Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

# Guidance Law Design via Variable Structure Control with Finite Time Sliding Sector

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**Abstract:** A new guidance law utilizing variable structure control with finite time sliding sector is proposed. First, a finite time sliding sector is defined. The finite time sliding sector is a subset of state space in which the Lyapunov function candidate satisfies the finite time stability condition, in contrast to the commonly used notion of asymptotic stability in conventional sliding sector. Then, based on the finite time sliding sector, a sliding sector control law is designed to move the system state in to the sector in finite time. The target acceleration is considered as an uncertainty. The proposed sliding sector guidance law is derived by supposing the target acceleration upper bound can be estimated as a priori. Simulation results show that the new guidance law is highly effective.

*Keywords:* Guidance law, Variable structure control, Lyapunov function, Finite time stability, Sliding sector.

## 1. INTRODUCTION

A large number of design methods have been applied to missile guidance problems, ranging from proportional navigation (PN) to robust control algorithms. The PN guidance law has been widely used due to its advantages such as simple form and easy implementation (Guelman, 1971; Zarchan, 2012). If the target does not maneuver, the PN guidance law can achieve high precision. When the targets acceleration information can be obtained, the augmented proportional navigation (APN) and the predictive guidance law (PGL) are proposed to intercept maneuvering targets (Ha et al., 1990; Talole & Ravi, 1998). However, the target acceleration is hard to be estimated precisely in practical applications. Therefore, some robust control algorithms have been applied to guidance problems such as the $H_\infty$ guidance law (Yang & Chen, 1998; Chen et al., 2002; Shieh, 2007), the $L_2$ gain guidance law (Zhou et al., 2001), the Lyapunov nonlinear guidance law (Lechevin & Rabbath, 2004) and the variable structure guidance law (Zhou et al., 1999; Moon et al., 2001; Zhou et al., 2009; Babu et al., 1994; Zhou et al., 2013).

The variable structure control is well known for its robustness properties, but it is suffering the chatting phenomena. To deal with the chatting phenomena existing in a VSC system, a sliding sector (Furuta & Pan, 2000) for a linear time invariant (LTI) system has been proposed instead of sliding mode. The sector is designed by the algebraic Riccati equation (ARE). For the nonlinear time varying (NTV) system with a matched uncertainty, the forward integration of state dependent differential Riccati equation (SDDRE) is used to design an NTV sliding sector (Pan et al., 2009). It has been show that the Lyapunov function decreases with a VSC law inside the sector. And outside the sector, a sliding sector control (SSC) law is designed to move the system state into the sector in finite time. But it can't ensure the finite time stability while inside the sector.

In recent years, the finite time stability (Hong, 2002; Huang et al., 2002) for feedback control systems has gained increased attention. It was demonstrated that finite time stable systems might enjoy not only faster convergence but also better robustness and disturbance rejection properties (Huang et al., 2002; Bhat & Bernstein, 1997; Bhat & Bernstein, 1998; Bhat & Bernstein, 2000). In this paper, the relative motion equations between the missile and the target are represented in a state dependent linear time variant (SDLTV) form. The finite time sliding sector is designed in which the Lyapunov function candidate satisfies the finite time stability condition. A SSC guidance law using the finite time sliding sector is proposed based on the solution of SDDRE.

The paper is organized as follows. In Sec. 2, the missile-target engagement problem is formulated. In Sec. 3, the finite time sliding sector is defined and the new guidance law is proposed utilizing the VSC with finite time sliding sector. Numerical simulation results are shown in Sec. 4, and conclusions are reported in Sec. 5.

## 2. PROBLEM FORMULATION

Considering the spherical line of sight (LOS) coordinates $(r, \theta, \phi)$ with origin fixed at the missile's gravity center. Let $e_r$, $e_\theta$ and $e_\phi$ be the unit vectors along the coordinate axes. Fig.1 is the three-dimensional pursuit-evasion geometry between the missile and the target.

 

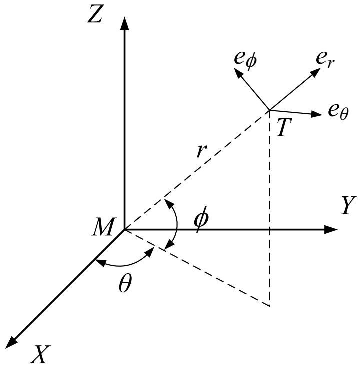


Fig.1. Three-dimensional interception geometry.

In Fig.1, the missile $M$ is attempting to intercept a target $T$. In the guidance process, the missile and the target are assumed as two point masses. By virtue of the principles of kinematics, the relative motion can be expressed by the following set of second-order nonlinear differential equations (Chen et al., 2002; Shieh, 2007):

$$\ddot{r} - r\dot{\phi}^2 - r\dot{\theta}^2\cos^2\phi = a_{Tr} - a_{Mr} \tag{1}$$

$$r\ddot{\theta}\cos\phi + 2\dot{r}\dot{\theta}\cos\phi - 2r\dot{\phi}\dot{\theta}\sin\phi = a_{T\theta} - a_{M\theta} \tag{2}$$

$$r\ddot{\phi} + 2\dot{r}\dot{\phi} + r\dot{\theta}^2\sin\phi\cos\phi = a_{T\phi} - a_{M\phi} \tag{3}$$

where $r$ is the relative distance between missile and target, $\phi$ and $\theta$ are LOS angles in elevation loop and the azimuth loop, respectively.

Let $V_r = \dot{r}$ , $V_\theta = r\dot{\theta}\cos\phi$ , $V_\phi = r\dot{\phi}$ , the relative velocity in the LOS coordinates can be expressed as

$$\boldsymbol{V}_{mt} = \begin{bmatrix} V_r & V_\theta & V_\phi \end{bmatrix}^{\mathrm{T}} \tag{4}$$

In the terminal guidance phase, the relative speed and distance satisfy the following condition:

$$V_r < 0,\ \ 0 < r < r(0) \tag{5}$$

The purpose of design a guidance law is to make sure that the tangential relative velocities $V_\theta$ and $V_\phi$ converge to zero. It means that the missile and target are in head-on condition. To design such a guidance law, (2)-(3) can be rewritten as

$$\dot{V}_\theta = -\frac{V_rV_\theta}{r} + \frac{V_\theta V_\phi\tan\phi}{r} - a_{M\theta} + a_{T\theta} \tag{6}$$

$$\dot{V}_\phi = -\frac{V_rV_\phi}{r} - \frac{V_\theta^2\tan\phi}{r} - a_{M\phi} + a_{T\phi} \tag{7}$$

The missile's acceleration $a_{M\theta}$ and $a_{M\phi}$ are chosen as a form of an extension PN guidance law, that is

$$a_{M\theta} = -\frac{NV_rV_\theta}{r} + u_{M\theta} \tag{8}$$

$$a_{M\phi} = -\frac{NV_rV_\phi}{r} + u_{M\phi} \tag{9}$$

where $N$ is a navigation constant, $N > 1$ . $u_{M\theta}$ and $u_{M\phi}$ will be designed in the sequel. Substituting (8)-(9) into (6)-(7), the nonlinear system can be represent in the SDLTV form

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(\boldsymbol{x},t)\boldsymbol{x} + \boldsymbol{B}(\boldsymbol{x},t)(\boldsymbol{u}+\boldsymbol{w}) \tag{10}$$

where,

$$\boldsymbol{x} = \begin{bmatrix} V_\theta \\ V_\phi \end{bmatrix},\ \boldsymbol{A}(\boldsymbol{x},t) = \begin{bmatrix} \frac{(N-1)V_r}{r} & \frac{V_\theta\tan\phi}{r} \\ -\frac{V_\theta\tan\phi}{r} & \frac{(N-1)V_r}{r} \end{bmatrix},$$

$$\boldsymbol{B}(\boldsymbol{x},t) = -\boldsymbol{I}\ ,\ \boldsymbol{u} = \begin{bmatrix} u_{M\theta} \\ u_{M\phi} \end{bmatrix},\ \boldsymbol{w} = \begin{bmatrix} -a_{T\theta} \\ -a_{T\phi} \end{bmatrix}$$

In practical applications, the target acceleration $\boldsymbol{w}$ is unknown and is usually difficult to estimate, but its upper bound can be estimated as a priori. Suppose the target acceleration is bounded as

$$\|\boldsymbol{w}\|_\infty \le f \tag{11}$$

with a positive constant $f$ , where $\|\boldsymbol{w}\|_\infty$ denotes infinity norm of $\boldsymbol{w}$ .

## 3. GUIDANCE LAW DESIGN

### *3.1 Finite time sliding sector*

An LTI sliding sector (Furuta & Pan, 2000) and an NTV sliding sector (Pan et al., 2009) are defined for the LTI system and the SDLTV system respectively. Considering the parameter uncertainties or external disturbances, a VSC law is implemented to ensure the decrease of the Lyapunov function candidate inside the sliding sector (Pan et al., 2009). In recent years, finite time stability of nonlinear systems has gained increased attention. The finite time stability theory for time-invariant nonlinear systems (Hong, 2002) is extended to time-varying nonlinear systems (Zhou, 2009) in the following lemma.

*Lemma 1* (Zhou, 2009): Consider the nonlinear system described as

$$\dot{x} = f(x,t),\ f(0,t) = 0\ ,\ x \in R^n$$

Suppose that there is a continuously differentiable function $V(x,t)$ defined in the neighborhood $\hat{U} \subset R^n$ of the origin, and that there are real numbers $\alpha > 0$ and $0 < \lambda < 1$ , such that $V(x,t)$ is positive definite on $\hat{U}$ and that $\dot{V}(x,t) + \alpha V^\lambda(x,t) \le 0$ on $\hat{U}$ . Then, the zero solution of the nonlinear system is finite time stable. The settling time, depending on initial state $x_0$ , is given by

$$T_r \le \frac{V^{1-\lambda}(x_0,0)}{\alpha(1-\lambda)}.$$

In this paper, based on the above lemma, a finite time sliding sector is defined as follows.

*Definition 1*: A finite time sliding sector for the SDLTV system (10) is defined as

$$\mathrm{S}(\boldsymbol{x},t) = \left\{\boldsymbol{x}\,\middle|\,\|\boldsymbol{\sigma}(\boldsymbol{x},t)\| - \|\boldsymbol{\delta}(\boldsymbol{x},t)\| \le 0, t \in R^+\right\} \tag{12}$$

inside which a Lyapunov function candidate satisfies the finite time stability condition

(1) $V(\boldsymbol{x},t)$ is positive define;

(2) $\dot{V}(\boldsymbol{x},t) + cV^{\alpha}(\boldsymbol{x},t) \le 0$ .
where the real numbers $c > 0$ and $0 < \alpha < 1$ .

In the case of the conventional sliding sector, it requires only $V(\boldsymbol{x},t)$ be positive definite and $\dot{V}(\boldsymbol{x},t)$ be negative definite for all $\boldsymbol{x}(t) \in \mathrm{S}(\boldsymbol{x},t)$ . On contrary, the finite time sliding sector above requires a much stronger condition. The control input $\boldsymbol{u}$ and the sliding sector (12) can be determined by the following SDDRE

$$-\dot{\boldsymbol{P}}(\boldsymbol{x},t) = \boldsymbol{A}^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{P}(\boldsymbol{x},t) + \boldsymbol{P}(\boldsymbol{x},t)\boldsymbol{A}(\boldsymbol{x},t) + \boldsymbol{Q}(\boldsymbol{x},t) - \boldsymbol{P}(\boldsymbol{x},t)\boldsymbol{B}(\boldsymbol{x},t)\boldsymbol{B}(\boldsymbol{x},t)\boldsymbol{P}(\boldsymbol{x},t) \tag{13}$$

Since (10) is an NTV system, the closed form solution $\boldsymbol{P}(\boldsymbol{x},t)$ cannot be obtained easily. Here, the matrix $\boldsymbol{Q}(\boldsymbol{x},t)$ is chosen as a special form

$$\boldsymbol{Q} = \frac{\left((1+br)r - 2(N-1)V_r(1+ar)\right)(1+br) + (a-b)rV_r}{r(1+ar)^2}\boldsymbol{I} \tag{14}$$

where the real numbers $0 \le a \le b$ . Since the navigation constant $N > 1$ , $0 < r < r(0)$ and $V_r < 0$ , it follows that $\boldsymbol{Q}(\boldsymbol{x},t) > 0$ . Then, fortunately, $\boldsymbol{P}(\boldsymbol{x},t)$ can be solved as

$$\boldsymbol{P}(\boldsymbol{x},t) = \frac{1+br}{1+ar}\boldsymbol{I} \tag{15}$$

It can be adjusted off-line to achieve desired performance, and the time-consuming online computation is avoid when solving the SDDRE (13). Thus, $\boldsymbol{\sigma}(\boldsymbol{x},t)$ and $\boldsymbol{\delta}(\boldsymbol{x},t)$ are obtained as

$$\boldsymbol{\sigma}(\boldsymbol{x},t) = \boldsymbol{S}(\boldsymbol{x},t)\boldsymbol{x} \tag{16}$$

$$\left\|\boldsymbol{\delta}(\boldsymbol{x},t)\right\| = \sqrt{\boldsymbol{x}^{\mathrm{T}}\Delta(\boldsymbol{x},t)\boldsymbol{x}} \tag{17}$$

where $\boldsymbol{S}(\boldsymbol{x},t) = \boldsymbol{B}^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{P}(\boldsymbol{x},t)$ , $\Delta(\boldsymbol{x},t) = \boldsymbol{Q}(\boldsymbol{x},t) - \boldsymbol{R}(\boldsymbol{x},t) \ge 0$ and $\boldsymbol{R}(\boldsymbol{x},t) \ge 0$ .The VSC law used inside the sliding sector is designed as

$$\boldsymbol{u} = -\left(k_1 + k_2\left\|\boldsymbol{\sigma}(\boldsymbol{x},t)\right\|^{2\alpha-1}\right)\operatorname{sgn}\left(\boldsymbol{\sigma}(\boldsymbol{x},t)\right) \tag{18}$$

The parameter $k_1$ and $\alpha$ satisfy

$$0 < \alpha = const. < 1 \tag{19}$$

$$k_1 \ge f \tag{20}$$

$$k_2 \ge \frac{c\lambda_{\max}\left(\boldsymbol{P}(\boldsymbol{x},t)\right)}{2\lambda_{\min}\left(\boldsymbol{S}(\boldsymbol{x},t)^{\mathrm{T}}\boldsymbol{S}(\boldsymbol{x},t)\right)}, \ c = const. > 0 \tag{21}$$

where $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum singular value of matrix respectively.

For clarity, from now on, we omit the arguments $\boldsymbol{x}$ and $t$ to simplify the notation. For all $\boldsymbol{x} \in \mathrm{S}$ , the derivative of the Lyapunov function $V = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}$ along the trajectory of system (10) satisfies

$$\begin{aligned}\dot{V} &= \boldsymbol{x}^{\mathrm{T}}\left(\dot{\boldsymbol{P}} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}\right)\boldsymbol{x} + 2\boldsymbol{\sigma}^{\mathrm{T}}(\boldsymbol{u}+\boldsymbol{w}) \\ &= \boldsymbol{x}^{\mathrm{T}}\left(\boldsymbol{P}\boldsymbol{B}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P} - \boldsymbol{Q}\right)\boldsymbol{x} + 2\boldsymbol{\sigma}^{\mathrm{T}}(\boldsymbol{u}+\boldsymbol{w}) \\ &= \boldsymbol{\sigma}^2 - \boldsymbol{\delta}^2 - \boldsymbol{x}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{x} + 2\boldsymbol{\sigma}^{\mathrm{T}}(\boldsymbol{u}+\boldsymbol{w}) \\ &\le 2\boldsymbol{\sigma}^{\mathrm{T}}(\boldsymbol{u}+\boldsymbol{w})\end{aligned} \tag{22}$$

Substituting (18) into (22), we have

$$\begin{aligned}\dot{V} &\le 2\boldsymbol{\sigma}^{\mathrm{T}}\left(-k_1\operatorname{sgn}(\boldsymbol{\sigma}) - k_2\left\|\boldsymbol{\sigma}\right\|^{2\alpha-1}\operatorname{sgn}(\boldsymbol{\sigma}) + \boldsymbol{w}\right) \\ &\le -2k_2\left\|\boldsymbol{\sigma}\right\|^{2\alpha}\end{aligned} \tag{23}$$

where we used the inequality $\boldsymbol{\sigma}\operatorname{sgn}(\boldsymbol{\sigma}) \ge \left\|\boldsymbol{\sigma}\right\|$ .

According to the property of singular value, we have

$$\left\|\boldsymbol{\sigma}\right\|^2 \ge \lambda_{\min}\left(\boldsymbol{S}^{\mathrm{T}}\boldsymbol{S}\right)\left\|\boldsymbol{x}\right\|^2 \tag{24}$$

$$V \le \lambda_{\max}(\boldsymbol{P})\left\|\boldsymbol{x}\right\|^2 \tag{25}$$

Combining (24) and (25) produces

$$\left\|\boldsymbol{\sigma}\right\|^2 \ge \frac{\lambda_{\min}\left(\boldsymbol{S}^{\mathrm{T}}\boldsymbol{S}\right)}{\lambda_{\max}(\boldsymbol{P})}V \tag{26}$$

Substitute (26) into (23) yields

$$\begin{aligned}\dot{V}(\boldsymbol{x},t) &\le -2k_2\frac{\lambda_{\min}\left(\boldsymbol{S}^{\mathrm{T}}\boldsymbol{S}\right)}{\lambda_{\max}(\boldsymbol{P})}V^{\alpha} \\ &\le -cV^{\alpha}(\boldsymbol{x},t)\end{aligned} \tag{27}$$

Therefore, the sector defined by (12) is a finite time sliding sector.

*Remark 1*. According to Lemma 1, if the state keeps inside the sector, the settling time, depending on the initial state is given by

$$T_{r1} \le \frac{V^{1-\alpha}(\boldsymbol{x}_0, 0)}{c(1-\alpha)}$$

It is shown that the convergent time relates to the parameters $c$ and $\alpha$ , so we can control the convergent rate by adjusting these parameters.

### *3.2 Guidance law with finite time sliding sector*

Based on the finite time sliding sector just given, the guidance law with finite time sliding sector is designed via finite stability theory in the following theorem.

*Theorem 1*. For the guidance system (10), if the guidance law is designed as an extension of traditional PN guidance law

$$\boldsymbol{a} = \begin{cases} -\dfrac{NV_r}{r}\boldsymbol{x} - \left(k_1 + k_2\left\|\boldsymbol{\sigma}\right\|^{2\alpha-1}\right)\operatorname{sgn}(\boldsymbol{\sigma}) \ \forall \boldsymbol{x} \in \mathrm{S} \\ -\dfrac{NV_r}{r}\boldsymbol{x} - \boldsymbol{h}^{-1}\left(\boldsymbol{g} + \left(k_3\boldsymbol{h} + k_4\left\|\boldsymbol{\sigma}\right\|^{2\beta-1}\right)\operatorname{sgn}(\boldsymbol{\sigma})\right) \ \forall \boldsymbol{x} \notin \mathrm{S} \end{cases} \tag{28}$$

where $\boldsymbol{a} = \left[a_{M\theta} \quad a_{M\phi}\right]^{\mathrm{T}}$ , $\boldsymbol{g} = \left(\boldsymbol{B}^{\mathrm{T}}\dot{\boldsymbol{P}} + \boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A}\right)\boldsymbol{x}$ , $\boldsymbol{h} = \boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{B}$ , the parameter $0 < \alpha = const. < 1$ , $k_3 \ge f$ ,and $k_4 = const. > 0$ . Then, for all $\boldsymbol{x} \in \mathrm{S}$ , the system states (i.e., tangential relative velocities) satisfy finite time convergence, and for all $\boldsymbol{x} \notin \mathrm{S}$ , the tangential relative velocities move into the sliding sector $\mathrm{S}$ in finite time.

*Proof:*

Case 1. For all $\boldsymbol{x} \in \mathrm{S}$ , from (27), we obtain that the derivative Lyapunov function $V = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}$ along the trajectory of system

(10) satisfies $\dot{V} + cV^{\alpha} \le 0$, which implies satisfaction of the finite time stability condition.

Case 2. Outside the sliding sector, since $\boldsymbol{B} = -\boldsymbol{I}$ and $\boldsymbol{P}$ is symmetric positive definite, $\boldsymbol{h}$ is invertible and $\boldsymbol{h} > 0$. The control input $\boldsymbol{u}$ is designed as

$$\boldsymbol{u} = -\boldsymbol{h}^{-1}\left(\boldsymbol{g} + \left(k_3\boldsymbol{h} + k_4\|\boldsymbol{\sigma}\|^{2\beta-1}\right)\operatorname{sgn}(\boldsymbol{\sigma})\right)$$

The derivation of the function $\boldsymbol{\sigma}(\boldsymbol{x},t)$ is obtained as

$$\begin{aligned}\dot{\boldsymbol{\sigma}} &= \frac{d}{dt}(\boldsymbol{S}\boldsymbol{x}) = \frac{d}{dt}\left(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}\right)\\ &= \boldsymbol{B}^{\mathrm{T}}\dot{\boldsymbol{P}}\boldsymbol{x} + \boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\dot{\boldsymbol{x}}\\ &= \left(\boldsymbol{B}^{\mathrm{T}}\dot{\boldsymbol{P}} + \boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{A}\right)\boldsymbol{x} + \boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{B}(\boldsymbol{u}+\boldsymbol{w})\\ &= \boldsymbol{g} + \boldsymbol{h}(\boldsymbol{u}+\boldsymbol{w})\end{aligned} \tag{29}$$

Defining $V = \boldsymbol{\sigma}^{\mathrm{T}}(\boldsymbol{x},t)\boldsymbol{\sigma}(\boldsymbol{x},t)$, it follows that

$$\begin{aligned}\dot{V} &= 2\boldsymbol{\sigma}^{\mathrm{T}}\Big(\boldsymbol{g} + \boldsymbol{h}\big(-\boldsymbol{h}^{-1}\big(\boldsymbol{g} + k_3\boldsymbol{h}\operatorname{sgn}(\boldsymbol{\sigma})\\ &\qquad + k_4\|\boldsymbol{\sigma}\|^{2\beta-1}\operatorname{sgn}(\boldsymbol{\sigma})\big) + \boldsymbol{w}\big)\Big)\\ &\le -2k_4\|\boldsymbol{\sigma}\|^{2\beta-1}\boldsymbol{\sigma}^{\mathrm{T}}\operatorname{sgn}(\boldsymbol{\sigma})\\ &\le -2k_4V^{\beta}\end{aligned} \tag{30}$$

which implies the system state will move into the sliding sector in finite time. □

*Remark 2*. From (30), the settling time can also be obtained as

$$T_{r2} \le \frac{V^{1-\beta}(\boldsymbol{x}_0, 0)}{2k_4(1-\beta)}$$

*Remark 3*. In the guidance law (28), it does not need the time-consuming online computations to solve the SDDRE (13), such that the guidance law can be implemented easily in practical applications. The closed form solution can be obtained by choosing proper parameters $N$ and $\boldsymbol{Q}$. These parameters can be adjusted off-line to achieve desired performance.

## 4. SIMULATION RESULTS

In this section, a space interception problem is investigated. The interceptor's initial position coordinates are $x_{M0} = 0\,\mathrm{m}$, $y_{M0} = 0\,\mathrm{m}$ and $z_{M0} = 0\,\mathrm{m}$. Its initial velocity is $V_{M0} = 1600\,\mathrm{m/s}$ and its initial flight-path and heading angles are $\varphi_{M0} = 30\,\mathrm{deg}$ and $\psi_{M0} = 0\,\mathrm{deg}$, respectively. The target's initial position coordinates are $x_{T0} = 8.8\,\mathrm{km}$, $y_{T0} = 12\,\mathrm{km}$ and $z_{T0} = 5.6\,\mathrm{km}$. Its initial velocity is $V_{T0} = 900\,\mathrm{m/s}$ and its initial flight-path and heading angels are $\varphi_{T0} = -10\,\mathrm{deg}$ and $\psi_{T0} = 120\,\mathrm{deg}$, respectively. It is easy to calculate the initial relative distance between the interceptor and the target: $r = 15.9\,\mathrm{km}$, and the initial LOS angles: $\theta = 36.4\,\mathrm{deg}$ and $\phi = 20.6\,\mathrm{deg}$. The measurements of LOS angular rates given by the target seeker involve a stochastic noise. The target's maximum acceleration in the azimuth loop and the elevation loop both are 100 m/s$^2$. Thus $k_1$ and $k_3$ is set to be $k_1 = 100$ and $k_3 = 100$, respectively.

In the guidance law (28), it is required that the navigation constant $N > 1$. $N$ is the parameter of PN guidance law, we are quite familiar with it. The effective navigation constant is usually $3$ □ $5$. We just set it as $N = 4$ .The other parameters of the guidance law are chosen as $a = 1$, $b = 2$, $c = 1$, $k_4 = 1$, $\alpha = \beta = 0.1$, $\boldsymbol{R} = 0.35\boldsymbol{Q}$. , For comparison purposes, we also design a guidance law via NTV sliding sector control method. If we let $k_2 = k_4 = 0$, the NTV sliding sector guidance (NTVSSG) law is obtained. The finite time sliding sector guidance (FTSSG) law is compared with NTVSSG law and the adaptive sliding mode guidance (ASMG) law (Zhou et al., 1999).

Case 1. Intercept a non-maneuvering target. In the first case, we just consider use the guidance law to intercept non-maneuvering target which moves along a straight line with a constant velocity and assume that the missile's initial position and velocity are in the flight plane of the target.

**Table 1. Miss distance for case 1**

| Guidance law | Miss distance, m |
|---|---|
| ASMG | 0.312 |
| NTVSSG | 0.145 |
| FTSSG | 0.042 |

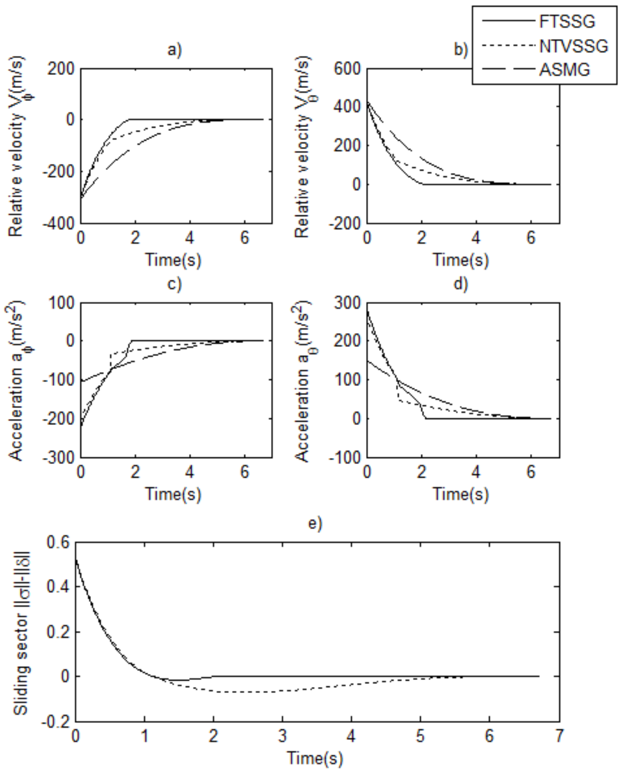


Fig. 2. Simulation results for case 1.

The tangential relative velocity and the missile acceleration in the elevation loop and the azimuth loop are plotted in Fig.

2a) and Fig. 2b), respectively. The tangential relative velocity and the missile acceleration in the azimuth loop are plotted in Fig. 2c) and Fig. 2d), respectively. Fig. 2e) is the evolution of $S(\boldsymbol{x},t)$. Table 1 summarizes the miss distances resulted.

Case 2. Intercept a maneuvering target. In the second case, the initial position and velocity of the target and the initial position and velocity of the missile are the same as those in case1. The target acceleration in the elevation loop and the azimuth loop are s $a_{T\phi}=-100$ m/s$^2$ and $a_{T\theta}=100$ m/s$^2$ , respectively.

The tangential relative velocity and the missile acceleration in the elevation loop and the azimuth loop are plotted in Fig. 3a) and Fig. 3b), respectively. The tangential relative velocity and the missile acceleration in the azimuth loop are plotted in Fig. 3c) and Fig. 3d), respectively. Fig. 3e) is the evolution of $S(\boldsymbol{x},t)$. Table 2 summarizes the miss distances resulted.

**Table 2. Miss distance for case 2**

| Guidance law | Miss distance, m |
|---|---|
| ASMG | 0.717 |
| NTVSSG | 0.303 |
| FTSSG | 0.075 |

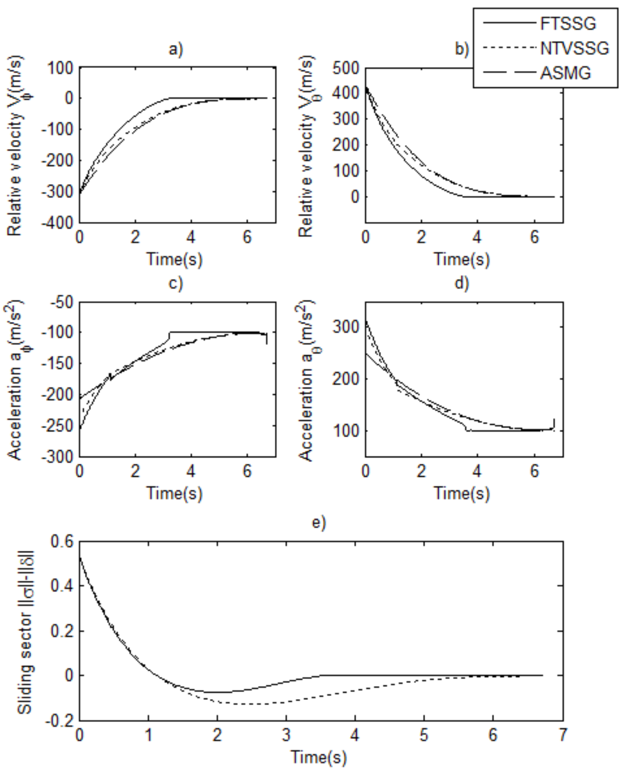


Fig. 2. Simulation results for case 2.

From Fig. 2-3, we can see that in three dimensional guidance dynamics, the ASMG and the NTVSSG law ensure that the tangential relative velocities converge to zero asymptotically. The missile translation acceleration converges to the target translation acceleration as the tangential relative velocities converge to zero. Both of the FTSSG and the NTVSSG send the system state into the sliding sector in about 1 s. When intercepting a non-maneuvering target, the tangential relative velocities converge to zero in 2 s under the FTSSG law in Fig. 2e). The time is 3.5s when intercept a maneuvering target in Fig. 3e). The tangential relative velocities converge faster in the finite time sliding sector than in the NTV sliding sector. As the tangential relative velocities converge to zero, the missile translation acceleration converges to the target acceleration. The missile acceleration under the FTSSG law ensures the tangential relative velocities enjoy fast convergence, resulting in a small miss distance.

## 5. CONCLUSION

In this paper, a new guidance law is proposed using variable structure control with finite time sliding sector. The sliding sector is designed base on the finite time stability theory. The relations between the finite time sliding sector and the NTV sliding sector are discussed. The guidance command is derived by regarding the target maneuvers as bounded disturbance inputs. Since the closed form solution of the SDDRE is obtained by choosing proper parameters, it does not need the time-consuming online computations. In practical applications, the guidance law can be implemented easily. The guidance law is able to provide a high guidance precision in intercepting a maneuvering target because it is robust against disturbances. For comparison purposes, we also designed the NTVSSG law. The proposed FTSSG law is compared with NTVSSG law and ASMG law. Numerical simulations show that the proposed guidance law offers better performance than other guidance laws.

## ACKNOWLEDGMENT

This work is supported in part by the Innovative Team Program of the National Natural Science Foundation of China (Grant No. 61021002), the National Natural Science Foundation of China (Grant No. 61174203) and the Program for New Century Excellent Talents in University (NCET-08-0153).

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