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Hybrid Model Predictive Control for Grinding Plants ⋆

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Abstract: Energy consumption reduction strategies in the mining industry have increased in the last years due to the energy prices increments. Control strategies in the mining processes are one of the many ways to optimize energy consumption, especially if the strategies consider global optimization and ensure the stability of the system. Mineral grinding accounts for as much as 50% of the energy consumption in a mineral concentrator plant. For this reason a centralized Hybrid model predictive control (HMPC) scheme is presented; this control minimizes the specific energy consumption and stabilizes the plant by ensuring an output particle size contained on the mesh 65. This work shows that more complex control solutions can be applied in the grinding process, substituting conventional control strategies in a successful way, and decreasing the control strategy implementation complexity, since conventional control strategies rely on expert systems to handle discrete variables and events, and HMPC strategies allow the inclusion of discrete events both in the model and in the controller.

Keywords: Hybrid model predictive control; Dynamic modelling; Hybrid modelling; Hybrid identification.

1. INTRODUCTION

Mineral grinding is one of the main components of a concentrator plant, its aim is to reduce the particle size by a combination of impact and abrasion effects (Wills and Napier-Munn, 2006). Control strategies for the mineral grinding process include global energy optimization, ensuring an output particle size suitable for the flotation process and ensuring the process stability. To achieve this, control techniques such as PID, multivariate, expert systems, fuzzy logic, neural networks, model predictive control (MPC), statistical process control, hybrid model predictive control (HMPC) and others (Wei and Craig, 2009) have been proposed.

The use of simple MPC strategies has increased in the last years because of good performance results in tracking and optimizing applications, the ability to handle restrictions and constraints, and a robust management of disturbances (Wei and Craig, 2009), the disadvantage is that these techniques do not allow the use of discrete variables, events, and dynamics; a problem solved by the HMPC control technique.

The Hybrid Model Predictive Control (HMPC) strategy is a technique that represents nonlinear dynamics under different operating modes, by creating a set of linear equations. Since a linearization process of nonlinear dynamics can be a difficult task, a hybrid identification procedure is used, allowing to obtain simple models suitable for a HMPC controller (Putz and Cipriano, 2013).

Given the existence of several operating modes in a grinding plant, regarding the ON/OFF status of secondary grinding circuits, ON/OFF status of conveyors, quantity of active hydrocyclones, and others. Hybrid modelling proposes an interesting and novel solution for the representation of the grinding process and the control. In this paper, we first develop a dynamic hybrid model for grinding stage, then we get a prediction model for the output particle size and the specific energy consumption through identification techniques, and last we develop an HMPC centralized controller to minimize the energy consumption and maintain the particle size output in certain ranges.

2. PLANT DESCRIPTION

The grinding plant consists on primary and secondary grinding. The primary grinding breaks the ore by using a SAG mill, and then its classified by a vibratory screen, the product goes to a sump or to a pebble crusher to reduce more the ore size. The secondary stage of the grinding starts after the sump, where the sump ore is sent to hydrocyclones batteries using pumps, and it is classified to go either to the flotation process or to a balls mill circuit that crushes the ore coming from the hydrocyclones and from the pebble crusher.

A schematic of the simulated plant is shown in Fig. 1, the secondary stage consists on a two line grinding mill circuit. Each hydrocyclone battery has 14 hydrocyclones.

3. MODELLING AND SIMULATION

The plant is modelled by including both dynamic models and discrete events/variables. This variables represent the state of plant components such as secondary grinding

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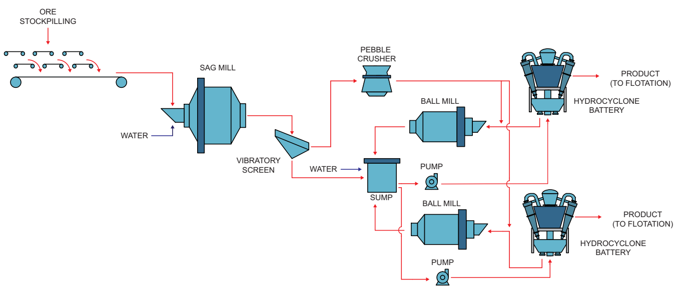

Fig. 1. Grinding schematic

circuits. They are also used to represent the behaviour of the stockpile feeding process.

3.1 Continuous Modelling

The continuous modelling of the grinding plant makes use of existing mathematical models of the machinery and processes involved. This models are taken from Orellana (2010), which are simplifications of the work developed in Weymont (1979), Austin et al. (1987) and Barahona (1984).

Mass flow vectors divided into intervals of ore size ranges (Orellana, 2010) are used to represent the mineral feeding from the stockpile. This can can be described by (1)

$$\mathrm{f_s} = F_t * \mathrm{f_{size}} \tag{1}$$

where F_t is the total mass flow and $\mathrm{f_{size}}$ is a vector dependent of the granulometric distribution of the mineral.

The flow of pulp that passes through the pumps is used to activate/deactivate the secondary grinding lines, together with other variables and processes. This can be described as in (2) (Orellana, 2010):

$$v_{tmp} = 1 + 4g_{cp} * [\kappa_{B1} V_b - \kappa_{B2} * \rho_{tP} * g * (h_h - h_p)]$$
$$\bar{f_{bc}} = \left[\frac{-1 + \sqrt{v_{tmp}}}{2 * g_{cP}}\right] \tag{2}$$

where κ_{B1} and κ_{B2} are pump constants, V_b is the pump speed, ρ_{tP} is the pulp density, g the gravity force, h_h the height difference between the pump and the hydrocyclones, h_p is the pulp height inside the sump, and g_{cP} is defined as (3):

$$g_{cP} = \kappa_{B2} * \left[\frac{\kappa_{H4}}{(1 - c_P)^{0.25}} + \kappa_{B3}\right] \tag{3}$$

with κ_{B3} representing a geometric parameters constant of the system and κ_{H4} a pressure constant in the input of the hydrocyclone battery.

3.2 Hybrid Modelling

The hybrid modelling was achieved by representing the variables with piecewise equations, or with the use of activation variables to certain equation terms.

The hybrid modelling in this paper was developed to achieve the modelling of two events of a grinding plant. The first event is the stockpile feeding process of the plant, and the second is the activation/deactivation of secondary grinding circuits.This section presents the equations involved in the development of the hybrid model.

The stockpile feeding process can be described as follows. In (Orellana, 2010) the particle size, f_{size}, can take three values depending on the granulometric distribution (big, medium or small), this is be described as in (4):

$$\mathrm{f_s} = \begin{cases} Ft * \mathrm{f_s}, & g_a(t) = 1 \\ Ft * \mathrm{f_m}, & g_a(t) = 2 \\ Ft * \mathrm{f_b}, & g_a(t) = 3 \end{cases} \tag{4}$$

where g_a is the variable that allows the selection of the granulometric distribution of the solids. This modelling allows the development of an independent equation for each conveyor feeding the SAG mill. This work makes the use of six conveyors. Two of each granulometric distribution. After this, the product of the conveyors is added, forming the feeding mass flow of the grinding plant.

As it is known, a usual stockpile has the finest material in the centre and the coarsest material on the sides, making the material discharge process feed the finest granulometric distribution ore first. This becomes a problem since the stockpile feeding rate is not infinite, and when the finest ore stops flowing, the grinding only processes medium and coarse ore, making it harder to crush. And in worst case scenario the grinding only processes coarse ore. This forces the operators to push the remaining material to the centre. The modelling of the primary feeding is shown in Fig. 2:

The state of the conveyors is modelled by adding an activation term to the sum as in (5):

$$\mathrm{f_{s_{total}}} = \alpha_1 * \mathrm{f_s} + \alpha_1 * \mathrm{f_m} + \alpha_3 * \mathrm{f_b} \tag{5}$$

where the activation terms are $\alpha_r \in \{0,1\}$ with $r = \{1,2,3\}$. Additionally $\mathrm{f_s}, \mathrm{f_m}$ and, $\mathrm{f_b}$ represent the conveyors mineral flow. In this work only three conveyors have this activation terms, one of each granulometric distribution, the other three conveyors feed the ore at a small constant rate, representing the material mix on the stockpile, allowing a percentage of mixed granulometries feed the process.

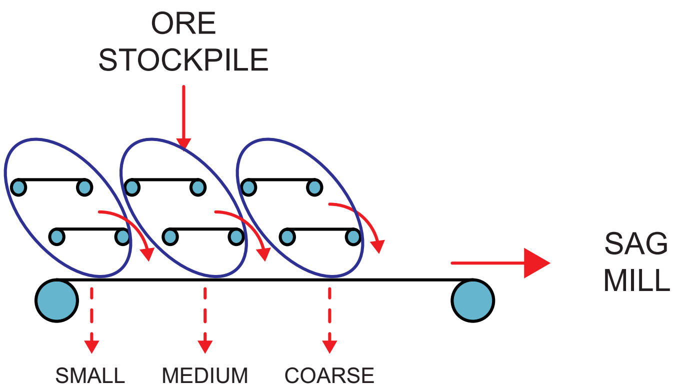

Fig. 2. Ore stockpile feeding modelling

The state of the secondary grinding is modelled by adding activation terms to the flow of solids and water that pass through the sump. The solid flow is described as in (6):

$$\bar{f_b} = s_{ls} * \bar{f_{bc}} \tag{6}$$

where $s_{ls} \in \{0,1\}$ is the activation term to stop the flow of solids, and $\bar{f_{bc}}$ represents the flow of pulp passing the pumps. The water flow is represented as (7):

$$q_{oP} = s_{ls} * [\bar{p_P}\rho_{tP}(1-c_P)] \tag{7}$$

with c_P representing the solids percentage in the pulp. ρ_{tP} is the pulp density and $\bar{p_P}$ is the material output volumetric flow from the sump.

4. HYBRID IDENTIFICATION

The identification process used in this paper is divided into two steps. First, hybrid identification of the system is performed, using data generated with the grinding simulator, which is tuned with industrial data. Then, the identified variables are transformed into a Mixed Logical Dynamical (MLD) environment, for HMPC strategies through the Toolbox HYSDEL (HYbrid Systems DEscription Language) developed by Jost and Torrisi (2002). As mentioned in Jost and Torrisi (2002), the MLD framework is a powerful tool for modeling discrete-time linear hybrid systems. Its main favorable feature is its ability to model logical parts of processes and heuristics knowledge about plant operation as integer linear inequalities. This framework allows the convenient modelling using HYSDEL, creating a well suited model for the formulation of HMPC strategies.

The hybrid identification was developed in a manual manner, generating one data set for each of the plant modes. In this case, 6 modes were taken into account. Three simulating the three states (all granulometries, medium-coarse granulometries, and coarse granulometries feeding rates) of the conveyors that represent the stockpile discharge, with both secondary circuits switched ON, and the other three with one of the secondary circuits switched OFF.

The procedure of identification consist on generating an AutoRegresive eXogenous (ARX) model for each of the modes, using the Linear Identification Toolbox of Matlab (Ljung, 2013). The structure of this models can be described as in (8)

$$A(q)y(t) = B(q)u(t-n_k) + e(t) \tag{8}$$

with q as a delay operator, represented as (9):

$$\begin{aligned} A(q) &= 1 + a_1 q^{-1} + ... + a_{n_a} q^{-n_a} \\ B(q) &= b_1 + b_2 q^{-1} + ... + b_{n_b} q^{-n_b+1} \end{aligned} \tag{9}$$

This allows the creating of a model suitable for HMPC, as the ARX models are equivalent to Mixed Logical Dynamical (MLD) models (Heemels et al., 2001) which are widely used in hybrid controllers. The MLD model form and development can be found in Bemporad and Morari (1999). The process of generating the matrices associated with the model was developed using HYSDEL (Jost and Torrisi, 2002) available for Matlab®. The HYSDEL models of the plant were translated from the ARX obtained from the identification, and then used to create the MLD.

5. CONTROL STRATEGIES

In this section a hybrid MPC is developed, showing that the plant can be controlled in different modes. This modes include the switching of the granulometric distribution of the feeding, and the change of ON/OFF state of the secondary grinding line.

Hybrid model predictive control (HMPC) uses models characterized by the interaction of dynamic behaviours, logical rules and operating constraints. Hybrid systems can be represented in a mixed logical dynamical (MLD) environment by a set of mixed integer inequalities, that is, inequalities that include states, inputs and auxiliary variables that may be continuous and/or discrete (Bemporad and Morari, 1999). An MLD system is completely represented by the following set of equations:

$$\begin{aligned} x(t+1) &= Ax(t) + B_1 u(t) + B_2 \delta(t) + B_3 z(t) \\ y(t) &= Cx(t) + D_1 u(t) + D_2 \delta(t) + D_3 z(t) \\ E_2\delta(t) + E_3 z(t) &\leq E_1 u(t) + E_4 x(t) + E_5 \end{aligned} \tag{10}$$

where x, y and u are the state, output and input of the system expressed by:

$$x = \begin{bmatrix} x_c \\ x_\ell \end{bmatrix}, x_c \in \mathbb{R}^{n_c}, x_\ell \in \{0,1\}^{n_\ell} \tag{11}$$

$$y = \begin{bmatrix} y_c \\ y_\ell \end{bmatrix}, y_c \in \mathbb{R}^{p_c}, y_\ell \in \{0,1\}^{p_\ell} \tag{12}$$

$$u = \begin{bmatrix} u_c \\ u_\ell \end{bmatrix}, u_c \in \mathbb{R}^{m_c}, u_\ell \in \{0,1\}^{m_\ell} \tag{13}$$

Also, $\delta \in \{0,1\}^{r_\ell}$ is the auxiliary logical variable and $z \in \mathbb{R}^{r_c}$ the auxiliary continuous variable.

The hybrid model predictive control problem can therefore be stated as follows (Bemporad and Morari, 1999):

$$\begin{aligned} &\min_{\{u_0^{N-1}, \delta_0^{N-1}, z_0^{N-1}\}} \sum_{k=0}^{N-1} \|u(k) - u_e\|^2_{\mathcal{Q}_1} + \|\delta(k|t) - \delta_e\|^2_{\mathcal{Q}_2} + \\ &+ \|z(k|t) - z_e\|^2_{\mathcal{Q}_3} + \|x(k|t) - x_e\|^2_{\mathcal{Q}_4} + \|y(k|t) - y_e\|^2_{\mathcal{Q}_5} \end{aligned} \tag{14}$$

subject to:

$$\begin{aligned} x(T|t) &= x_e \\ x(k+1|t) &= Ax(k|t) + B_1 u(k) + ... \\ &... + B_2\delta(k|t) + B_3 z(k|t) \\ y(k|t) &= Cx(k|t) + D_1 u(k) + ... \\ &... + D_2\delta(k|t) + D_3 z(k|t) \\ E_2\delta(k|t) + E_3 z(k|t) &\leq E_1 u(k) + E_4 x(k|t) + E_5 \end{aligned} \tag{15}$$

where $u_0^{N-1} = \{u(0),\ldots,u(N-1)\}$, $\mathcal{Q}_1 = \mathcal{Q}_1' > 0$, $\mathcal{Q}_2 = \mathcal{Q}_2' \geq 0$, $\mathcal{Q}_3 = \mathcal{Q}_3' \geq 0$, $\mathcal{Q}_4 = \mathcal{Q}_4' > 0$, $\mathcal{Q}_5 = \mathcal{Q}_5' \geq 0$, $\|x\|_{\mathcal{Q}}^2 = x'\mathcal{Q}x$, $x(k|t) \triangleq x(t+k, x(t), u_0^{k-1}, \delta_0^{k-1}, z_0^{k-1})$, and $\delta(k|t)$, $z(k|t)$, $y(k|t)$ are defined in similar manner. The prediction $x(k|t)$ represents the state future value at $t+k$ given the state information at time t and the future values of the optimization variables.

Assuming the solution to the problem exist, the receding horizon control strategy can be applied, by setting $u(t) = u^*(0)$, disregarding the optimal sequence for future periods and repeating the same procedure for $t+1$. The HMPC problem can be solved using mixed integer quadratic programming (MIQP). From (15) we have

$$x(k|t) = A^k x_0 + \sum_{i=0}^{k-1} A^i \left[B_1 u(k-1-i|t) + B_2\delta(k-1-i|t) + B_3 z(k-1-i|t)\right] \quad (16)$$

Then we add (16) to the objective function and the constraints, and define the vectors in (17)

$$\Omega \triangleq \begin{bmatrix} u(0) \\ \vdots \\ u(T-1) \end{bmatrix}, \Psi \triangleq \begin{bmatrix} \delta(0) \\ \vdots \\ \delta(T-1) \end{bmatrix}, \Xi \triangleq \begin{bmatrix} z(0) \\ \vdots \\ z(T-1) \end{bmatrix}, \mathcal{V} \triangleq \begin{bmatrix} \Omega \\ \Psi \\ \Xi \end{bmatrix} \quad (17)$$

The problem formulated by (14) and (15) can be rewritten as in (Bemporad and Morari, 1999)

$$\begin{aligned} &\min_{\mathcal{V}} \mathcal{V}' S_1 \mathcal{V} + 2(S_2 + x_0' S_3)\mathcal{V} \\ &\text{subject to: } F_1 \mathcal{V} \leq F_2 + F_3 x_0 \end{aligned} \quad (18)$$

6. RESULTS

6.1 Hybrid Identification

The hybrid identification procedure was developed for the two controlled variables, specific energy consumption and product particle size (defined as the percentage of product retained by a 65 mesh sieve). Data was obtained with the industrial data tuned grinding simulator, with a sample time of 3.6 seconds. The variables taken into account in the identification process were the feed rate of the controlled conveyors, the water feeding the sump, the SAG mill speed, and the product hardness. Each set of data was generated by changing the inputs on a PRBS sequence.

The results are shown in Fig.3 and Fig. 4. As it can be seen, the identification procedure successfully represents the real data. Each set was generated by changing the granulometric distribution of the product and switching OFF one of the secondary grinding lines.

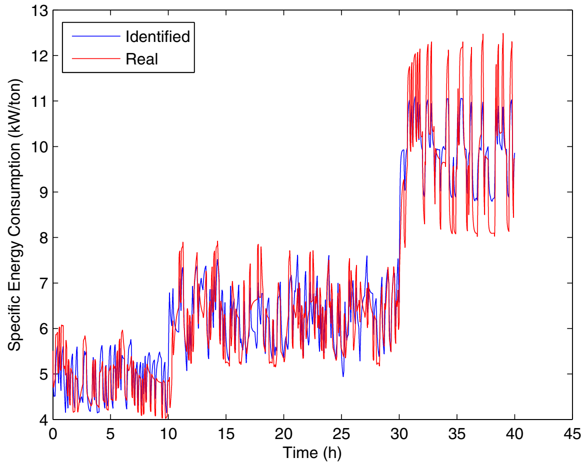

Fig. 3. Specific Energy Consumption

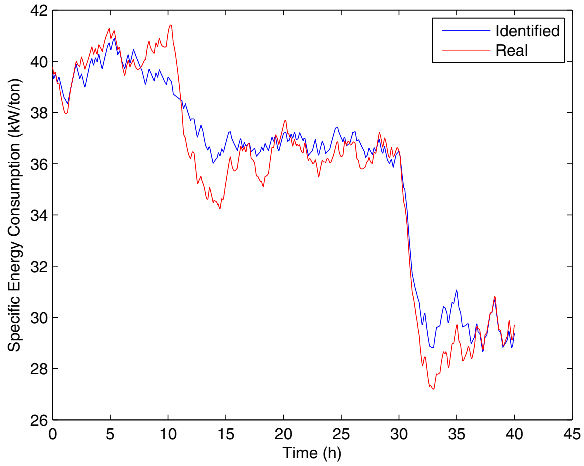

Fig. 4. Product Particle Size

6.2 Hybrid Model Predictive Control

On this work, two different control scenarios were tested. The first one consist on simulating the stockpile discharge behaviour by turning the conveyors ON/OFF. The results for this scenario are shown in Fig. 5, Fig. 6, and Fig. 7.

The stockpile granulometric distribution feeding was switched every 5 hours. This feeding was switched accordingly to the next: on the first 5 hours full granulometric distribution (fine, medium, and coarse), from 5 to 10 hours medium-coarse granulometric distribution, from hour 10 to 15 only coarse granulometric distribution, from hour 15 to 20 the state was returned to medium-coarse granulometric distribution, and on the last 5 hours the stockpile state was returned to its original value. From 10 to 15 hours it is noticeable that the controller could not follow the reference, the reason of this is because of the physical restrictions taken into account in the HMPC strategy. On the other states, the controller successfully followed the reference, and the energy consumption was minimized.

The second scenario consists in simulating the secondary line activation/deactivation. The results for this scenario are shown in Fig. 8, Fig. 9, and Fig. 10. The reference value

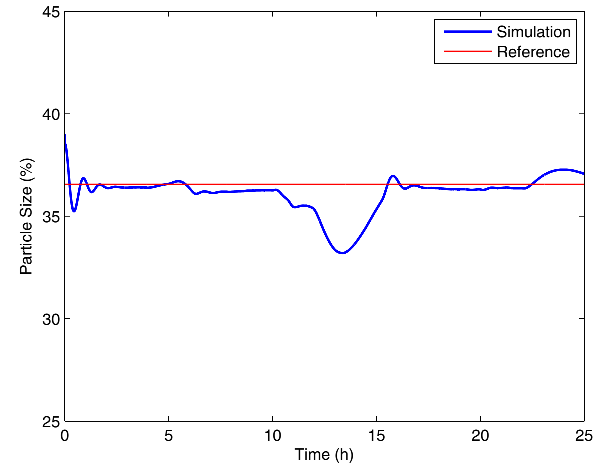

Fig. 5. Scenario 1: Particle Size

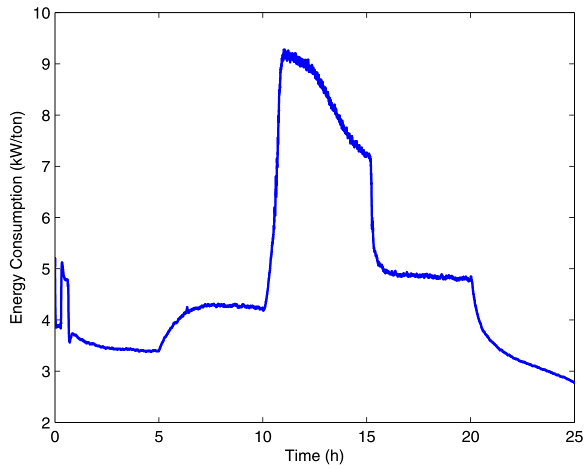

Fig. 6. Scenario 1: Energy Consumption

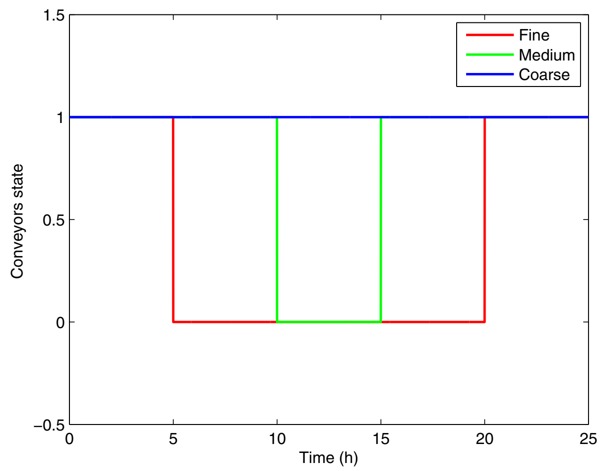

Fig. 7. Scenario 1: Stockpile

was changed when turning ON the secondary grinding line, since if maintained, the physical restrictions of the variables would not have allowed a successful result. For this scenario, the controller successfully follows the given

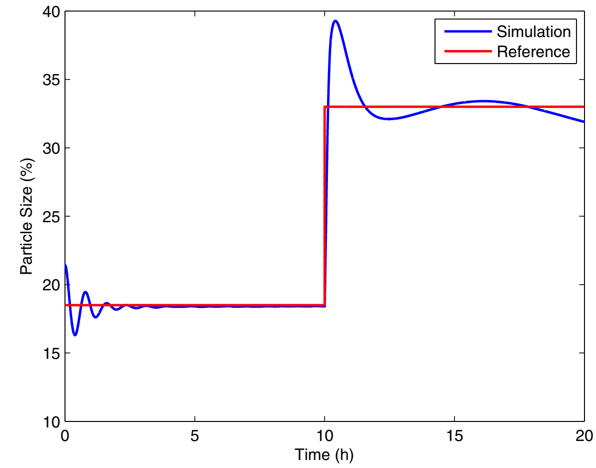

Fig. 8. Scenario 2: Particle Size

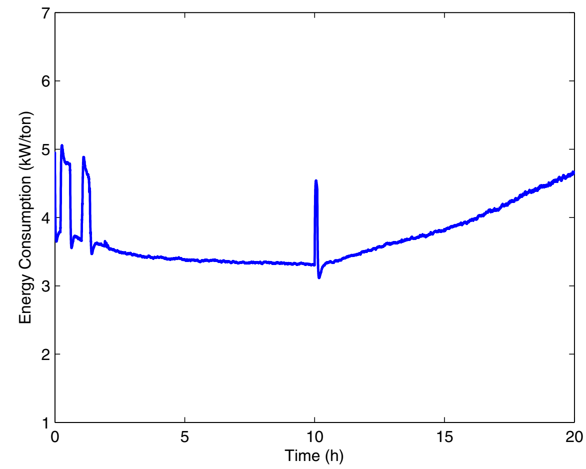

Fig. 9. Scenario 2: Energy Consumption

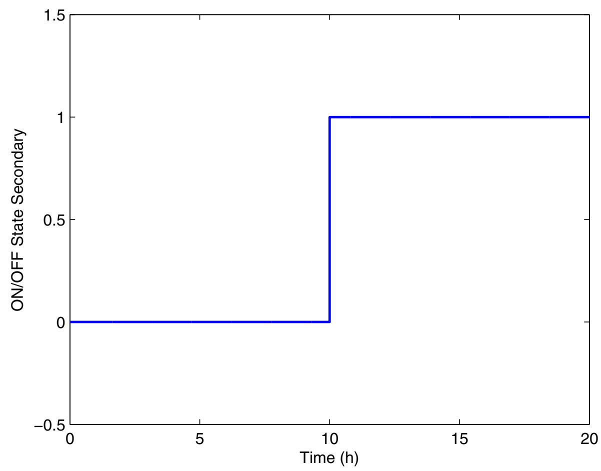

Fig. 10. Scenario 2: Secondary Grinding Stage

reference on both cases, with the secondary grinding circuit turned ON and OFF.

7. CONCLUSIONS AND FUTURE WORK

This paper presents the identification of piecewise ARX models for mineral grinding and the design of a hybrid model predictive controller using of the identified hybrid plant. The plant consist of a primary and secondary grinding circuits, involving the modelling of the stockpile discharge to the plant and the ON/OFF state of the secondary grinding lines.

The hybrid identification procedure was performed by driving the plant to each one of its operating modes, generating data sets with PRBS input sequences and using identification software to develop ARX models. These models were then converted to MLD systems, a representation that is suitable for the design of a HMPC control strategy. Two case scenarios were tested with the designed controller, in each one of them the results were successful. On the controller design the physical restrictions of the variables were included, and the scenarios were tested by switching the discrete events of the model.

The procedure used in this paper offers a highly systematized methodology for the analysis, modelling and development of hybrid model predictive controllers, suitable for complex applications such as grinding circuits.

As future work, this control strategy will be compared to commonly used control strategies used in the mineral grinding profess such as centralized model predictive control (non hybrid) and descentralized model predictive control.

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