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Opinion Dynamics of Modified Hegselmann-Krause Model with Group-based Bounded Confidence ⋆

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Abstract: The continuous opinion dynamics with group-based heterogeneous bounded confidences is considered in this paper. Firstly, a slightly modified Hegselmann-Krause model is proposed, and the agents are divided into open-minded-, moderate-minded-, and close-minded-subgroups according to the corresponding confidence intervals. Then numerical simulations are carried out to analyze the influence of the close-minded and open-minded agents, as well as the population size, on the opinion dynamics. It is observed that (1) for the fixed population size, the larger proportion of close-minded agents, the more opinion clusters; (2) open-minded agents cannot contribute to forging different opinions, instead, the existence of them maybe diversify final opinions; also interestingly the relative size of the largest cluster varies along concave-parabola-like curve as the proportion of open-minded agents increases; (3)for the same proportion of the three subgroups, as population size increases, the number of final opinion clusters will increase at the beginning and then reach a stable level, which is quite different from the previous studies.

Keywords: opinion dynamics, modified Hegselmann-Krause model, heterogeneous, group-based bounded confidence

1. INTRODUCTION

Does gene-modified food do harm to our health and will you eat such food? What kind of fashion style would you like to follow? When you go to the polls, which candidate are you going to support? All such things are determined by our own opinions. It can be even said that almost all social interactions are shaped by our beliefs and opinions (Acemoglu et al. (2011)). Thus it is of high value to study the opinion dynamics, and up to now many researchers with different background have proposed various models to analyze the evolution of the opinion dynamics from various aspects, see, for example Clifford et al. (1973); Holley et al. (1975); Sznajd-Weron et al. (2000); Hegselmann et al. (2002); Deffuant et al. (2000); Lorenz (2008); Martins et al. (2010); Altafini (2012); Ghaderi et al. (2013). One of the most commonly used models is Hegselmann-Krause (HK) model proposed by Hegselmann et al. (2002), which is agent-based and considers the continuous opinion dynamics under bounded confidence, i.e. agents only interact when they are close in opinion to each other, and where every agent synchronously updates their opinion by averaging all the opinions of their neighbours.

HK models can be classified into agent-based and density-based model (Lorenz (2008)). The stability and convergence of HK model have been studied in Lorenz (2005); Blondel et al. (2009); Nedic et al. (2012); Mirtabatabaei et al. (2011). Previous studies have also investigated the opinion dynamics based on HK model from various aspects. Fortunato (2005) studied the consensus threshold of the bounded confidence of homogeneous HK model and revels that the consensus threshold is general for all the network topology, but it can only take two possible values $\epsilon_c \sim 0.2$ *or* ~ 0.5 that depends on the average degree of the graph when the number of agents approaches infinity. Liang et al. (2013) considered the impact of both the bounded confidence and influence radius of agents on the opinion dynamics, and they found that heterogeneity did not always promote consensus and there is an optimal heterogeneity under which the relative size of the largest opinion cluster reaches its peak point. Kou et al. (2012) studied the opinion dynamics of HK model with multi-level confidences by dividing the agents into three subgroups: close-minded, moderate-minded and open-minded respectively based on social differentiation theory, which is similar to Weisbuch et al. (2002) and analyze the influence of the fractions of each subgroup as well as the population size on opinion formation. However, all the agents in the same subgroup share the same bounded confidence.

In this paper, we consider the continuous opinions dynamics in the space [0 1] in the population level. Firstly, we slightly modify HK model based on non-Bayesian learning rule (Jadbabaie et al. (2012)) so that every agent can take into account their own opinion independently with different weight rather than the same weight as their neighbors'. Inspired by the work Kou et al. (2012), we

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also divide the agents into three subgroups according to their confidence levels and investigate the impact of each subgroups on the opinion dynamics. But unlike the work by Kou et al. (2012), we assume the confidence levels of the individuals in the same subgroup is heterogeneous, i.e. uniformly distributed in the corresponding interval. Another important aspect of our work we want to stress here is that the influence of the population size on the number of final opinion clusters through numerous simulations is not the same as is stated in the previous study(Kou et al. (2012)).

The rest of this paper is organized as follows. Section 2 reviews HK model briefly and then proposes modified HK (MHK) model; main results are presented in section 3, devoted to the study of the opinion dynamics according to MHK with respect to different proportion of subgroups and population size by numerous simulations. Finally summaries and conclusions are given in section 4.

2. MODEL FORMULATION

We will review the original HK model first, and then propose our model by slight modification.

2.1 Introduction of Hegselmann-Krause (HK) *Model*

Consider a system of n agents, whose opinions are located in one-dimensional Euclidean space $\mathbb{R}$ and can be expressed by a real number. Denote the set of n agents as $V = \{1, 2, \ldots, n\}$ and for agent $i \in V$, its opinion at time t is represented by $x_i(t) \in \mathbb{R}$. Given the bounded confidence set $\epsilon = \{\epsilon_1, \epsilon_2, \cdots, \epsilon_n\}$, each agent i only interacts with his neighbors whose opinions differ from his own not more than the certain confidence level ϵ_i, then the discrete-time HK model (Lorenz (2008)) can be described by Eq.(1):

$$x_i(t+1) = \frac{1}{|N_i(t)|} \sum_{j \in N_i(t)} (x_j(t) - x_i(t)) \quad (1)$$

where $N_i(t) = \{j \in V \mid |x_i(t) - x_j(t)| \le \epsilon_i\}$ is the neighbor set of agent i at time t and $|N_i(t)|$ is the cardinality of $N_i(t)$. When $\epsilon_1 = \epsilon_2 = \cdots = \epsilon_n$, the HK model is homogeneous, otherwise, it is heterogeneous.

Obviously, Eq.(1) inherently indicates that agent i is always his own neighbor and updates his opinion by simply averaging all the opinions of his neighbors with the same weights $\frac{1}{|N_i(t)|}$.

2.2 Modified HK (MHK) *Model*

As mentioned above, the agent i considers his own opinion and his neighbors' equally, and this may be not very practical in the real world. We thus claim it would be more reasonable to consider his own opinion independently when agent i updates his opinion. To this end, we redefine agent i's neighbor set as $\bar{N}_i(t) = \{j \in V \cap j \ne i \mid |x_i(t) - x_j(t)| \le \epsilon_i\}$ and based on non-Bayesian updating rule, we propose MHK model as Eq.(2):

$$\begin{aligned} &\textit{if } \bar{N}_i(t) \ne \emptyset \\ &x_i(t+1) = \alpha_i x_i(t) + (1-\alpha_i) \frac{1}{|\bar{N}_i(t)|} \sum_{j \in \bar{N}_i(t)} x_j(t) \\ &\textit{otherwise} \\ &x_i(t+1) = x_i(t) \end{aligned} \quad (2)$$

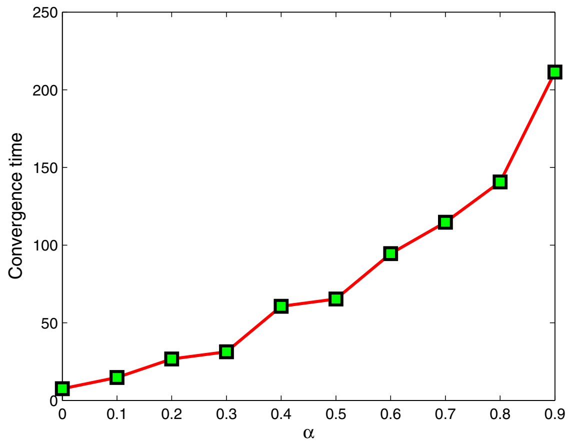

Fig. 1. Convergence time of MHK (2) with with different α. The result refers to $n = 100$ and $\epsilon = 0.5$ and is an average over 50 realizations with the same initial opinion.

where $\alpha_i \in [0\ 1]$ is the weight that agent i assigns to his own opinion, which we can refer to as the measure of self-belief and here without loss of generality, we assume $\alpha_1 = \alpha_2 = \cdots = \alpha_n = \alpha$. We can also refer α to convergence parameter, and usually the convergence time of MHK with large α is much longer than that with small α. As is shown in Fig.1, where we consider the number of agents $n = 100$ and assume the homogeneous confidence level $\epsilon = 0.5$, and which is the result by averaging over 50 realizations with the same computer configuration and initial opinion profile, it can be observed that the larger α, the longer convergence time. When the bounded confidences are heterogeneous, it displays qualitatively similar trend, thus we omit its result here.

Obviously when $\alpha_i = \frac{1}{|\bar{N}_i(t)|+1}$, MHK model is equal to HK model. Furthermore, when $\alpha_i = 1$, it means agent i will never consider other people's opinion and we can treat it as stubborn agent. All throughout the paper, we will choose $\alpha = 0.5$.

Fig.2 shows the time evolution of 100 agents' opinion dynamics according to MHK (2), where the initial opinion is uniformly distributed in the opinion space, and confidence levels ϵ_i are uniformly distributed in the intervals [0.01 0.05], [0.2 0.3], [0.4 0.9], respectively. Since the initial opinion can also influence the dynamics of the final opinion (see Lorenz (2008)), we also realize the result under different initial profiles. In Fig.2, the left three (a), (b) and (c) have the same initial opinion, and so do the right three (d),(e) and (f). It can observed from Fig.2 that opinion consensus, polarization and fragmentation emerges respectively under different confidence levels. Moreover, under different initial profiles, some of the results would remain similar while some would be quite different from each other, for example, Fig.2(b) and Fig.2(e). The reason lies in both initial opinion profile and confidence level. Since the initial opinion is generated randomly, it will always occur that the difference of the opinions among several agents happens randomly to be very small or very large. If confidence level of these agents is large enough, even though the difference of the initial opinion among several

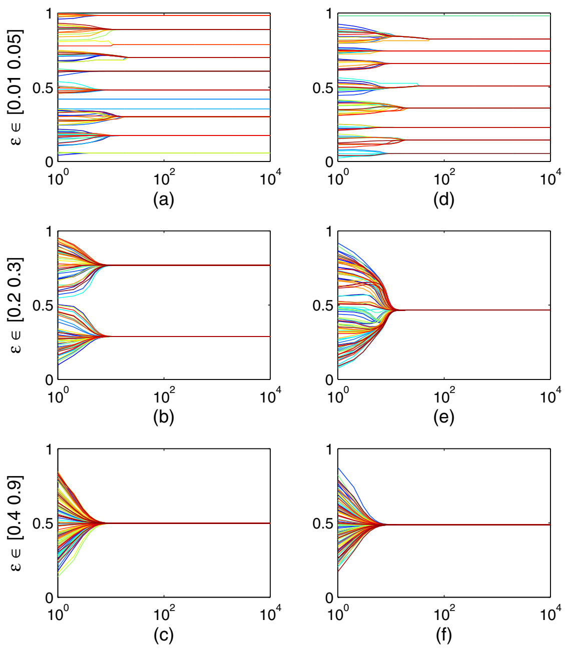

Fig. 2. Time evolution of 100 agent opinions according to MHK model Eq.(2) with different bounded confidence. ϵ_i are uniformly distributed in the corresponding interval. (a), (b), (c) have the same initial opinion, while (d),(e),(f) have another same initial opinion.

agents is large, they are more likely to exchange their opinions in next time step; on the contrary, if confidence level is very small, no matter how similar the initial opinions among several agents are, the probability that they will consider each other's opinion is still very small. Thus in the case when the confidence level $\epsilon_i \in [0.01\ 0.05]$ or $[0.4\ 0.9]$, the results remain the same; But when the confidence level is neither small nor large enough, as is the case when $\epsilon_i \in [0.2\ 0.3]$, the result would be influenced by the initial opinion profile to a large extent.

3. SIMULATION ANALYSIS AND RESULTS

In the real world, there are always some people who are much more willing to accept the new things or much more easily influenced by others, while some would be less likely to be influenced by his friends or other contacts. Thus similar to Kou et al. (2012), we divide the agents into open-minded-, moderate-minded-, and close-minded-subgroups respectively depending on the different confidence levels, but unlike the work by Kou et al. (2012), where the agents in the same sub-group share the same confidence level, we assume the bounded confidence ϵ_i of each subgroup is heterogeneous.

By considering the result of Fortunato (2005), which shows that the consensus threshold for homogeneous HK model is about $\epsilon_c \sim 0.2$ for the complete graph, and the confidence level that was chosen by Kou et al. (2012), as well as the result of time evolution of MHK(2) shown in Fig.2, we suppose the confidences of close-minded agents uniformly distributed in the interval [0.01 0.05], moderate-minded

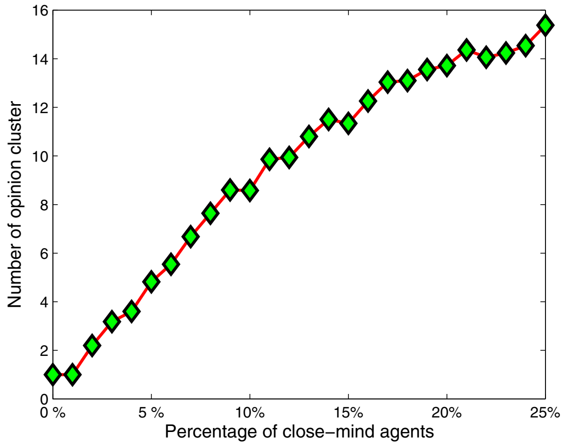

Fig. 3. The number of final opinion clusters with different proportion of close-minded agents p_c. The result refers to 100 agents, $p_o = 20\%$ and $p_m = 1 - p_c - p_o$, and is an average over 50 realizations which have the same initial opinion profiles

agents in [0.2 0.3] and open-minded agents in [0.4 0.9], respectively.

In what follows, we will investigate how the proportion of close-minded and open-minded agents, as well as the population size, can influence the opinion dynamics according to MHK (2). We denote the proportion of close-minded-, moderate-minded- and open-minded-agent as p_c, p_m, p_o, respectively. Obviously, it always holds that $p_c + p_m + p_o = 1$. Assume that the agents for each subgroup are chosen randomly; the bounded confidence ϵ_i follows uniform distribution in the corresponding intervals; initial opinions are generated from the opinion space [0 1] with uniform distribution and the simulations are carried out in the complete network. All of the following results are an average over 50 realizations.

3.1 Proportion of close-minded agents

We consider a population of $n = 100$ agents and the proportion of close-minded agents p_c varies from 0 to 30%. The relation between p_c and the number of final opinion clusters when the proportion of open-minded agents $p_o = 20\%$ is shown in Fig.3, from which we can observe that the number of final opinion clusters is nearly proportional to the percentage of close-minded agents p_c under the same population size. This is easy to understand. Since the close-minded agents are less likely to consider other agents' opinions, more opinion will be kept unchanged with more close-minded agents, as a result, the number of the final opinion clusters becomes larger.

3.2 Proportion of open-minded agents

Will a group with more open-minded agents result in less different opinions, or make an agreement more quickly? We will investigate such problem in this subsection. We assume the proportion of close-minded agents p_c is fixed and p_o varying from 0, i.e. only close-minded and moderate-minded agents are considered, to $1 - p_c$, i.e. only close-minded and open-minded agents are considered. We have

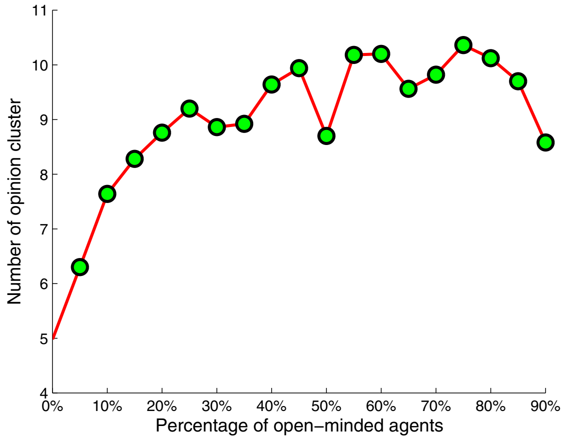

Fig. 4. The number of final opinion clusters with different proportion of open-minded agents p_o. The result refers to $n = 100, p_c = 10\%$ and $p_m = 1 - p_c - p_o$ and is an average over 50 realizations.

done the simulations for $p_c = 5\%, 10\%, 20\%$ respectively. In order to eliminate the influence of initial opinion, the same initial profile as previous subsection is used. The three cases have the qualitatively similar results, thus only the result of $p_c = 10\%$ is presented here.

Fig.4 shows the number of final opinion clusters with different proportion of open-minded agents p_o. It can be seen that on one hand, when $p_o = 0$, the number of final opinion clusters is much smaller compared to that when $p_o > 0$; on the other hand, as p_o increases from certain level, which is about 15% in this case, to 90%, the number of final opinion clusters remains almost stable. Thus it can be conjectured that open-minded agent cannot contribute to forging different opinions and the existence of open-mined agent may even diversify the final opinions.

The relative size of the largest cluster with different p_o are shown in Fig.5. The relative size is the ratio between the number of agents in the largest final opinion cluster and the whole population size. It is interesting to find that the relative size of the largest cluster varies along concave-parabola-like curve as p_o increases. There is a minimum point of the relative size, which occurs at about $p_m = p_o$.

Fig.6 demonstrates the convergence time as a function of the proportion of open-minded agents p_o, where $p_c = 0.1$. It can be observed that the convergence time increases dramatically at the beginning and then decreases gradually, as p_o increases from 0 to 90%. Thus it can be drawn that open-minded agent may accelerate an subgroup agreement in the mixed population, where close-minded, open-minded, and moderate-minded agents coexist, however, compared to the population without open-minded agent, the existence of such agent will result in increasing the convergence time, especially when p_o is small.

3.3 Population Size

The effect of the population size on the opinion dynamics will be investigated in this subsection. We consider the population size n increasing from 100 to 2500, and assume

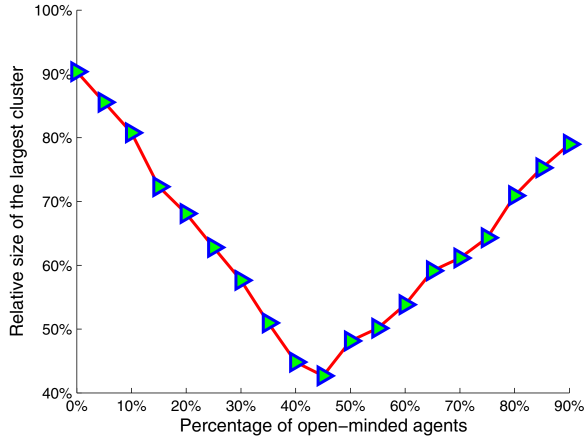

Fig. 5. Relative size of the largest cluster with different p_o. The result refers to $n = 100, p_c = 10\%$ and $p_m = 1 - p_c - p_o$ and is an average over 50 realizations

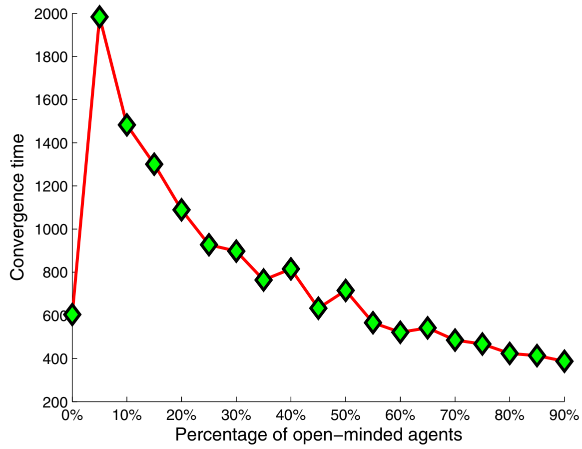

Fig. 6. Convergence time with different p_o. The result refers to $n = 100, p_c = 10\%$ and $p_m = 1 - p_c - p_o$ and is an average over 50 realizations.

the proportion of each subgroup as $p_c = 0.1, p_m = 0.7, p_o = 0.2$ for all the population size.

The average number of final opinion clusters with different population size is shown in Fig.7, from which it can be seen that the average number of final opinion clusters increases dramatically at the beginning when population size increases from 100 to about 500 and then it reaches the stable level even though n continues to increase. This result is quite different from that in the work (Kou et al. (2012)), which claimed that when p_c is fixed, the number of final opinion clusters is proportional to the total number of agents and can be approximated by a linear function.

4. CONCLUSIONS

In this paper, we investigate the continuous opinion dynamics in the population level with group-based heterogeneous confidences. First, a slightly modified HK model is proposed based on non-Bayesian rules, and then the agents are divided into three subgroups: open-minded-, moderate-minded-, and close-minded-subgroups, respec-

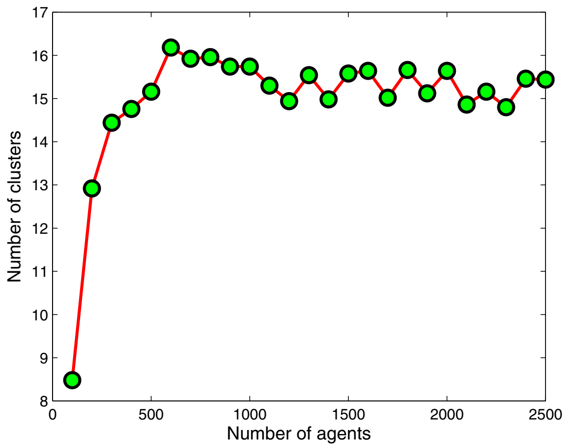

Fig. 7. The number of final opinion clusters with different population size. The result refers to $p_c = 0.1, p_o = 0.2, p_m = 0.7$ and is an average over 50 realizations.

tively, according to their corresponding confidence levels. It is assumed that the confidence level of the agents in the same subgroup is heterogeneous and follows uniform distribution in the corresponding intervals, which is different from Kou et al. (2012) and Weisbuch et al. (2002), where agents in the same subgroup have the bounded confidence level. Numerical simulations are given to investigate the influence of open-minded agents, close-minded agents, and the population size on the opinion dynamics. The results can be summarised as follows:
(1) Intuitively, the number of final opinion clusters is dominated by the proportion of the close-minded agents, and for the fixed population size, the larger p_c, the more final opinion clusters.
(2) Open-minded agent cannot contribute to forging the different opinions, instead, the existence of open-minded agents will diversify the final opinions; it is also interesting to find that the relative size of the largest cluster varies along concave-parabola-like curve, as p_o increases. In addition, open-minded agent may promote an subgroup agreement in the mixed population, however, when p_o is too small, the existence of such agent will increase the convergence time, compared to the population without open-minded agent.
(3) For the fixed proportion p_c, p_o, p_m, as the population size n increases, the average number of final opinion clusters increases at the beginning and then reaches the stable level, which is quite different from that in the work (Kou et al. (2012)).

However, in this paper, we only carry out the simulations on the complete network, and the influence of network topology is still unveiled here. We assume the initial opinion profiles and the heterogenous bounded confidences follow uniform distribution in the corresponding intervals, thus we can analyze the influence by the initial opinion as well as the confidences with different distribution.Another aspect we would like to point out is that the confidence intervals for each subgroup are chosen tentatively and the reasons behind some of the results are not disclosed as well.

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