Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Baseline vibration attenuation in helicopters: robust MIMO-HHC control

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Abstract: Active control techniques aimed at reducing helicopter vibrations have been extensively studied in the last few decades. The most studied control law is the so-called T-matrix algorithm; its implementation requires knowledge of the frequency response relating the control input to the output measurements at the disturbance frequency, which is very hard to characterise analytically. Adaptive schemes have been employed in literature to handle this problem. Surprisingly, however, very little effort has been devoted to the analysis of the T-matrix algorithm and in particular to the trade-off between robustness and adaptation in its deployment. In this paper an H_∞ approach to the design of a robust T-matrix algorithm is proposed, with the aim of developing a systematic approach to the design of active vibration control laws for helicopters.

1. INTRODUCTION

Among the main problems affecting modern helicopters, vibrations generated by the main rotor are possibly the most important one. Higher Harmonic Control (HHC) has been considered for many years as a valid approach for the design and implementation of control laws aimed at rotor vibration attenuation and the improvement of rotor performance. Its basic idea is to attenuate the vibratory components at the blade-passing frequency in the fuselage accelerations or in the rotor hub loads (N/rev, N being the number of rotor blades) by adding suitably phased harmonic components to the rotor controls. Several studies have been carried out to determine the feasibility of HHC both from the theoretical and the experimental point of view, see, *e.g.*, the survey papers Friedmann and Millott (1995); Kessler (2011b,a) where the used actuation technologies, the considered performance criteria (*e.g.*, noise, vibrations, power, loads etc.) and the achieved performance are reviewed. As for the control implementation, a discrete-time adaptive algorithm known in the rotorcraft literature as the *T-matrix* algorithm (see Shaw and Albion (1981), where this approach was originally proposed) is typically used by defining the problem in the frequency-domain and tuning the controller using an LQ-like cost function. Surprisingly, however, very little effort has been placed to its analysis and in particular to the trade-off between robustness and adaptation in its deployment. Moreover, besides the robustness analysis of the T-matrix algorithm carried out in Chandrasekar et al. (2006), to the best knowledge of the Authors the problem has never been tackled in a robust control framework. In view of this, the aim of this paper (see also the preliminary results in Mura et al. (2014)) is to propose an approach to the design of robust HHC control laws which can be useful to reduce the need of adaptation and guarantee, more specifically:

- nominal stability of the closed-loop system;
- robustness to model uncertainty due to, *e.g.*, changes in the flight condition, configuration etc;
- guaranteed performance for the closed-loop system, *i.e.*, a guaranteed level of vibration attenuation.

Besides allowing to account for model uncertainty in the control design problem (which is already a significant advantage with respect to the LQ-like approach), the H_∞ formulation of the HHC problem provides an additional benefit when dealing with the tuning problem. Indeed, vibrations are typically measured on the fuselage in a large number of locations, so the control problem is strongly multivariable, with different performance requirements associated to the vibration attenuation on the individual outputs. From this point of view, the tuning of an LQ-like, possibly adaptive, algorithm, can turn out to be extremely challenging. Requirement specifications in terms of steady state attenuation levels and desired transient performance, on the other hand, can be immediately "encoded" in an H_∞ problem statement and the properties of the optimal solution can provide information about the actual distance between the desired and the achievable performance level.

The paper is organized as follows: in Section 2 an brief overview of HHC in terms of architecture, capabilities and limitations is provided. The following Section 3 deals with the approach from the point of view of control algorithms, with specific reference to the classical T-matrix algorithm. In Section 4 the problem of robust stability analysis and design for HHC control loops is discussed. Finally in Section 5 simulation results are shown and discussed.

2. ROBUST VIBRATION CONTROL - PROBLEM STATEMENT

The main rotor of a helicopter generates the thrust and the moments necessary to fly the aircraft. The main rotor, however, is also responsible for a number of side effects, mainly mechanical vibrations which are transmitted to the fuselage (and passengers). More precisely, the effect on the fuselage of vibratory loads generated by the main rotor is essentially periodic (as long as the rotor angular rate is constant; this is the case for most present day helicopters, even though variable speed rotors are becoming increasingly widespread), with angular frequency equal to $N\Omega$, where N is the number of blades and Ω is the rotor angular frequency. Furthermore, the main goal in terms of vibration attenuation is the first harmonic ($N\Omega$), denoted in the following as N/rev. To deal with this problem, in the last few years the research has focused on improving the characteristics of the helicopter through the addition of (suitably scaled and phased) higher harmonics to the collective and cyclic controls of the main rotor. In view of this, the problem can be formulated as the one of compensating a periodic disturbance of frequency $N\Omega$ acting at the output of an uncertain (possibly time-periodic) linear system. HHC algorithms are developed on basis of a representation of the coupled rotor-fuselage as a linear quasi-static model constructed in the frequency domain, which is applicable during steady-state flight conditions. Feedback is based on the measurements provided by sensors located across the fuselage or on the main rotor hub. The T-matrix algorithm, described in greater detail in the following Section, is basically a discrete, frequency-domain technique aimed at using estimates of the disturbance from the previous cycle (rotor revolution) to cancel it during the current revolution, assuming that the frequency response relating the control inputs to the measured outputs at the frequency of interest is either known exactly or estimated online. Obviously uncertainty in the elements of the T-matrix, due to varying flight condition and changing configuration, has to be considered. Typically offline or online estimation is used in the literature to this purpose (see Figure 1), and least-squares error methods like the RLS algorithm seem the natural choice for this task, given the linear nature of the model.

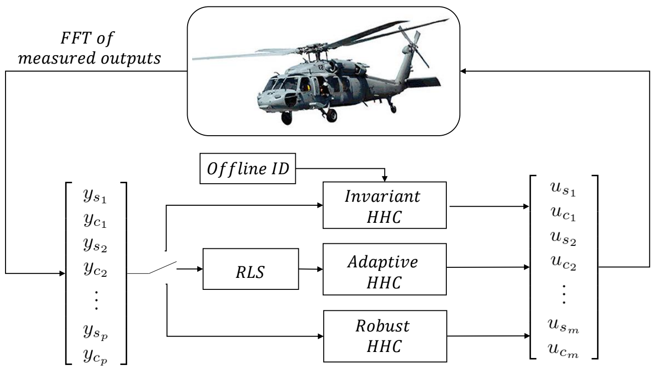

Fig. 1. Overview of possible HHC architectures for baseline vibration attenuation.

From the experimental results available in the literature, however, the need for adaptive control does not appear to be a strong requirement for the successful deployment of the control system (particularly so when control of the structural response, rather than rotor control, is considered, see the discussion of this point in Kessler (2011b,a)). To the contrary, vibration control systems based on the T-matrix approach seem to exhibit a fair degree of robustness. In this respect, it is surprising that the problem of robustness of active rotor control systems has received very little attention and deserves to be investigated in greater depth.

3. THE T-MATRIX ALGORITHM

Let $u \in \mathbb{R}^m$ be the vector of control inputs and $y \in \mathbb{R}^p$ the vector of measured outputs; assume also that u is a vector of piece-wise periodic functions of period $T = \frac{2\pi}{\Omega}$ (where Ω is the rotor's angular frequency), let be $\psi = \Omega t$, and define

$$y_{Nc}^{(i)} = \frac{2}{T}\int_0^T y^{(i)}(\psi)\cos(N\psi)d\psi \tag{1}$$

$$y_{Ns}^{(i)} = \frac{2}{T}\int_0^T y^{(i)}(\psi)\sin(N\psi)d\psi \tag{2}$$

$$y_N = \begin{bmatrix} y_{Nc}^{(1)} \\ y_{Ns}^{(1)} \\ \vdots \\ y_{Nc}^{(p)} \\ y_{Ns}^{(p)} \end{bmatrix} \tag{3}$$

and similarly for u_N. Assume now that under steady state conditions the above defined N/rev harmonics of u_N and y_N are related by the linear equation

$$y_N = T_{N,N}u_N + w \tag{4}$$

where $T_{N,N}$ is a $2p \times 2m$ constant coefficient matrix and w represents the N/rev component of the vibration affecting the system. Notice that if the dynamics relating u to y can be assumed to be linear time-invariant (which is a reasonable assumption if one considers a fixed steady flight condition and a configuration for the vibration control system with actuators and sensors located in the fuselage), then it can be described by the frequency response matrix $G(j\omega)$ defined as

$$G(j\omega) = \begin{bmatrix} G^{(1,1)}(j\omega) & \cdots & G^{(1,m)}(j\omega) \\ \vdots & & \vdots \\ G^{(p,1)}(j\omega) & \cdots & G^{(p,m)}(j\omega) \end{bmatrix}. \tag{5}$$

$T_{N,N}$ is clearly related to $G(jN\Omega)$, as

$$T_{N,N}^{(i,j)} = \begin{bmatrix} \mathrm{Real}(G^{(i,j)}(jN\Omega)) & \mathrm{Imag}(G^{(i,j)}(jN\Omega)) \\ -\mathrm{Imag}(G^{(i,j)}(jN\Omega)) & \mathrm{Real}(G^{(i,j)}(jN\Omega)) \end{bmatrix} \tag{6}$$

and, consequently,

$$T_{N,N} = \begin{bmatrix} T_{N,N}^{(1,1)} & \cdots & T_{N,N}^{(1,m)} \\ \vdots & & \vdots \\ T_{N,N}^{(p,1)} & \cdots & T_{N,N}^{(p,m)} \end{bmatrix}. \tag{7}$$

Then, the discrete time algorithm for the attenuation of the effect of w on y_N, known in the rotorcraft literature as the T-matrix algorithm, can be derived by minimizing at each discrete-time step k the LQ-like cost function

$$J(k) = {y_N}^T(k)Qy_N(k) + {u_N}^T(k)Ru_N(k) \tag{8}$$

where $Q = Q^T \geq 0$, $Q \in \mathbb{R}^{2p\times 2p}$ and $R = R^T > 0$, $R \in \mathbb{R}^{2m\times 2m}$.

The optimal control law is thus found by differentiating (8) with respect to $u_N(k)$

$$\frac{\partial J(k)}{\partial u_N(k)} = 0, \tag{9}$$

leading to the open-loop control algorithm

$$u_N(k+1) = -D^{-1}\big(T_{N,N}^T Q\big)w(k), \tag{10}$$

where

$$D = T_{N,N}^T Q T_{N,N} + R.$$

As for the implementation of the above discrete control algorithm, the following operation need to be carried out:

(1) the determination of the N/rev component of y_N, namely the computation of the modulated integrals (1) and (2) of the output;
(2) the update of the N/rev component of u_N using equation (10);
(3) the determination of the time domain value of the control input u via a modulation of the N/rev sine and cosine components.

The control law (10) can be written in closed-loop form as

$$u_N(k+1) = K_M u_N(k) + K_N y_N(k), \qquad (11)$$

where

$$K_M = -K_N T_{N,N}. \qquad (12)$$

In this respect it is interesting to point out that the structure of matrix $T_{N,N}$ (see (7)) implies a similar structure of matrices K_M and K_N. In particular, it can be shown that the structure of every submatrix $T_{N,N}^{(i,j)}$, being of the form

$$T_{N,N}^{(i,j)} = \begin{bmatrix} a^{(i,j)} & b^{(i,j)} \\ -b^{(i,j)} & a^{(i,j)} \end{bmatrix}, \qquad (13)$$

is extended to matrix K_N (its submatrices) by means of equation (10) and consequently also K_M inherits the same type of structure. This means that, in the face of $2m\cdot(2m+2p)$ entries in the matrices defining the control law, only $2p$ free parameters are exploited in the LQ control law. In the following Sections a different approach will be presented and a control law based on full parametrized matrices will be discussed.

The main drawback of the control law in the introduced form (10) is based on the assumption that exact knowledge of matrix $T_{N,N}$ is available. Clearly, an erroneous model of it can result in degraded performance and possible instability. To deal with model uncertainty, *a posteriori* analysis can be carried out to prove robustness qualities. An interesting derivation from Chandrasekar et al. (2006) provides upper bounds on the maximum singular value of the additive uncertainty for which robust stability is guaranteed by using the above described LQ-like control law. While such an analysis is of course informative, its main limitation lies in the difficulty in relating back bounds on the uncertainty on matrix $T_{N,N}$ to the actual model uncertainty in the dynamics of the helicopter. Based on these considerations, a robust framework could be used to design a control law by incorporating all the uncertainties during the synthesis process. Conventional HHC control deals with performance degradation in presence of model uncertainty by introducing adaptation, which takes into account the variation of the matrix $T_{N,N}$ between the different flight conditions. Many different algorithms have been developed in this sense in the last few years, mainly based on the estimation of $T_{N,N}$ at specific time steps during the flight operations. In this respect, the interest in investigating a robust control design approach is motivated by the possibility to relax the need for continuous update of the T-matrix.

4. H_∞ ROBUST VIBRATION CONTROL DESIGN

In this section an H_∞ approach to the design of the robust MIMO-HHC control law is presented. In the formulation of the robust H_∞ control synthesis, we choose the following output multiplicative representation of uncertainty in the T-matrix (see Figure 2)

$$T_{N,N} = \left(I_{2p} + W_m \Delta\right)\bar{T}_{N,N}, \quad \|\Delta\| < 1, \qquad (14)$$

where I_{2p} is the identity matrix of dimension $2p$, Δ is a normalized representation of the uncertainty and W_m represents the matrix of the uncertainty ellipsoids affecting the frequency responses $G^{(i,j)}(jN\Omega)$, $i = 1...p$, $j = 1...m$, at the disturbance frequency. Note that matrix W_m has a block diagonal structure due to the collection of uncertainties related to each of the matrices $T_{N,N}^{(i)}$, $i = 1...p$,

$$W_m = blkdiag\left(W_m^{(1)} \; W_m^{(2)} \; \cdots \; W_m^{(p)}\right) \qquad (15)$$

where

$$W_m^{(i)} = r^{(i)} \begin{bmatrix} \alpha^{(i)} & \beta^{(i)} \\ -\beta^{(i)} & \alpha^{(i)} \end{bmatrix}, \quad i = 1, ..., p \qquad (16)$$

with r^i a scalar scale factor and α, β the parameters relating to the considered uncertainty of the specific output.

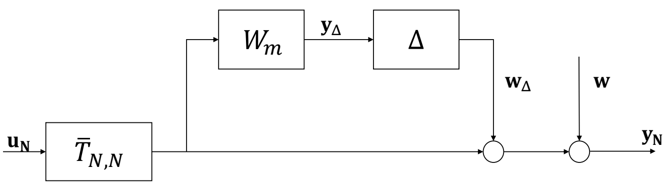

Fig. 2. Block diagram of the uncertainty representation.

A block diagram of the uncertain feedback system corresponding to the model (4), its uncertainty (14) and the controller (11) is represented in Figure 3, where Δ is defined as

$$\Delta = blkdiag\left(I_2\delta^{(1)} \; I_2\delta^{(2)} \; \cdots \; I_2\delta^{(p)}\right). \qquad (17)$$

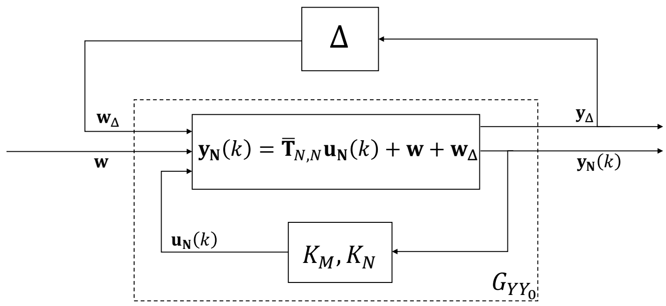

Fig. 3. Block diagram of the uncertain feedback system.

With reference to Figure 3, variables w_Δ and y_Δ can be defined as

$$\begin{aligned} y_\Delta(k) &= W_m \bar{T}_{N,N} u_N(k) \\ w_\Delta(k) &= \Delta y_\Delta(k), \end{aligned} \qquad (18)$$

leading to the uncertain closed-loop system

$$\begin{aligned} u_N(k+1) &= K_M u_N(k) + K_N y_N(k) \\ y_N(k) &= \bar{T}_{N,N} u_N(k) + w(k) + w_\Delta(k) \\ y_\Delta(k) &= W_m \bar{T}_{N,N} u_N(k) \\ w_\Delta &= \Delta y_\Delta. \end{aligned} \qquad (19)$$

Letting $Y = [y_N \quad y_\Delta]^T$ and $Y_0 = [w \quad w_\Delta]^T$, the uncertain system (19) can be represented in input-output form as

$$Y = \mathcal{G}_{YY_0}(z)Y_0 \tag{20}$$

where $\mathcal{G}_{YY_0}(z)$ is defined in (21). The second step of an H_∞ synthesis problem is the definition of the weighting functions $W_y^{(i)}$, $i = 1...p$ and $W_u^{(j)}$, $j = 1...m$ used, respectively, on the output y_N and the control variable u_N according to the block diagram for the augmented plant model in Figure 4.

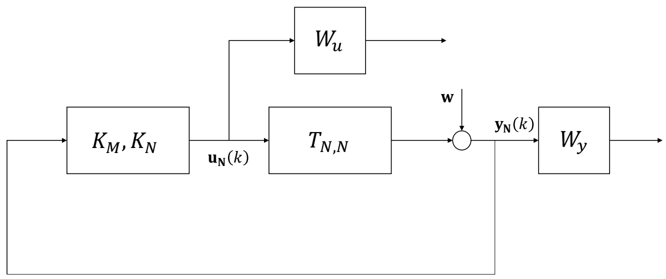

Fig. 4. Augmented plant model.

In particular the control design should be focused on the closed-loop steady-state performance, therefore, the shape of the frequency response of $W_y(z)$ can be intuitively designed on this basis (see, *e.g.*, Figure 5 where the frequency responses of continuous-time templates suitable to generate $W_y(z)$ are depicted). Besides the possibility of taking model uncertainty into account in the formulation of the control problem (which already represents a significant advantage *per se* with respect to the LQ-like approach), the H_∞ formulation of the HHC problem provides an additional, significant, benefit when dealing with the tuning problem. Indeed, the vibration control problem is a strongly multivariable one, with different performance requirements associated to vibration attenuation levels in different locations on the fuselage. From this point of view, the tuning of an LQ-like, possibly adaptive, algorithm, can turn out to be extremely time consuming. Requirement specifications in terms of steady state attenuation levels and desired transient performance, on the other hand, can be immediately "encoded" in the problem statement through weighting functions, while the properties of the optimal solution can provide information about the actual distance between the desired and the achievable performance level. More precisely, different options are available:

- one could design the same weighing function for all the considered outputs, so as to define a uniform performance bound for the entire system;
- one can define as many functions as the number of output considered, meaning that it could be possible to give higher penalties to specific outputs which can present, for example, more critical characteristics in terms of vibration (*e.g.*, pilot and co-pilot seats).

Given that, the robust synthesis problem can be formulated as

$$\begin{aligned} &Find \quad K_M,\ K_N \\ &s.t. \\ &\left\| \begin{matrix} \mathcal{G}_{YY_0} W_y \\ \mathcal{G}_{uy_0} W_u \end{matrix} \right\|_\infty \leq \gamma, \end{aligned} \tag{22}$$

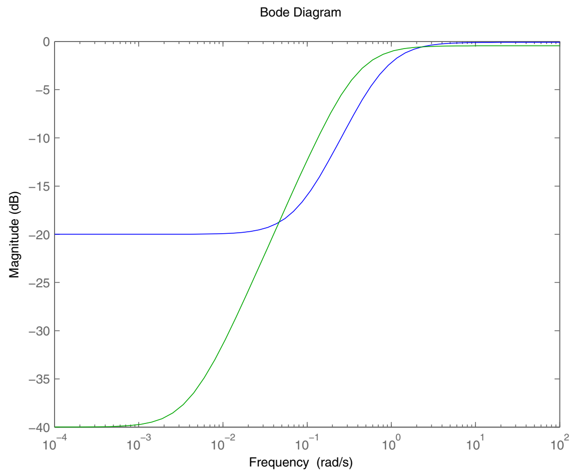

Fig. 5. Frequency response of possible $W_y(s)$ weighting functions.

where $\mathcal{G}_{uy_0}$ is the control sensitivity function, obtained by re-opening the closed-loop from the disturbance w to the control variable u_N in the nominal case:

$$\mathcal{G}_{uy_0} = (zI - (K_M + K_N\bar{T}_{N,N}))^{-1}K_N. \tag{23}$$

Problem (22) is a structured H_∞ problem, which is known to be both non-convex and non-smooth. This means that on one hand the convergence of the algorithm may depend on the initial controller and global optimality of the computed solution cannot be guaranteed, and on the other hand, for example, gradient-based descent algorithms could fail to converge. In view of this, a randomized method can be used to solve the optimization problem: the key point is that a control law is optimal if in its neighborhood a better control law cannot be found (or, equivalently, can be found with null probability, see Zanchettin et al. (2013)). To this purpose, assume that an initial stabilizing controller, based on the classical LQ T-matrix algorithm, is available. To compute a new controller $K^{(i+1)}$ the basic idea is to test randomly sampled controllers in a neighborhood of $K^{(i)}$ and select the best one in terms of minimization of the cost function in (22). When it is no longer possible to find better controllers, the algorithm stops and the (locally) optimal controller is obtained. The obtained controller is actually a pair (K_M, K_N) of random matrices, and to check convergence the procedure has been iterated as many times as the standard deviation in the cost function becomes below a threshold small enough to guarantee a very small tolerance on the minimality of the cost function.

Remark 1. As mentioned in the previous sections, the gains of an LQ T-matrix controller are not fully parameterized matrices, but inherit the structure of the T-matrix itself. This structure of the control law can be either retained in the design of the robust controller, or, on the contrary, can be relaxed leading to a fully parameterized control law. Both possibilities have been considered, but particular attention has been given to the second case, the results of which are reported in the following Section.

$$\begin{bmatrix} y \\ y_\Delta \end{bmatrix} = \begin{bmatrix} \bar{T}_{N,N}(zI-(K_M+K_N\bar{T}_{N,N}))^{-1}K_N + I_{2p} & \bar{T}_{N,N}(zI-(K_M+K_N\bar{T}_{N,N}))^{-1}K_N + I_{2p} \\ W_m\bar{T}_{N,N}(zI-(K_M+K_N\bar{T}_{N,N}))^{-1}K_N & W_m\bar{T}_{N,N}(zI-(K_M+K_N\bar{T}_{N,N}))^{-1}K_N \end{bmatrix} \begin{bmatrix} w \\ w_\Delta \end{bmatrix} \quad (21)$$

5. SIMULATION RESULTS

In this Section a numerical example is used to illustrate the main properties of the proposed robust HHC scheme. The T-matrix considered in this example is based on numerical values extracted from a simplified single rotor blade model of the Agusta A109 helicopter (the parameters of which are given in Table 1). Although very simple, the model describes the out-of-plane bending dynamics of a helicopter rotor blade, as derived in Johnson (1980) and retains the main characteristics of the full blade dynamics (refer to Bittanti and Lovera (1996) for more details). The considered T-matrix is characterised by 2 inputs and

Table 1. Characteristics of the rotor of the Agusta A109 helicopter.

Number of blades	4
Rotor angular frequency	40.32 $\frac{rad}{s}$
Rotor radius	5.5 m
Mass per unit	48.8
Stiffness	$1.8 \cdot 10^3$ N
Lift curve slope	5.7 rad^{-1}
Lock number	7.8
Blade chord	0.3 m

3 outputs and is therefore composed by six submatrices relating each input to each output. The nominal matrix $\bar{T}_{N,N}$, obtained by evaluating the model for a given flight condition, is given by

$$\bar{T}_{N,N} = \begin{bmatrix} 0.3476 & 0.0378 & 0.2206 & 0.0240 \\ -0.0378 & 0.3476 & -0.0240 & 0.2206 \\ 0.2318 & 0.0252 & 0.3389 & 0.0368 \\ -0.0252 & 0.2318 & -0.0368 & 0.3389 \\ 0.1738 & 0.0189 & 0.3323 & 0.0361 \\ -0.0189 & 0.1738 & -0.0361 & 0.3323 \end{bmatrix},$$

and the relative uncertainty associated with the system and defined consistently with the definition in equation (15) is

$$W_m = \begin{bmatrix} 0.1025 & 0.0278 & 0 & 0 & 0 & 0 \\ -0.0278 & 0.1025 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0683 & 0.0185 & 0 & 0 \\ 0 & 0 & -0.0185 & 0.0683 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0512 & 0.0139 \\ 0 & 0 & 0 & 0 & -0.0139 & 0.0512 \end{bmatrix}.$$

On the basis of the above uncertain model, two control scenarios are considered. The first one aims at attenuating vibrations on the whole set of outputs to more than 95%, while the second one separates the required performance between outputs y_1 and y_2 (again 95% attenuation required) and y_3 (only 90% attenuation required), in order to mostly penalize specific outputs because considered more critical in terms of vibration. Thus, two weighting functions have been introduced,

$$W_y^{(low)}(z) = \frac{0.99z - 0.968}{z - 0.7797} \qquad W_y^{(high)}(z) = \frac{0.95z - 0.949}{z - 0.8875}, \quad (24)$$

for which continuous-time equivalent frequency responses are represented in Figure 5. As for $W_u(z)$, no weight on the control action has been included in the problem. Finally, in order to compare the resulting performance, a second LQ tuning is obtained guaranteeing a similar level of nominal attenuation, in both the considered scenarios. With reference to Figure 6, Figure 7 and Table 2, LQ weight matrices Q and R are defined, for the first and second scenario respectively, as

$$\begin{array}{ll} Q = blkdiag(5.23 \cdot I_2,\ 2.28 \cdot I_2,\ 3.16 \cdot I_2) & R = 0.1 \cdot I_4 \\ Q = blkdiag(4.22 \cdot I_2,\ 2.31 \cdot I_2,\ 5.49 \cdot I_2) & R = 0.1 \cdot I_4 \end{array}. \quad (25)$$

For the feedback system (subject to a unit norm disturbance) a Monte Carlo study was carried out by randomly choosing 500 values for the normalized uncertainty Δ. In the following, results of this Monte Carlo procedure in terms of steady-state attenuation are depicted. Figure 6

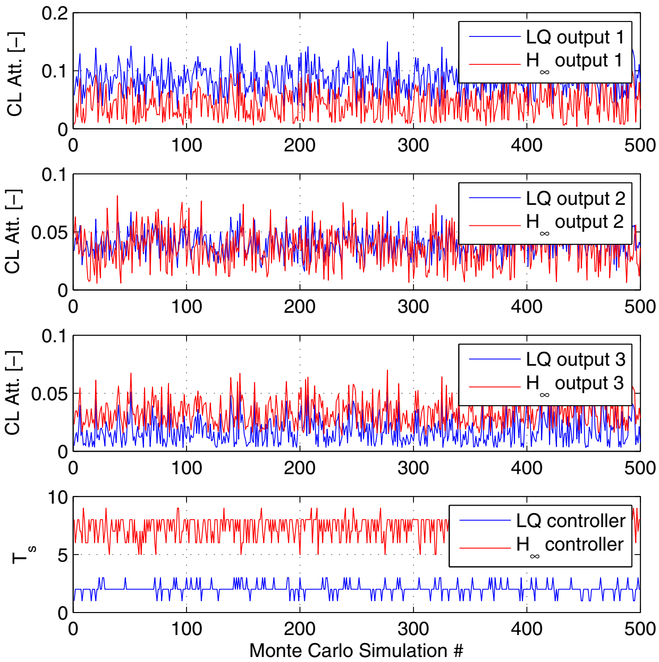

Fig. 6. Closed-loop performance comparison in the first scenario.

and Figure 7 show the steady-state attenuation level for each output. The same figures show also the difference between the two control laws in terms of settling time T_s to reach the steady-state (calculated in terms of discrete time steps), and clear evidence is posed upon the conservative property of the H_∞ regulation in this sense. In both scenarios a comparable level of attenuation is obtained with both controllers, but it is also apparent that the LQ-like controller fails to find a correct tradeoff to get similar levels of attenuation for each of the measured outputs. The robust design proposed in this paper shows another substantial advantage over a classical LQ-based synthesis: considering a general "large" MIMO system, defining weighting functions W_y that resume specific control objectives seems to be much more convenient than iterating different Q and R matrices until control requirements are being satisfied. Table 2 confirms these considerations, and

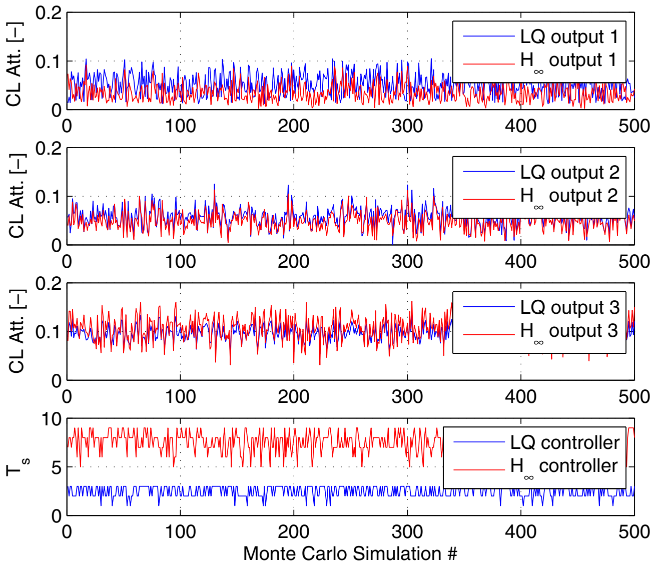

Fig. 7. Closed-loop performance comparison in the second scenario.

moreover shows how the cost of increased robustness is a slightly slower response in dynamic terms. As mentioned in the previous Section, the comparison has been extended also to a structured version of the H_∞ controller. To this purpose, the synthesis procedure has been modified on the basis of the structure constraint, meaning that the cost function is optimized with respect to 20 independent parameters instead of 40, as in the fully parameterized formulation. Table 3 summarizes the results obtained in the first scenario.

Based on these results, the H_∞ approach could be beneficial in the sense of reducing the need for adaptation in the operation of the HHC system, which would make the system itself easier to implement and operate, while allowing a more predictable closed-loop behavior both in terms of stability and performance.

Table 2. Simulation results. *LQ* vs H_∞.

	Scenario 1		Scenario 2	
	LQ	H_∞	LQ	H_∞
mean y_1	0.088	0.043	0.052	0.034
max y_1	0.149	0.099	0.105	0.095
mean y_2	0.041	0.037	0.058	0.049
max y_2	0.068	0.082	0.125	0.113
mean y_3	0.017	0.033	0.096	0.106
max y_3	0.054	0.069	0.135	0.152
T_s	3.30	7.98	2.48	7.61

Table 3. Simulation results. Structured vs Unstructured H_∞.

	struct	*unstruct*
mean y_1	0.075	0.043
max y_1	0.151	0.099
mean y_2	0.044	0.037
max y_2	0.087	0.082
mean y_3	0.023	0.033
max y_3	0.075	0.069
T_s	9.31	7.98

6. CONCLUSION

In this paper the problem of robust design of HHC control laws has been considered and an H_∞ approach to the problem has been proposed and compared to the classical LQ solution. Simulation results show that a closed-loop performance similar to the LQ one can be achieved with the additional benefit of a reduced sensitivity of the feedback system to uncertainty in the knowledge of the T-matrix. This property can be exploited to reduce the need for adaptation in the actual implementation of HHC systems.

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