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Rotorcraft system identification: a time/frequency domain approach ⋆

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Abstract: The availability of accurate models for helicopter aeromechanics is becoming more and more important, as rotorcraft flight control systems have to meet progressively more stringent performance requirements: as the required control bandwidth increases, model accuracy becomes a vital part of the design problem. In this paper, the results of a joint industry-academia research project aimed at developing a novel time/frequency domain approach to rotorcraft model identification will be presented and discussed. The proposed approach will be illustrated by means of a case study based on data collected during piloted simulations of a state-of-the-art flight dynamics code.

1. INTRODUCTION

Model identification has been exploited extensively in the rotorcraft community (see for example the recent books Tischler and Remple [2006], Jategaonkar [2006] and the references therein), to overcome the difficulties associated with the accurate description of the most sensitive issues in helicopter dynamics, *i.e.*, the complexity of the interaction between dynamics and aerodynamics. In particular, from a control perspective accurate dynamic models for helicopter aeromechanics are becoming more and more important, as the requirements for rotorcraft control systems become more and more stringent. The identification of rotorcraft flight dynamics poses many challenges, which lead to specific requirements in the choice of a suitable method. First of all, it is customary to work with continuous-time models rather than with discrete-time ones, mainly because they are more intuitive, so that identification methods for continuous-time models are needed. In addition, rotorcraft systems are often open-loop unstable, so that identification experiments have to be carried out in closed-loop, either under pilot feedback or under automatic control. As a consequence, model identification methods capable of coping with this situation would be preferable. Furthermore, in view of both the open-loop instability and the complexity of some piloting tasks, data for identification are frequently collected in separate experiments in which each input channel is excited separately. The capability to handle easily such separate datasets in a single identification procedure is therefore desirable. The rotorcraft system identification literature appears to be characterised by a strong dichotomy opposing frequency-domain methods on one hand and time-domain methods on the other hand. The former assume that identification-oriented flight testing consists in the execution of (manual or automatic) frequency sweeps, through which reliable estimates of the frequency response functions associated with the flight dynamics of the helicopter can be reconstructed. The latter, on the other hand, rely on excitation inputs such as, *e.g.*, the 3211 sequence, developed at the German Aerospace Center DLR for flight dynamics testing (see Hamel and Kaletka [1997] for details), which excites a wide frequency band within a short time period.

Both approaches have advantages and disadvantages, but surprisingly enough no significant effort has been made so far to combine the two viewpoint to develop a unified approach allowing the refinement of parametric models using information coming from the two domains (see for example Panzani and Lovera [2012] for a preliminary discussion of joint time/frequency domain approaches to parameter estimation). In view of the above discussion, in this paper the results of a joint industry-academia research project aimed at developing a novel time/frequency domain approach to rotorcraft model identification will be presented and discussed. The proposed approach consists of a multi-step procedure exploiting the best features of a number of existing methods and tools to achieve identified models of improved accuracy.

The paper is organized as follows. A discussion of the state of the art in rotorcraft model identification is presented in Section 2. Then, the proposed approach to handle such requirements, based on the integration of time-domain and frequency-domain ideas to make the most, in terms of model quality, of the available methods for rotorcraft flight testing (see also Bergamasco and Lovera [2011a]) is illustrated in Section 3. Finally, results obtained in the analysis of data collected during piloted simulations are presented and discussed in Section 4, with specific emphasis on the achieved performance with respect to state of the art methods and tools from the rotorcraft community.

2. ROTORCRAFT SYSTEM IDENTIFICATION

In rotorcraft engineering system identification (both in time and frequency domain) has emerged as a useful complement to first principle modelling because physical models for rotorcraft dynamics include a number of uncertain parameters which are hard to determine analytically (see, *e.g.*, Tischler and Kaletka [1987]). Rotorcraft dynamics usually involves multiple inputs and multiple outputs (MIMO) models, indeed it is described by the interaction of inertial and aerodynamic forces as well as control forces acting on the rotor and the airframe. The interactions and interferences between these forces and their effects on the dynamic response of the helicopter change both with flight condition and configuration. Wind-tunnel data can provide only limited insight in the dynamics because of aerodynamic scale effects, model deficiencies and constrained free flight capabilities. Therefore, flight tests are necessary to overcome such limitations and reduce uncertainty in models of rotorcraft aeromechanics (see the discussion in Hamel and Kaletka [1997]).

Frequency-domain system identification in helicopter engineering has been developed during the last three decades, so there is a number of contributions in the literature describing the

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relevant approaches and case studies (see the recent book Tischler and Remple [2006]). As is common practice in frequency-domain methods, the procedure starts with the estimation of a nonparametric model for the frequency response function of the helicopter. Subsequently, a parametric frequency response curve is matched with the nonparametric model to compute estimates of the parameters of interest (stability and control derivatives, time delays). First applied to lightly coupled, low bandwidth articulated rotors (Tischler and Kaletka [1987]), this technique has been applied to more highly coupled, higher bandwidth rotor systems (Tischler and Cauffman [1992], Tischler and Tomashoski [2002], Lawler et al. [2006]).

Time-domain system identification is another approach to rotorcraft system identification, which has been intensely developed. In this technique the model, written in state-space form, is matched directly with the test data using least squares and/or maximum likelihood methods (see Hamel and Kaletka [1997], Jategaonkar [2006]) or, more recently, subspace model identification (SMI) methods (see Lovera [2003], Bergamasco and Lovera [2011b]).

3. TIME/FREQUENCY ROTORCRAFT IDENTIFICATION

As already mentioned in the Introduction, with the exception of the preliminary results presented in Bergamasco and Lovera [2013], no systematic method for joint time/frequency domain identification has been considered in the literature. In the following, the proposed approach will be presented.

3.1 Problem statement

Consider the linear, time-invariant continuous-time system

$$\mathscr{M}_s(\lambda): \begin{cases} \dot{x}(t) &= A(\lambda)x(t) + B(\lambda)u(t) + w(t),\; x(0) = x_0 \\ y(t) &= C(\lambda)x(t) + D(\lambda)u(t) + v(t) \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$ are, respectively, the state, input and output vectors and $w \in \mathbb{R}^n$ and $v \in \mathbb{R}^p$ are the process and the measurement noise, respectively, with covariance given by

$$E\left\{ \begin{bmatrix} w(t_1) \\ v(t_1) \end{bmatrix} \begin{bmatrix} w(t_2) \\ v(t_2) \end{bmatrix}^T \right\} = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \delta(t_2 - t_1).$$

The system matrices $A(\lambda)$, $B(\lambda)$, $C(\lambda)$, and $D(\lambda)$ are dependent on the constant parameter vector $\lambda \in \mathbb{R}^{n_\lambda}$ such that $(A(\lambda), C(\lambda))$ is observable and $(A(\lambda), [B(\lambda), Q^{1/2}])$ is controllable. In the following the model structure $\mathscr{M}_s(\lambda)$ is considered globally identifiable. Assume now that a dataset $\{u(t_i), y(t_i)\}$, $i \in [1,N]$ of sampled input/output data (possibly associated with a non equidistant sequence of sampling instants) obtained from system (1) is available, together with a set of samples of the frequency response function, $G(j\omega_k)$, $k \in [1,K]$. Then, the problem is to provide an estimate of the parameter λ on the basis of the available time and frequency data.

The problem can be faced using a multi-step approach: in the first step an unstructured black-box model $\mathscr{M}_{ns}$ of the form

$$\mathscr{M}_{ns}: \begin{cases} \dot{x}(t) &= \hat{A}x(t) + \hat{B}u(t) + \hat{K}e(t),\; x(0) = \hat{x}_0 \\ y(t) &= \hat{C}x(t) + \hat{D}u(t) + e(t) \end{cases} \tag{2}$$

is identified using time-domain data only, by means of a continuous-time SMI method, capable of dealing with data generated under feedback; in the subsequent step *a-priori* information on the model structure and frequency-domain data are enforced in the model using a frequency-domain model matching method; finally, an Output-Error (OE) time-domain step completes the approach.

3.2 Predictor-based subspace model identification

The continuous-time algorithm considered in the present study is based on the results first presented in Ohta and Kawai [2004], Ohta [2005], and further expanded in Kinoshita and Ohta [2010], Ohta [2011], Bergamasco and Lovera [2011a], which allow to obtain a discrete-time equivalent model starting from the continuous-time system (1), as briefly outlined in the following. Let $\mathscr{L}_2(0,\infty)$ denote the space of square integrable and Lebesgue measurable functions of time $0 < t < \infty$. Consider the family of Laguerre filters, defined as

$$\mathscr{L}_i(s) = \sqrt{2a}\frac{(s-a)^i}{(s+a)^{i+1}}, \quad i = 0, 1, \ldots \tag{3}$$

and denote with $\ell_i(t)$ the impulse response of the i-th Laguerre filter. Then, it can be shown that the set $\{\ell_0, \ell_1, \ldots, \ell_i, \ldots\}$ is an orthonormal basis of $\mathscr{L}_2(0,\infty)$, *i.e.*, all signals in $\mathscr{L}_2(0,\infty)$ can be represented by means of the set of their projections on the Laguerre basis. Under the assumptions stated in Section 3.1, a discrete-time model equivalent to (2) can be defined by considering the sequence of sampling instants t_i, $i = 1, \ldots, N$ and applying to the input u, the output y and the innovation e of (2) the the transformations

$$\begin{aligned} \tilde{u}_i(k) &= \int_0^\infty (\Lambda_w^k \ell_0(\tau))u(t_i+\tau)d\tau \\ \tilde{e}_i(k) &= \int_0^\infty (\Lambda_w^k \ell_0(\tau))de(t_i+\tau) \\ \tilde{y}_i(k) &= \int_0^\infty (\Lambda_w^k \ell_0(\tau))y(t_i+\tau)d\tau \end{aligned} \tag{4}$$

where $\tilde{u}_i(k) \in \mathbb{R}^m$, $\tilde{e}_i(k) \in \mathbb{R}^p$ and $\tilde{y}_i(k) \in \mathbb{R}^p$. Then (see Ohta and Kawai [2004] for details) the transformed system has the state space representation

$$\begin{aligned} \xi_i(k+1) &= A_o\xi_i(k) + B_o\tilde{u}_i(k) + K_o\tilde{e}_i(k),\; \xi_i(0) = x(t_i) \\ \tilde{y}_i(k) &= C_o\xi_i(k) + D_o\tilde{u}_i(k) + \tilde{e}_i(k) \end{aligned} \tag{5}$$

where the state space matrices are given by

$$\begin{aligned} A_o &= (\hat{A} - aI)^{-1}(\hat{A} + aI) \\ B_o &= \sqrt{2a}(\hat{A} - aI)^{-1}\hat{B} \\ C_o &= -\sqrt{2a}\hat{C}(\hat{A} - aI)^{-1} \\ D_o &= \hat{D} - C(\hat{A} - aI)^{-1}\hat{B} \\ K_o &= \sqrt{2a}(I - \hat{C}(\hat{A} - aI)^{-1}\hat{K})^{-1}(\hat{A} - aI)^{-1}\hat{K}. \end{aligned} \tag{6}$$

By applying to the transformed data a PBSID-like subspace identification approach, estimates of the state space matrices A_o, B_o, C_o, D_o, K_o can be worked out, as discussed in detail in Bergamasco and Lovera [2011a]. Finally, straightforward inversion of the bilinear system transformations defined in (6) allow the recovery of the state space matrices of the original continuous-time system on the basis of the discrete-time ones. The identified model $\mathscr{M}_{ns}$ corresponds to a full state space parameterisation, and therefore does not match the model structure $\mathscr{M}_s(\lambda)$. This problem, as well as the one of incorporating frequency-domain data in the identified model, is handled in the subsequent step.

3.3 From unstructured to structured models

Consider now the model class $\mathscr{M}_s(\lambda)$ introduced in Section 3.1; the parameter vector λ should be tuned so as to ensure that $\mathscr{M}_{ns}$ and $\mathscr{M}_s(\lambda)$ have the same input-output behavior. This problem can be faced in a computationally effective way by defining the transfer functions $\hat{G}_{ns}(s)$ and $G_s(s;\lambda)$ associated with $\mathscr{M}_{ns}$ and $\mathscr{M}_s(\lambda)$, respectively, and seeking the values of the parameters λ corresponding to the solution of the optimization problem

$$\lambda^\star = \arg\min_\lambda \|\hat{G}_{ns}(s) - G_s(s;\lambda)\|, \tag{7}$$

for a suitable choice of norm. The most straightforward approach is to formulate the problem in terms of a quadratic norm penalising deviations, both in magnitude and in phase, between the frequency responses associated with $\hat{G}_{ns}(s)$ and $G_s(s;\lambda)$ (along the lines of the approach discussed in Tischler and Remple [2006]). If, on the other hand, the $\mathscr{H}_\infty$ norm is considered, then the corresponding optimization problem has been studied extensively in recent years in the framework of the fixed-structured robust controller design problem and reliable tools (see Gahinet and Apkarian [2011]) for the computation of local solutions are presently available. Note that the open-loop dynamics of a helicopter is unstable in most flight conditions and so the $\mathscr{H}_\infty$ norm may be undefined. In this case the eigenvalues of $\mathscr{M}_s(\lambda)$ and $\mathscr{M}_{ns}$ are shifted on the real axis by a suitable value μ as follows

$$\tilde{G}_s(s;\lambda) = C(\lambda)((s-\mu)I - A(\lambda))^{-1}B(\lambda) + D(\lambda) \quad (8)$$

$$\tilde{G}_{ns}(s) = \hat{C}((s-\mu)I - \hat{A})^{-1}\hat{B} + \hat{D}, \quad (9)$$

where μ is chosen such that all eigenvalues of $\mathscr{M}_{ns}$ have negative real part. Then the model matching problem is reformulated as

$$\lambda^\star = \arg\min_{\lambda} \|\tilde{G}_{ns}(s) - \tilde{G}_s(s;\lambda)\|_\infty. \quad (10)$$

It is important to point out that the above outlined approach allows to take into account the available frequency-domain data, by including them in the model matching problem in exactly the same way as the frequency response of the unstructured model $\mathscr{M}_{ns}$. Therefore, the optimal structured model $\mathscr{M}_s(\lambda^\star)$ is effectively based on information from the two domains.

3.4 Time-domain output-error refinement

Finally, if the applications of the structured identified model include time-domain simulation, so that its ability to provide a satisfactory time-domain fit of measured data, a final refinement of the model using a time-domain performance criterion is in order. In the present study, the classical output-error approach has been considered (see, *e.g.*, Klein and Morelli [2006] for details).

4. IDENTIFICATION FOR THE AW189 HELICOPTER

In this Section the problem of identifying a six-DOF state-space model for the dynamics of the AW189 helicopter is considered, with specific reference to a forward flight condition at 80 knots. The test data for this case study were generated in piloted simulations based on a state-of-the-art coupled rotor-fuselage model (55th order FlightLab model). The input vector for the model is

$$u = [\ \delta_{col}\ \ \delta_{lat}\ \ \delta_{lon}\ \ \delta_{ped}\ ],$$

corresponding to the four pilot controls (collective, lateral and longitudinal cyclic, pedal). The output vector is

$$y = [a_x\ a_y\ a_z\ p\ q\ r\ \phi\ \theta], \quad (11)$$

where a_x, a_y, a_z are the body components of the linear acceleration, p, q, r denote the body components of the angular rate of the aircraft and ϕ, θ are the roll and pitch angles. Finally, for the structured models the state vector is given by

$$x = [\ u\ v\ w\ p\ q\ r\ \phi\ \theta\ ],$$

where u, v, w denote the body components of the aircraft velocity. The identification experiments have been performed in closed-loop (pilot feedback only) because of the instability of the aircraft. More precisely, in each experiment the primary input channel has been manually excited to generate either a frequency sweep or a 3211 sequence with prescribed characteristics, while the secondary input channels were manually controlled to maintain the helicopter around the nominal trim condition. Nonparametric frequency response estimates have been computed on the basis of the responses to the frequency sweeps using the FRESPID, MISOSA and COMPOSITE modules of the CIFER tool (see Tischler and Remple [2006] for details). The time domain responses to the 3211 sequences and the nonparametric frequency response estimates then served as inputs for the time-frequency domain approach outlined in the previous Section.

The obtained results are illustrated in Figures 1-8 (axis scales have been removed for confidentiality reasons), which provide a comparison between the frequency response of the high-order simulation model (black solid lines in the figures) and the frequency responses of identified parametric models obtained using the proposed time-frequency domain approach (grey solid lines in the figures) and using the CIFER DERIVID module for parametric frequency-domain identification (dashed grey lines in the figures) [1] . As can be seen from the figures, the agreement is quite satisfactory. In particular, it is apparent that the inclusion of time-domain data in the identification process leads to a significant improvement in model accuracy with respect to a pure frequency-domain approach. A time-domain validation of the identified model has been also carried out, by measuring the accuracy of the model in response to random inputs applied on all input channels simultaneously. The time histories for the outputs are presented in Figures 9-10 (axis scales have been removed for confidentiality reasons). Again, even though the open-loop system is unstable, the simulated outputs obtained from the model identified using the time-frequency domain approach (dashed lines) match very well the simulator response (solid lines). Finally, the time-domain performance of the identified models has been also assessed in terms of the RMS simulation error on the individual output variables considered in the study. The result of the comparison is presented in Figure

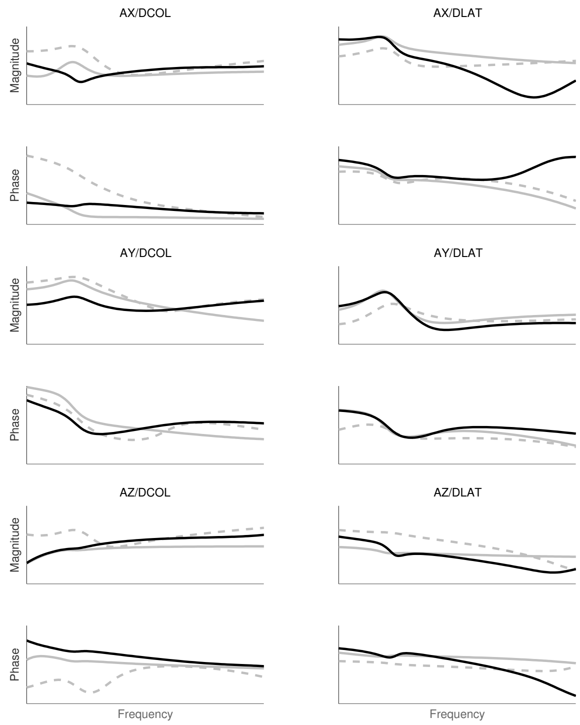

Fig. 1. Frequency responses from collective and lateral inputs to linear accelerations (real: black line; TD/FD model: grey solid line; CIFER: grey dashed line).

[1] For the initialisation of the DERIVID module of CIFER and of the iterative optimization procedure outlined in Section 3.3 reduced order linearised models obtained from FlightLab have been used.

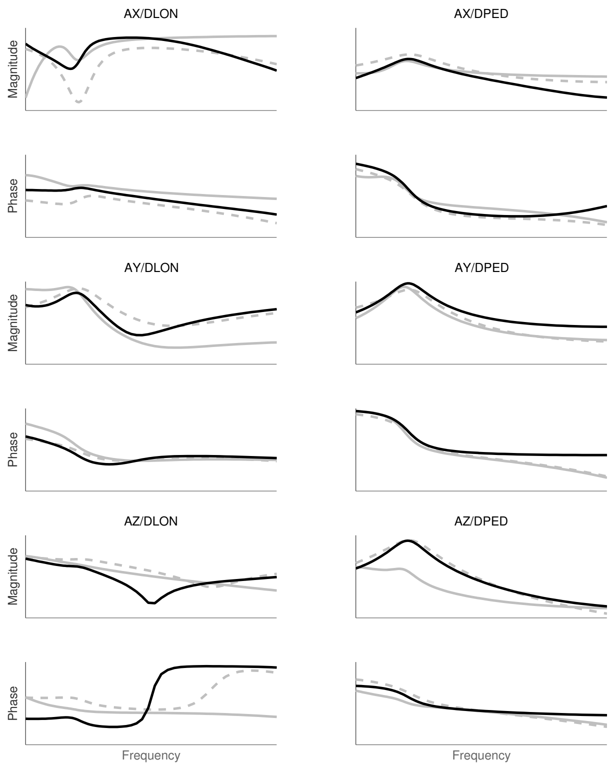

Fig. 2. Frequency responses from pedal and longitudinal inputs to linear accelerations (real: black line; TD/FD model: grey solid line; CIFER: grey dashed line).

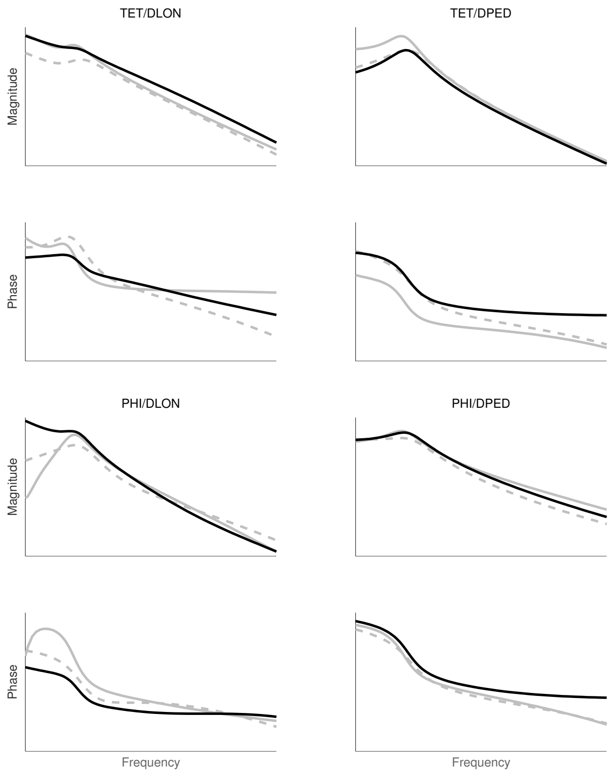

Fig. 4. Frequency response from pedal and longitudinal inputs to pitch and roll angles (real: black line; TD/FD model: grey solid line; CIFER: grey dashed line).

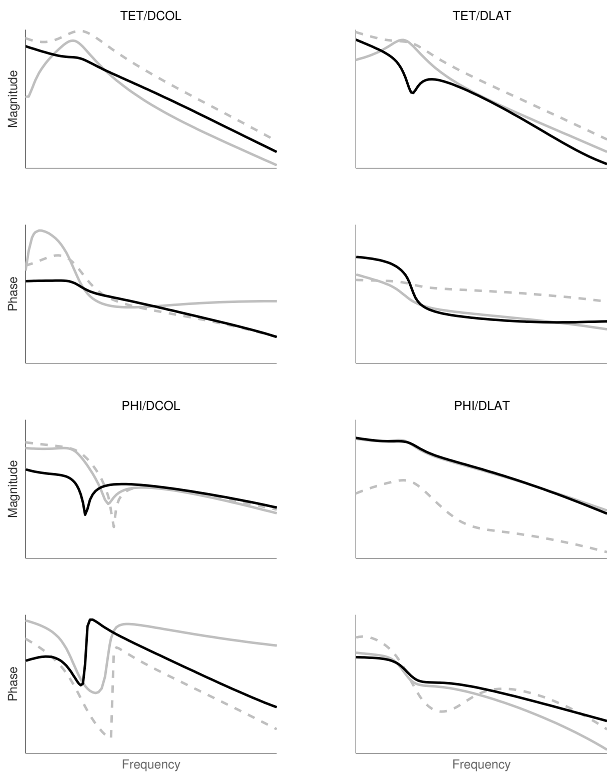

Fig. 3. Frequency responses from collective and lateral inputs to pitch and roll angles (real: black line; TD/FD model: grey solid line; CIFER: grey dashed line).

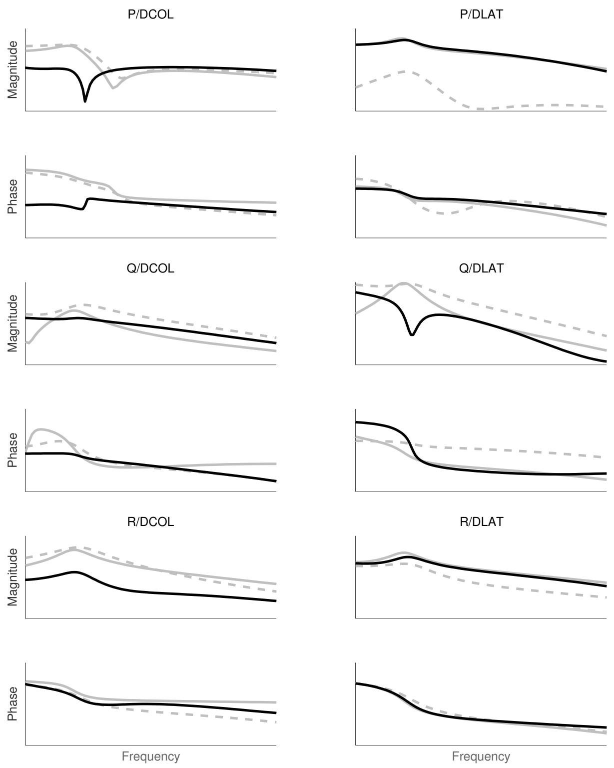

Fig. 5. Frequency responses from collective and lateral inputs to body angular rates (real: black line; TD/FD model: grey solid line; CIFER: grey dashed line).

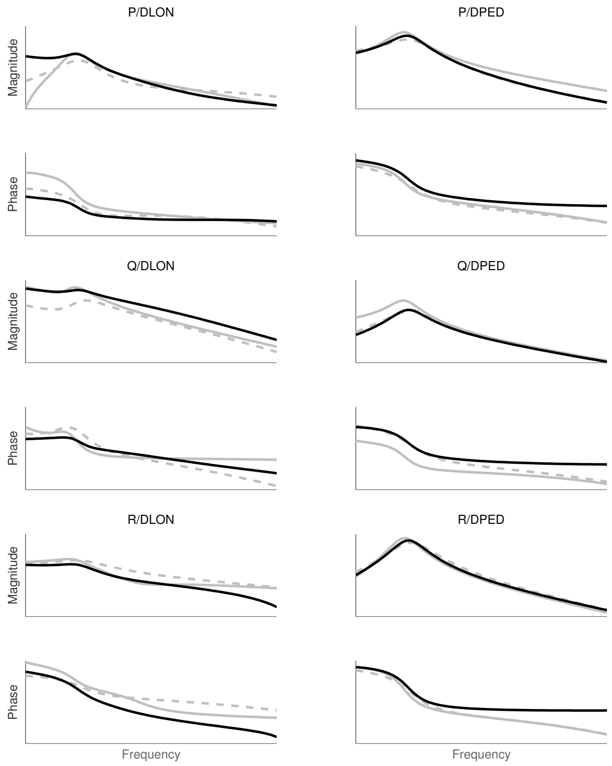

Fig. 6. Frequency response from pedal and longitudinal inputs to body angular rates (real: black line; TD/FD model: grey solid line; CIFER: grey dashed line).

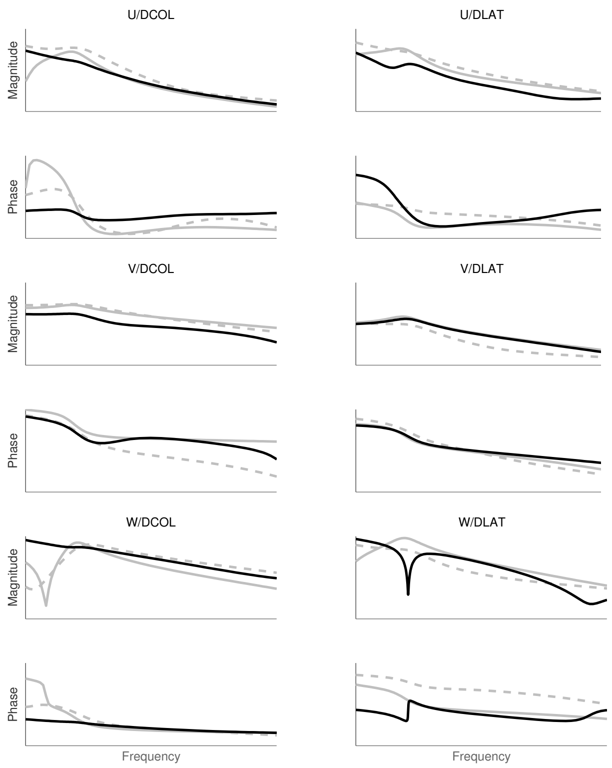

Fig. 7. Frequency responses from collective and lateral inputs to linear body rates (real: black line; TD/FD model: grey solid line; CIFER: grey dashed line).

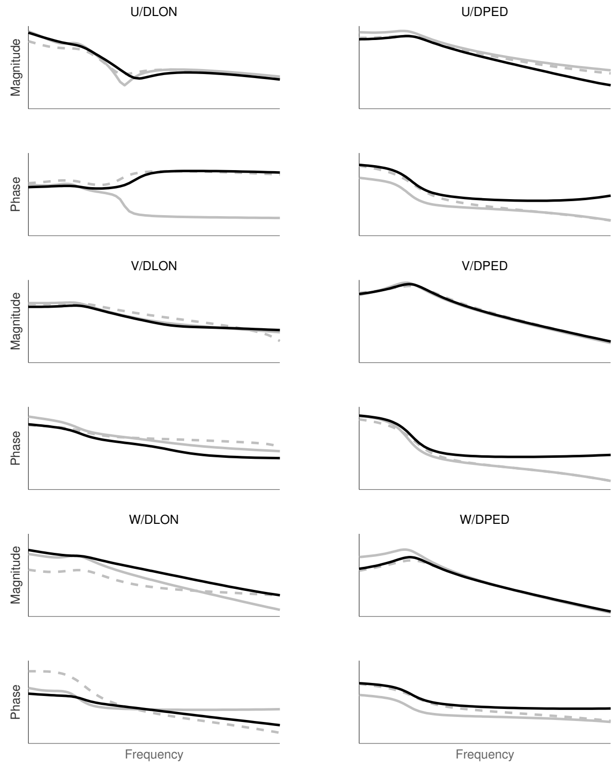

Fig. 8. Frequency response from pedal and longitudinal inputs to linear body rates (real: black line; TD/FD model: grey solid line; CIFER: grey dashed line).

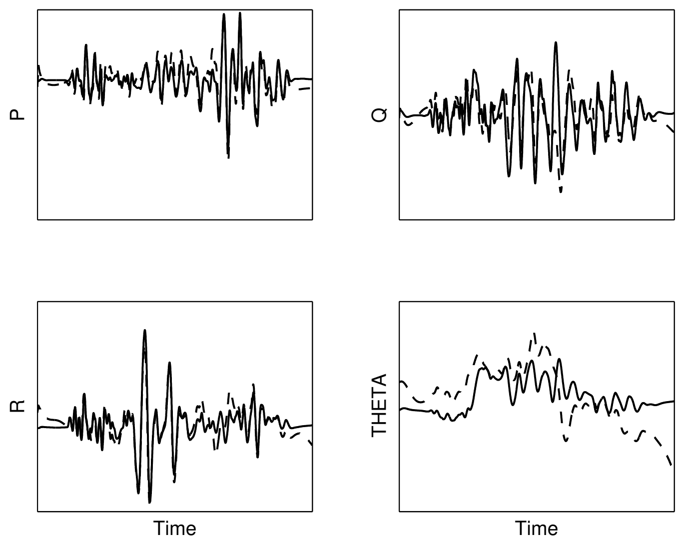

Fig. 9. Time domain validation: body rates and pitch attitude (data: solid line; TD/FD model: dashed line).

11. As can be seen from the Figure, both the black-box model identified using the $PBSID_o$ algorithm and the gray-box one obtained by means of the entire procedure outlined in Section 3 outperform the model identified using only frequency-domain data with CIFER.

CONCLUDING REMARKS

The problem of rotorcraft model identification has been considered and a novel approach combining time and frequency domain data has been presented and discussed. Preliminary results

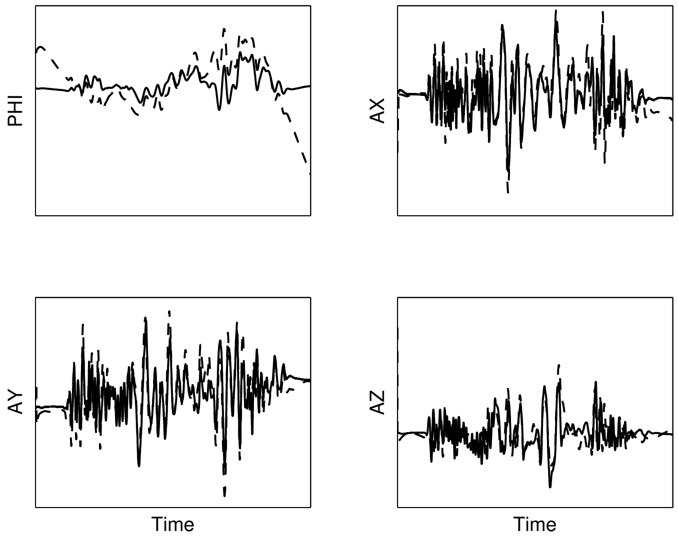

Fig. 10. Time domain validation: acceleration and roll attitude (data: solid line; TD/FD model: dashed line).

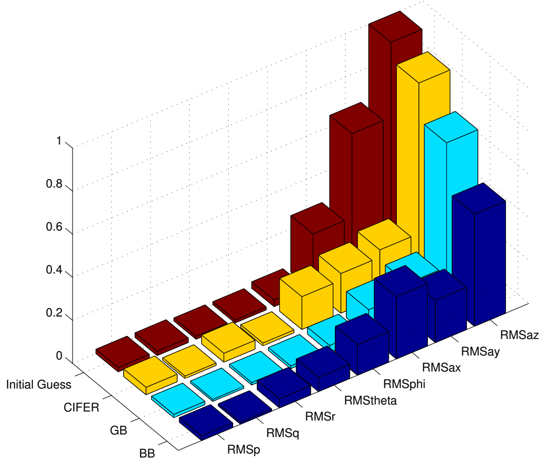

Fig. 11. RMS simulation error (normalised to the maximum value) for the identified models.

based on piloted simulations of a high-order simulation model show that the proposed approach is viable and can provide both black-box and grey-box models with improved accuracy with respect to state-of-the-art tools in the field.

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