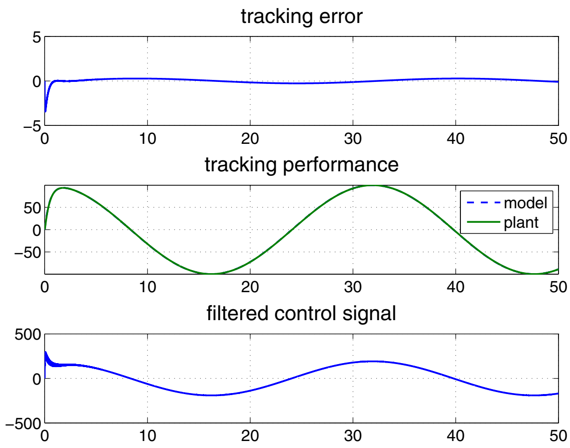

Fig. 10. eBMRAC: Example 2 with $r(t) = 100cos(0.2t)$

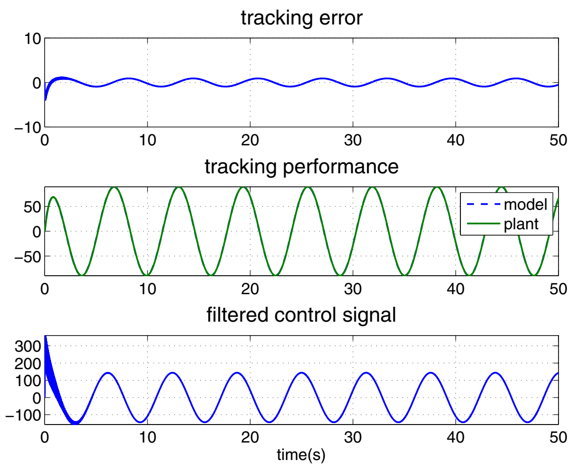

Fig. 11. eBMRAC: Example 2 with $r(t) = 100cos(t)$

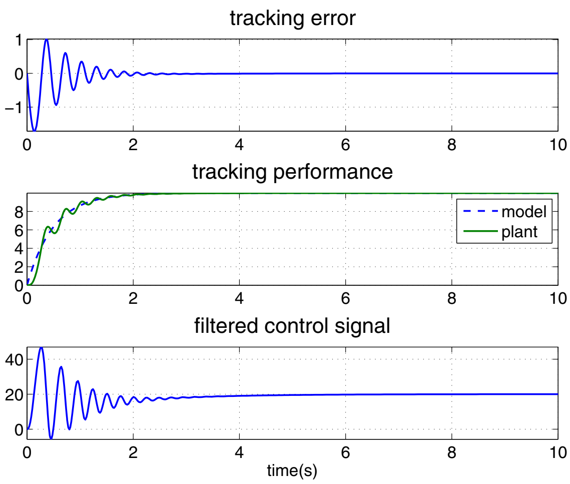

Fig. 12. eBMRAC step response for unknown K_p in Example 2

excessively large gain. Also, the new controller is simpler and its architecture does not need to be structurally re-designed in order to be applied to problems of different complexity, in contrast to the L1-AC.

9. PROOF OF THEOREM 1

Proof: Property (i) is derived by considering the Lyapunov candidate: $2V_\theta = \theta^T\theta$. The derivative is:

$$\dot{V} = (\sigma_{eq} - \sigma)\,\|\theta\|^2 = (\sigma_{eq} - \sigma)V/2 \tag{32}$$

from (8) it follows that $(\sigma_{eq} - \sigma) \leq 0$ for $\|\theta\| \geq M_\theta$, such that $\|\theta\| \leq M_\theta$ is positively invariant and therefore $\tilde{\theta}^T\tilde{\theta}$ is uniformly bounded.

Property (ii) is obtained using through the following Lyapunov candidate: $V = x_\varepsilon^T P x_\varepsilon + \frac{1}{\gamma}\tilde{\theta}^T\tilde{\theta}$. The time derivative is

$$\dot{V} = -x_\varepsilon^T Q x_\varepsilon - \frac{2\sigma}{\gamma}\tilde{\theta}^T\theta \tag{33}$$

without the Since $\|\theta\|$ is uniformly bounded $V \leq x_\varepsilon^T P x_\varepsilon + \mathcal{O}(\gamma^{-1})$ From which is possible to establish that $\dot{V} \leq -\lambda_1\left[V - \mathcal{O}(\gamma^{-1})\right]$ where $\lambda_1 = \lambda_{min}(Q)\ \lambda_{max}(P)$, with $\lambda_{min}(Q)$ and $\lambda_{max}(P)$ being the minimum and maximum eigenvalues of Q and P, respectively. The proof of (ii) is completed using a comparison lemma.

Following the same arguments presented in [Ioannou and Sun, 1996] (pp. 205) one can conclude that $\varepsilon_0 \to 0$. Thus, since $\varepsilon_0 = k^*M(s)[u_0 - u^*]$, it is possible to establish that $u_0 \to u^*$. Consequently, referring to the tracking error: $e_0 = k^*M(s)\left[\frac{u^*}{\tau s + 1} - u^*\right]$. Which is equivalent to $e_0 = k^*M(s)\left[\frac{-\tau s}{\tau s + 1}\right]u^*$. Following similar steps presented in the proof of Theorem 2 in [Hsu, 1997], it can be shown that ω is bounded and hence u^* is also bounded. Since $M(s)$ is minimum-phase, it follows that $\left\|k^*M\frac{-\tau s}{\tau s + 1}\right\| \leq \tau K_1$. thus $\|e_0\| \leq \tau K + c_2 e^{-\lambda_2 t}$ for some positive constants c_2 and λ_2

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