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State Feedback Controller Design for A Class of Linear Switched Systems with Uncontrollable Subsystems ⋆

Jie Zhang * **Xingxuan Wang** **

* *Department of Electronic Engineering, Fudan University, Shanghai, (e-mail: 11210720037@fudan.edu.cn).*
** *Department of Electronic Engineering, Fudan University, Shanghai, (e-mail: wxx07@fudan.edu.cn)*

Abstract: This paper investigates the state feedback controller design for a class of linear switched systems with uncontrollable subsystems. After stating the problem discussed in this paper and some preliminaries, we design a controller that makes the given linear switched closed-loop system states converging to zero and show its effectiveness by analyzing the system states norm. Furthermore, we interpret that the presented controller design can be turned into a LMIs(linear Matrices Inequations) feasible problem which can be solved via existing softwares. In addition, an illustrative numerical example is presented to demonstrate the utility of the proposed state feedback controller.

Keywords: Linear system, switched system, uncontrollable subsystem, controller design, state feedback controller, LMIs.

1. INTRODUCTION

Switched systems consist of a finite number of subsystems. And there are logical rules that orchestrate switching between these subsystems. Such systems are common across a diverse range of application areas. For example, switched systems modeling plays a major role in the field of power systems where interactions between continuous dynamics and discrete events are an intrinsic part of power system dynamic behavior. One convenient way to classify switched systems is based on the dynamics of their subsystems, for instance, continuous-time or discrete-time, linear or nonlinear and so on. In this paper, we focus on the stability of a class of linear switched systems, that is, their subsystems are linear systems.

In recent years, switched systems have attracted a growing interest(see Serres et al.(2011), Mitra et al.(2001), Lin et al.(2009), Zhai et al.(2000), and Zhao et al.(2012) and references therein). For example, sufficient conditions for the convergence to zero of the trajectories of linear switched systems are investigated in Serres et al.(2011). A collection of results that use weak dwell-time, dwell-time, strong dwell-time, permanent and persistent activation hypothesis are provided. Mitra et al.(2001) addresses the issue of structural stability results of switched linear systems and provide sufficient and non-conservative results for stability of such systems. Lin et al.(2009) briefly surveys recent results in the field of stability analysis and switching stabilization for switched systems. The stability and stabilization problems for a class of switched linear systems with mode-dependent average dwell time are investigated by Zhao et al.(2012). However, most of these papers focus on the properties of linear switched systems with subsystems that have similar structures and properties. Noting that in realistic applications, the subsystems of a switched system may be very different. Here we consider the stability problem for a class of linear switched systems that some of the subsystems are uncontrollable and the others are controllable. In particular, we consider the problem of designing a feedback controller for the given linear switched systems whose switch consequences and switch instants are fixed. The reason for considering uncontrollable subsystems and linear switched systems with fixed switch consequences and switch instants is theoretical as well as the fact that such models cannot be avoided in many applications. Specially, in some applications, the switch instants and switch consequences are essential properties of systems and cannot be changed by us.

We note that there are also a few papers that study linear switched systems consisting of different subsystems. For instance, in Zhai et al.(2000), the authors study the stability properties of linear switched systems consisting of both Hurwitz stable and unstable subsystems using an average dwell time approach. Then they derive a switching law that incorporates an average dwell time approach so that the switched system is exponentially stable. However, the widely-used average dwell time approach aims to specify activation time period ratio of different subsystems and it is obvious that such average dwell time approach is hard to be applied here because of the fixed switch consequence and switch instants. Furthermore, the fact that common quadratic Lyapunov functions for all subsystems exist only in a few situations also leads to the difficulty in our controller design. Hence, we consider to design in another way.

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Inspired primarily by the works in Amato et al.(2006) and Amato et al.(2001), instead of analyzing a Lyapunov-like function or state norms as the most general ways do, in our controller design we analyze a scalar consequence which denotes the state bounds imposed on a given scalar function $x^{\mathrm{T}}(t)Rx(t),\ t \geq 0$ where $x(t),\ t \geq 0$ denotes the system states and R is a positive defined matrix. Specially, in Amato et al.(2006) a sufficient condition for the design of a dynamic output feedback controller with which the linear closed-loop system states do not exceed a certain threshold of a given bound during a given time interval is presented, and it inspires us designing a controller that guarantees the system states inside required bounds over given time intervals. We consider different control strategies for different kinds of subsystems and we design a state feedback controller which makes the closed-loop system states converging to zero although over some time intervals the system states are not convergent, but inside given bounds. And by analyzing the system state norms, we show its effectiveness. Furthermore, we show that the design of such controllers can be turned into a LMIs(linear Matrices Inequations) feasible problem which can be solved via existing softwares(for example the LMI Control Toolbox of MATLAB™) and it is not a hard problem in realistic applications. Finally, an illustrative numerical example is presented to demonstrate the utility of the proposed state feedback controller.

The contents of the paper are as follows. In Section 2 we state the problem discussed here and some preliminaries. In Section 3 we present our main results, including design of a state feedback controller that makes the given linear switched closed-loop system states converging to zero and showing its effectiveness by analyzing the system states norm. In Section 4, an illustrative numerical example is presented to demonstrate the utility of the proposed controller. Finally, in Section 5 we draw some conclusions.

The notation used in this paper is fairly standard. Specifically, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the set of $n \times 1$ real column vectors, $\mathbb{I}$ denotes the set of integers, $\mathbb{N}_0$ denotes the set of nonnegative integers, $(\cdot)^{\mathrm{T}}$ denotes transpose. Furthermore, we write $\mathrm{d}x$ for the differential of x, $V^{'}(x)$ for the *Fréchet* derivative of V at x, $\| \cdot \|$ for a vector norm, $\| \cdot \|_F$ for the Frobenius matrix norm, $(\cdot)^{\dagger}$ for Moore-Penrose inverse, $\lambda_{max}(\cdot)$ (resp., $\lambda_{min}(\cdot)$) for the maximum (resp., minimum) eigenvalue of a Hermitian matrix.

2. PROBLEM STATEMENT AND PRELIMINARIES

In this paper we consider the linear switch system $\mathcal{G}$ described by

$$\dot{x}(t) = A_i x(t) + Bu(t),\ x(t_0) = x_0,\ t_i \leq t < t_{i+1},\ i \in \bar{\mathbb{Z}}_+, \tag{1}$$

where $x(t) \in \mathbb{R}^n, t \geq t_0$ is the state vector, $u(t) \in \mathbb{R}^m, t \geq t_0$ is the control input, $A_i \in \mathcal{A}$ and $B \in \mathbb{R}^{n\times m}$ are known matrices where $\mathcal{A} = \{\Gamma_j | \Gamma_j \in \mathbb{R}^{n\times n},\ j = 1, 2, \ldots, p\}$. And some of $[\Gamma_j, B],\ j = 1, 2, \ldots, p$ are controllable, while the others are not controllable. $t_i,\ i \geq 1$ denotes the time instant when ith switch happens. Without loss of generality, assume that $[\Gamma_j, B],\ j = 1, 2, \ldots, r$ are not controllable where $1 \leq r < p$ and the control input $u(\cdot)$ in (1) is restricted to the class of *admissible controls* consisting of measurable functions. Moreover, the number of switch times can be either infinite or finite. Furthermore, for the given linear switch system $\mathcal{G}$ we assume that the required properties for the existence and uniqueness of solutions are satisfied. In addition, we assume that the system state $x(t), t \geq t_0$ is available for feedback.

Here let $T_i = t_{i+1} - t_i \geq 0,\ i \in \bar{\mathbb{Z}}_+,\ \mathcal{S}_1 = \{\Gamma_j | \Gamma_j \in \mathbb{R}^{n\times n},\ j = 1, 2, \ldots, r\}$, and $\mathcal{S}_2 = \{\Gamma_j | \Gamma_j \in \mathbb{R}^{n\times n},\ j = r+1, r+2, \ldots, p\}$.

Our goal here is to develop a control input $u(t),\ t \geq t_0$ such that $\| x(t) \| \to 0$ as $t \to \infty$.

Remark 1 It is very important to note that the matrix pairs $[A_i, B],\ i \in \bar{\mathbb{Z}}_+$ may be not controllable, that is, there does not exist $K \in \mathbb{R}^{m\times n}$ such that $A_i + BK$ is stable. The uncontrollable matrix pairs compound the difficulty in applying some off-the-shelf control laws and make it complex to analyze a Lyapunov function. This is because of the system may not be Lyapunov stable if $A_i \in \mathcal{S}_1,\ i \in \bar{\mathbb{Z}}_+$. As a result, the time derivative of a Lyapunov function can not be nonpositive for all $t \geq 0$ with a widely-used control law given by $u(t) = K(t)x(t),\ t \geq t_0$.

Problem 1 Given the linear switch system $\mathcal{G}$ described by (1), develop a control input $u(t) = u_i(t),\ t_i \leq t < t_{i+1}$ such that $\| x(t) \| \to 0$ as $t \to \infty$.

However, *Problem 1* may not be solvable in some situations. For example, if there exists $l > 0$ such that $A_i \in \mathcal{S}_1$ for all $i \geq l$, there does not exist $K(t),\ t \geq t_l$ such that the control input $u(t) = K(t)x(t),\ t \geq t_l$ guarantees the closed-loop system stable. This is result from the fact that the system given by

$$x(t) = A(t)x(t) + Bu(t),\ x(t_l) = x_l,\ t \geq t_l,$$
$$A(t) = A_i,\ t_i \leq t < t_{i+1},\ i \geq l,$$

is uncontrollable. Hence, we need another two assumptions to ensure that *Problem 1* is solvable.

Assumption 1 $\mathcal{S}_2 \neq \emptyset$.

Assumption 2 If the number of switch times is infinite, for any $l \in \bar{\mathbb{Z}}_+$, there exist $q > l,\ q \in \bar{\mathbb{Z}}_+$ such that $A_q \in \mathcal{S}_2$. If the number of switch times is finite, $A_\rho \in \mathcal{S}_2$, where ρ denotes the number of switch times.

Remark 2 *Assumption 1* is equivalence to the condition $r < p$ in the definition of $\mathcal{S}_1$. Hence, it is satisfied. And it is appropriate in realistic applications. In addition, *Assumption 2* is also appropriate in realistic applications. It is equivalence to the condition that there doesn't exist $\rho_1 \in \bar{\mathbb{Z}}_+$ such that $[A_i, B]$ is uncontrollable for all $i \geq \rho_1$.

Here we consider the problem of developing a piecewise continuous control input given by $u(t) = u_i(t) = K_i x(t),\ t_i \leq t < t_{i+1}$ such that $\| x(t) \| \to 0$ as $t \to \infty$. And the controller design is divided in two situations. In the first situation, $A_i \in \mathcal{S}_1,\ i \in \bar{\mathbb{Z}}_+$. Since there does not exist $K \in \mathbb{R}^{m\times n}$ such that $A_i + BK,\ i \in \bar{\mathbb{Z}}_+$ is stable, our goal here is to design a control input such that the system state does not exceed a certain threshold of a given bound during this time interval. In the second situation, $A_i \in \mathcal{S}_2,\ i \in \bar{\mathbb{Z}}_+$. And our goal here is design a control input that guarantees the system state converge to a given bound at the end of this time interval. And it is important to note that the control strategy here is different from the classic control law that guarantees Lyapunov asymptotic stable of the closed-loop system.

The next theorem is needed for the statement of our main results presented in the next section.

Theorem 1. Consider the linear system given by

$$\dot{x}_f(t) = A_f x_f(t) + B_f u_f(t),\ x_f(t_{f0}) = x_{f0},\ t \geq t_{f0}, \quad (2)$$

where $x_f(t) \in \mathbb{R}^{n_f}, t \geq t_{f0}$ is the state vector, $u_f(t) \in \mathbb{R}^{m_f}, t \geq t_{f0}$ is the control input, $A_f \in \mathbb{R}^{n_f\times n_f}$ and $B_f \in \mathbb{R}^{n_f\times m_f}$ are known matrices. For three given positive scalars c_{f1}, c_{f2}, T_f, with $c_{f1} < c_{f2}$, and a given positive define matrix R_f such that

$$x_{f0}^{\mathrm{T}} R_f x_{f0} \leq c_{f1}, \quad (3)$$

if there exist a nonnegative scalar α_f, a positive definite matrix $Q_f \in \mathbb{R}^{n_f\times n_f}$ and a matrix $N_f \in \mathbb{R}^{m_f\times n_f}$ such that

$$A_f\tilde{Q}_f + \tilde{Q}_f A_f^{\mathrm{T}} + B_f N_f + N_f^{\mathrm{T}} B_f^{\mathrm{T}} - \alpha_f \tilde{Q}_f < 0, \quad (4)$$

$$\mathrm{cond}(Q_f) < \frac{c_{f2}}{c_{f1}} e^{-\alpha_f T_f}, \quad (5)$$

$$\tilde{Q}_f = R_f^{-\frac{1}{2}} Q_f R_f^{-\frac{1}{2}}, \quad (6)$$

where $\mathrm{cond}(Q_f) = \lambda_{max}(Q_f)/\lambda_{min}(Q_f)$ denotes the condition number of Q_f, the linear system is FTS with respect to $(c_{f1}, c_{f2}, T_f, R_f)$, that is, there exist

$$x_f^{\mathrm{T}}(t) R_f x_f(t) < c_{f2},\ \forall t \in [t_{f0}, t_{f0} + T_f], \quad (7)$$

with a state feedback controller given by

$$u_f(t) = K_f x_f(t),\ t \geq t_{f0}, \quad (8)$$

where $K_f = N_f \tilde{Q}_f^{-1}$.

Proof. It is cited from Amato et al.(2006) and is a direct consequence of *Theorem 5* in Amato et al.(2006), hence, is omitted.

Remark 3 It is very important to note that $[A_f, B_f]$ is not required controllable here. Furthermore, the closed-loop linear system presented in *Theorem 1* may not be Lyapunov asymptotic stable. And *Theorem 1* provides a sufficient condition for designing a state feedback controller such that once a time interval is fixed, the system state does exceed some bounds during this time interval.

3. MAIN RESULTS

In this section, we develop a control input $u(t) = u_i(t) = K_i x(t),\ t_i \leq t < t_{i+1}$ such that $\| x(t) \| \to 0$ as $t \to \infty$. In the situation that $A_i \in \mathcal{S}_1,\ i \in \bar{\mathbb{Z}}_+$, our goal here is to design a state feedback controller such that $x^{\mathrm{T}}(t)Rx(t) < \gamma_{i2},\ \forall t \in [t_i, t_{i+1})$ with $x^{\mathrm{T}}(t_i)Rx(t_i) \leq \gamma_{i1}$ where $\gamma_{i1},\ \gamma_{i2}$ are given positive scalars such that $\gamma_{i2} > \gamma_{i1} > 0$ and R is a given positive definite matrix. And in the situation that $A_i \in \mathcal{S}_2,\ i \in \bar{\mathbb{Z}}_+$, our goal here is to design a state feedback controller such that $x^{\mathrm{T}}(t_{i+1})Rx(t_{i+1}) \leq \gamma_{i3}$ where γ_{i3} is a given positive scalar.

The following theorem presents a control law that guarantees the stable of the linear switch system $\mathcal{G}$ given by (1).

Theorem 2. Consider the linear switch system $\mathcal{G}$ given by (1), and assume that *Assumption 1* and *Assumption 2* hold. In addition, assume there exist a positive definite matrix R and a positive scalar sequence $\{c_0, c_1, c_2, \ldots\}$ such that $x_0^{\mathrm{T}} R x_0 \leq c_0$ and the given positive definite matrix R and the given positive scalar sequence $\{c_0, c_1, c_2, \ldots\}$ satisfy the following conditions:

(I) If $A_i \in \mathcal{S}_1,\ i \in \bar{\mathbb{Z}}_+$, there exist nonnegative scalars $\alpha_i,\ i \in \bar{\mathbb{Z}}_+$ and positive definite matrices $Q_i \in \mathbb{R}^{n\times n},\ i \in \bar{\mathbb{Z}}_+$ and matrices $N_i \in \mathbb{R}^{m\times n},\ i \in \bar{\mathbb{Z}}_+$ such that

$$A_i\tilde{Q}_i + \tilde{Q}_i A_i^{\mathrm{T}} + BN_i + N_i^{\mathrm{T}} B^{\mathrm{T}} - \alpha_i \tilde{Q}_i < 0,\ A_i \in \mathcal{S}_1,\ i \in \bar{\mathbb{Z}}_+, \quad (9)$$

$$\mathrm{cond}(Q_i) < \frac{c_{i+1}}{c_i} e^{-\alpha_i T_i},\ i \in \bar{\mathbb{Z}}_+, \quad (10)$$

$$\tilde{Q}_i = R^{-\frac{1}{2}} Q_i R^{-\frac{1}{2}},\ i \in \bar{\mathbb{Z}}_+, \quad (11)$$

where $\mathrm{cond}(Q_i) = \lambda_{max}(Q_i)/\lambda_{min}(Q_i),\ i \in \bar{\mathbb{Z}}_+$ denotes the condition number of $Q_i,\ i \in \bar{\mathbb{Z}}_+$;

(II) If $A_i \in \mathcal{S}_2,\ i \in \bar{\mathbb{Z}}_+$, there exist matrices $F_i \in \mathbb{R}^{m\times n},\ i \in \bar{\mathbb{Z}}_+$ and positive scalars $\beta_i,\ i \in \bar{\mathbb{Z}}_+$ such that

$$R^{-\frac{1}{2}} A_i^{\mathrm{T}} + R^{-\frac{1}{2}} F_i^{\mathrm{T}} B^{\mathrm{T}} + A_i R^{-\frac{1}{2}} + BF_i R^{-\frac{1}{2}} + \beta_i I_{n\times n} < 0,\ A_i \in \mathcal{S}_2,\ i \in \bar{\mathbb{Z}}_+, \quad (12)$$

$$\frac{c_i}{c_{i+1}} < e^{T_i \beta_i},\ i \in \bar{\mathbb{Z}}_+. \quad (13)$$

In addition, if the number of switch times is infinite, there exists $\lim_{j\to+\infty} c_j = 0$. Then with the state feedback controller given by $u(t) = u_i(t) = K_i x(t),\ t_i \leq t < t_{i+1}$ where

$$K_i = \begin{cases} N_i \tilde{Q}_i^{-1}, & \text{if } A_i \in \mathcal{S}_1,\ i \in \bar{\mathbb{Z}}_+, \\ F_i, & \text{if } A_i \in \mathcal{S}_2,\ i \in \bar{\mathbb{Z}}_+, \end{cases} \quad (14)$$

the closed-loop linear switch system is stable, that is, $\| x(t) \| \to 0$ as $t \to \infty$.

Proof. Firstly, we proof that

$$x^{\mathrm{T}}(t_i) R x(t_i) \leq c_i,\ i \in \bar{\mathbb{Z}}_+. \quad (15)$$

Noting that (15) holds for $i = 0$, assume that (15) holds for all $i \leq N$ where $N \geq 0,\ N \in \bar{\mathbb{Z}}_+$.

If $A_N \in \mathcal{S}_1$, it follows from $\alpha_N > 0,\ T_N > 0,\ c_N > 0$ and (10) that

$$c_{N+1} > c_N \frac{\lambda_{max}(Q_N)}{\lambda_{min}(Q_N)} e^{\alpha_N T_N} > c_N. \quad (16)$$

Since $x^{\mathrm{T}}(t_N) R x(t_N) \leq c_N$ and $T_N = t_{N+1} - t_N$, it follows from (9)-(11), (16) and *Theorem 1* that

$$x^{\mathrm{T}}(t) R x(t) \leq c_{N+1},\ \forall t \in [t_N, t_{N+1}],\ A_N \in \mathcal{S}_1, \quad (17)$$

which implies that (15) holds for $i = N + 1$ if $A_N \in \mathcal{S}_1$.

If $A_N \in \mathcal{S}_2$, it follows from (1),(12), and (14) that

$$x(t_{N+1}) = x(t_{N+1}^-) = e^{T_N(A_i + BF_i)} x(t_N). \quad (18)$$

Noting that $x^{\mathrm{T}}(t_N) R x(t_N) \leq c_N$, (13) and (18) implies that

$$
\begin{aligned}
&x^{\mathrm{T}}(t_{N+1})Rx(t_{N+1})\\
&= x^{\mathrm{T}}(t_N)e^{T_N(A_N+BF_N)^{\mathrm{T}}}Re^{T_N(A_N+BF_N)}x(t_N)\\
&= x^{\mathrm{T}}(t_N)e^{T_N(R^{\frac{1}{2}}A_N+R^{\frac{1}{2}}BF_N)^{\mathrm{T}}}e^{T_N(R^{\frac{1}{2}}A_N+R^{\frac{1}{2}}BF_N)}x(t_N)\\
&= x^{\mathrm{T}}(t_N)e^{T_N[(R^{\frac{1}{2}}A_N+R^{\frac{1}{2}}BF_N)^{\mathrm{T}}+R^{\frac{1}{2}}A_N+R^{\frac{1}{2}}BF_N]}x(t_N)\\
&= x^{\mathrm{T}}(t_N)e^{T_N R^{\frac{1}{2}}[R^{-\frac{1}{2}}A_N^{\mathrm{T}}+R^{-\frac{1}{2}}F_N^{\mathrm{T}}B^{\mathrm{T}}+A_NR^{-\frac{1}{2}}+BF_NR^{-\frac{1}{2}}]R^{\frac{1}{2}}}x(t_N)\\
&< x^{\mathrm{T}}(t_N)e^{T_N R^{\frac{1}{2}}[-\beta_N I_{n\times n}]R^{\frac{1}{2}}}x(t_N) = e^{-T_N\beta_N}x^{\mathrm{T}}(t_N)Rx(t_N)\\
&< e^{-T_N\beta_N}c_N < c_{N+1},\ A_N\in\mathcal{S}_2.
\end{aligned}
\tag{19}
$$

Obviously, (19) implies that (15) holds for $i = N + 1$ if $A_N \in \mathcal{S}_2$.

Since (15) also holds for $i = N + 1$, it is straightforward that (15) holds for all $i \in \bar{\mathbb{Z}}_+$.

Next, we proof that $\| x(t) \| \to 0$ as $t \to \infty$.

If the number of switch times is finite, it is obvious that $A_\rho + BK_\rho$ is Hurwitz stable where ρ denotes the number of switch times. Since $x(t) = e^{(A_\rho+BK_\rho)(t-t_\rho)}x(t_\rho)$ holds for all $t \geq t_\rho$, it is straightforward that $\| x(t) \| \to 0$ as $t \to \infty$.

Then we consider the situation when the number of switch times is infinite. Noting that $x(t),\ t \geq t_0$ is continuous, it follows from (15) and the definition of the positive scalar sequence $\{c_0, c_1, c_2, \ldots\}$ that there exists a continuous function $c : \mathbb{R} \to \mathbb{R}$ such that $c(t_i) = c_i,\ i \in \bar{\mathbb{Z}}_+$ and

$$0 \leq x^{\mathrm{T}}(t)Rx(t) \leq c(t),\ t \geq t_0. \tag{20}$$

Since $\lim\limits_{j\to+\infty} c_j = 0$, it follows that

$$\lim_{t\to+\infty} c(t) = 0. \tag{21}$$

It follows from (20) and (21) that

$$\lim_{t\to+\infty} x^{\mathrm{T}}(t)Rx(t) = 0, \tag{22}$$

which implies that $\| x(t) \| \to 0$ as $t \to \infty$, that is, the closed-loop linear switch system is stable.

This completes the proof.

Remark 4 Since $[A_i, B],\ i \in \bar{\mathbb{Z}}_+$ is controllable if $A_i \in \mathcal{S}_2,\ i \in \bar{\mathbb{Z}}_+$, (12) is feasible.

Remark 5 It is very important to note that there does not exist such positive scalar sequence $\{c_0, c_1, c_2, \ldots\}$ described in *Theorem 2* if one of *Assumption 1* and *Assumption 2* does not hold. Hence *Assumption 1* and *Assumption 2* are necessary conditions for the existence of the given positive scalar sequence $\{c_0, c_1, c_2, \ldots\}$.

Remark 6 The positive scalar sequence $\{c_0, c_1, c_2, \ldots\}$ denotes the state bounds imposed on the given scalar function $x^{\mathrm{T}}(t)Rx(t),\ t \geq 0$. If the number of switch times is infinite, the condition $\lim\limits_{j\to+\infty} c_j = 0$ is necessary for the convergence of the system states because the existence of uncontrollable subsystems may collapse the stability of the systems if the state bounds do not convergent. Furthermore, this condition is not very hard to be satisfied in realistic applications. For example, in some industry applications, the system switches in a circular order, that is, there exists $\theta \in \bar{\mathbb{Z}}_+$ such that $A_{i+\theta} = A_i,\ i \in \bar{\mathbb{Z}}_+$. Choosing appropriate gains according to the given control laws, one can guarantees that $c_\theta = \lambda c_0$ where $0 \leq \lambda < 1$. Then there exists $c_{i+\theta} = \lambda c_i,\ i \in \bar{\mathbb{Z}}_+$, which implies that $\lim\limits_{j\to+\infty} c_j = 0$.

Remark 7 As mentioned in Amato et al.(2006), (9)-(11) can be turned into LIMs by using some simple algebra. Furthermore, (12)-(13) can be turned into LMIs in this way, too. Therefore the design of our state feedback controller presented here, once values for $\alpha_i,\ \beta_i,\ i \in \bar{\mathbb{Z}}_+$ are fixed, is an LMI feasibility problem which can be solved via existing software(for example the LMI Control Toolbox of MATLAB$^{\mathrm{TM}}$). In addition, the parameter search may be necessary here. Nevertheless this does not represent a hard computational problem.

4. ILLUSTRATIVE NUMERICAL EXAMPLE

In this section we present a numerical example to demonstrate the utility of the proposed state feedback controller. Specially, consider the linear switched system given by

$$\dot{x}(t) = A_ix(t) + Bu(t),\ x(t_0) = x_0,\ t_i \leq t < t_{i+1},\ i \in \bar{\mathbb{Z}}_+, \tag{23}$$

where $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$, $x_0 = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} -2 \\ -5 \end{bmatrix}$ and $A_{2j} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = R,\ A_{2j+1} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix},\ j \in \bar{\mathbb{Z}}_+,\ B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Obviously, since $rank[B, A_{2j}B] = 1 < 2,\ rank[B, A_{2j+1}B] = 2,\ j \in \bar{\mathbb{Z}}_+,\ [A_{2j}, B],\ j \in \bar{\mathbb{Z}}_+$ are uncontrollable and $[A_{2j+1}, B],\ j \in \bar{\mathbb{Z}}_+$ are controllable. Here we consider the problem of utilizing the presented state feedback controller with time intervals given by $T_{2j} = 1$(sec), $T_{2j+1} = 3$(sec), $j \in \bar{\mathbb{Z}}_+$.

The controller design is divided in two steps. Firstly, we design the feedback gain $K_{2j} = N_{2j}Q_{2j}^{-1},\ j \in \bar{\mathbb{Z}}_+$. Since $c_0 \geq x_0^{\mathrm{T}}Rx_0 = 29$, we define $c_0 = 29$. Then we fix $\alpha_{2j} = 3 > 0,\ j \in \bar{\mathbb{Z}}_+$, solve the inequations given by (9)-(11). We obtain

$$
\begin{gathered}
N_{2j} = [1, 1],\ Q_{2j} = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix},\ Q_{2j}^{-1} = \begin{bmatrix} 0.6 & -0.4 \\ -0.4 & 0.6 \end{bmatrix},\\
K_{2j} = N_{2j}\tilde{Q}_{2j}^{-1} = [0.2, 0.2],\ j \in \bar{\mathbb{Z}}_+,
\end{gathered}
\tag{24}
$$

with

$$
\begin{gathered}
\lambda_{max}(Q_{2j}) = 5,\ \lambda_{min}(Q_{2j}) = 1,\\
c_{2j+1} = 100.5c_{2j} > 5e^3c_{2j} = \mathrm{cond}(Q_{2j})e^{\alpha_{2j}T_{2j}},\ j \in \bar{\mathbb{Z}}_+.
\end{gathered}
$$

Next, solving the inequations given by (12)-(13) with fixed $\beta_{2j+1} = 2,\ j \in \bar{\mathbb{Z}}_+$, obtain

$$F_{2j+1} = [-12, 6],\ A_{2j+1} + BF_{2j+1} = \begin{bmatrix} -10 & 6 \\ -12 & 7 \end{bmatrix},\ j \in \bar{\mathbb{Z}}_+, \tag{25}$$

with

$$
\begin{gathered}
\lambda_{max}(A_{2j+1}+BF_{2j+1}) = -1,\ \lambda_{min}(A_{2j+1}+BF_{2j+1}) = -2,\\
c_{2j+2} = \frac{c_{2j+1}}{403} > e^{-6}c_{2j+1} = c_{2j+1}e^{-T_{2j+1}\beta_{2j+1}},\ j \in \bar{\mathbb{Z}}_+\\
.
\end{gathered}
$$

It is very important to note that

$$\lim_{j\to+\infty} c_{2j} = 0,\ \lim_{j\to+\infty} c_{2j+1} = 0$$

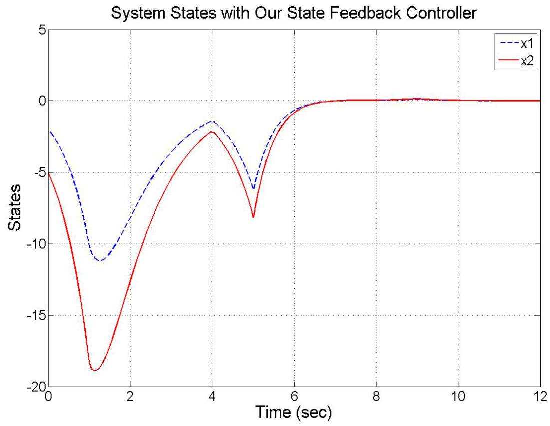

Fig. 1. States trajectories versus time with our state feedback controller

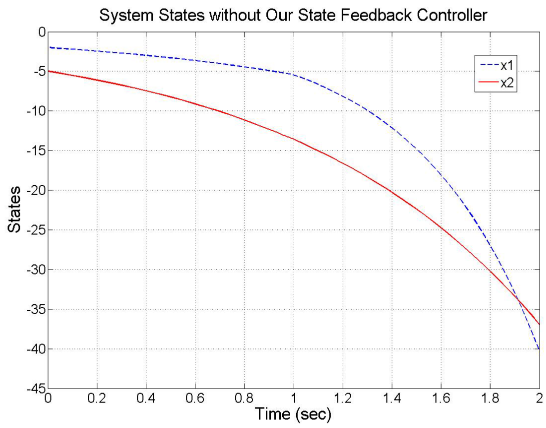

Fig. 2. States trajectories versus time without our state feedback controller

since $c_{2j+2} = \frac{100.5}{403} c_{2j}$, $c_{2j+3} = \frac{100.5}{403} c_{2j+1}$, $j \in \bar{\mathbb{Z}}_+$. It implies that $\lim_{j \to +\infty} c_j = 0$. Hence, the state feedback controller given by

$$K_{2j} = [0.2, 0.2], K_{2j+1} = [-12, 6], \ j \in \bar{\mathbb{Z}}_+ \tag{26}$$

is feasible.

The system states trajectories versus time with our state feedback controller is shown in Fig.1. And Fig.2 shows states trajectories versus time without our state feedback controller. Obviously, the system states trajectories in Fig.2 diverge in a short time. The system states trajectories versus time in Fig.1 converge to 0. And it is obvious that the system switches at t=1(sec),4(sec),5(sec) in Fig.1. However, since the $\| x(t) \|$, $t \geq t_0$ is sufficiently small when t≥7(sec), the switches at t=8(sec) and t=9(sec) is not obvious.

Table 1. The values for c_j, $j \in \bar{\mathbb{Z}}_+$

c_0	c_1	c_2	c_3	c_4	c_5	c_6	...
29	2914.5	7.232	726.8	1.803	181.25	0.450	...

Next, define the states norm as $V(t) = \| x(t) \|_R = x^{\mathrm{T}}(t) R x(t)$, $t \geq t_0$. Fig.3 shows the states norm trajectories versus time with our state feedback controller. Noting the definition of c_j, $j \in \bar{\mathbb{Z}}_+$, obtain the values for c_j, $j \in \bar{\mathbb{Z}}_+$ given in TABLE I.

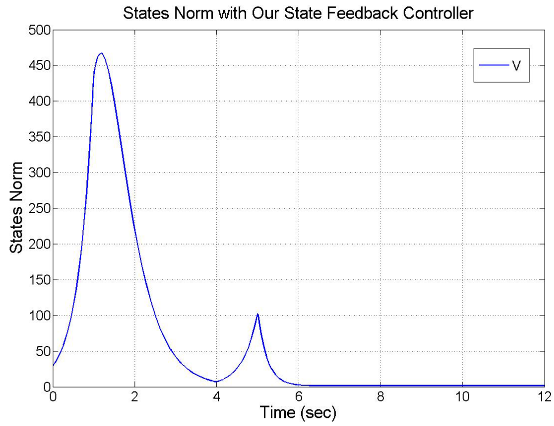

Fig. 3. States norm trajectories versus time with our state feedback controller

Fig.3 shows that the states norm $V(t)$, $t \geq t_0$ satisfies the conditions given by

$V(t) < c_{2j+1}$, $\forall t \in [t_{2j}, t2j+1)$, $V(t_{2j}) \leq c_{2j}$, $j = 0, 1, 2$.

However, since we do not impose any bound on the state norm over the time intervals $[t_{2j+1}, t_{2j})$, $j \in \bar{\mathbb{Z}}_+$, the states norm may be very large over these intervals. This may lead to some undesirable phenomena in realistic applications. The state feedback controller with sates norm bound for all $t \geq t_0$ is included in our future work.

5. CONCLUSION

In this paper we investigate the state feedback controller design for a class of linear switched systems with uncontrollable subsystems. And we design a controller that makes the given linear switched closed-loop system states converging to zero and show its effectiveness by analyzing the system states norm. Furthermore, we interpret that the presented controller design can be turned into a LMIs feasible problem which can be solved via existing softwares(for example the LMI Control Toolbox of MATLAB$^{\mathrm{TM}}$) and it is not a hard problem in realistic applications. In addition, an illustrative numerical example is presented to demonstrate the utility of the proposed state feedback controller.

As mentioned in Section 4, the state feedback controller with sates norm bound for all $t \geq t_0$ is included in our future work. Furthermore, the problem of designing the controller for nonlinear switched uncertain systems with uncontrolled subsystems is worth investigating.

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