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Disturbance-adaptive stochastic optimal control of energy harvesters, with application to ocean wave energy conversion ⋆

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Abstract: This paper proposes a theory for optimizing the power generated from stationary stochastic vibratory disturbances, using a resonant energy harvester. Although the theory is general, the target application of the paper concerns ocean wave energy harvesting. The control technique involves the use of a causal discrete-time feedback algorithm to dynamically optimize the power extracted from the waves. The theory assumes that the input impedance of the converter is known precisely, but that *a priori* models are unavailable for the characterization of the stochastic behavior of the waves, as well as their hydrodynamic excitation of the system. For these assumptions, we develop an adaptive control technique, which iteratively re-optimizes the feedback law for the controller based on recursive subspace identification of the stochastic disturbance dynamics. The technique is demonstrated on a simulation example pertaining to a cylindrical surface-floating wave energy converter in heave.

Keywords: Energy harvesting, wave energy, adaptive regulation, subspace identification

1. INTRODUCTION

It has long been recognized that control theory can be used to optimize the power generated by ocean wave energy converters [Evans, 1981, Falnes, 2002, Salter et al., 2002, Falc̃ao, 2010]. The determination of the optimal controller for a wave energy converter (WEC) system is predicated on knowledge of its dynamic behavior, as well as a characterization of the sea state to which it is to be subjected. For WECs with linear dynamic models, control designs typically presume harmonic waves, and are designed according to the same network-theoretic impedance-matching principles used in the design and operation of antenna arrays and waveguides [Falnes, 1980].

However, true sea states are stochastic, with standardized power spectra (such as Pierson-Moskowitz or JONSWAP spectra [Faltinsen, 1990]) which exhibit significant available energy over a nontrivial band of frequencies. For such cases, controllers derived via impedance matching theory must impose a feedback law which is the Hermitian adjoint (i.e., complex-conjugate transpose) of the driving-point impedance matrix for the WEC, at all frequencies [Nebel, 1992]. Such controllers are always anticausal, and thus require some anticipatory technique in which present decisions are made with future wave information. This can be accomplished, for example, with the use of deployable wave elevation sensors.

Alternatively, WEC controllers can be optimized subject to the constraint of causality. It was recently shown in [Scruggs et al., 2013] that under the assumptions of linear dynamics, a stationary stochastic sea state, and unconstrained generator controllability, the optimal WEC control problem is a special case of the Linear Quadratic Gaussian (LQG) control problem, which has a well-known solution. The optimal causal controller has a number of features (besides, of course, causality) that differentiate it from the optimal anticausal controller. Most importantly, while the optimal anticausal controller does not depend on the power spectrum of the sea state, the optimal causal controller does. Moreover, the optimal causal controller also depends on the hydrodynamic forcing functions for the WEC; i.e., the transfer functions that characterize the mapping from wave elevation to system forces.

In most realistic applications of control to wave energy conversion, there will be uncertainty about the nature of the wave excitation, both in terms of its stochastic spectrum, as well as its propagatory direction. Causal controllers that are optimized under an assumed disturbance model, which is markedly different from the true disturbance, may perform quite poorly – so much so that they may exhibit negative average power generation. It is therefore essential that causal controllers be capable of accommodating disturbance model uncertainties, either through robust control techniques, adaptation, or some combination of both.

In this paper, we consider the design of controllers that are disturbance-adaptive; i.e., which presume a precise *a priori* model for the driving point impedance of the

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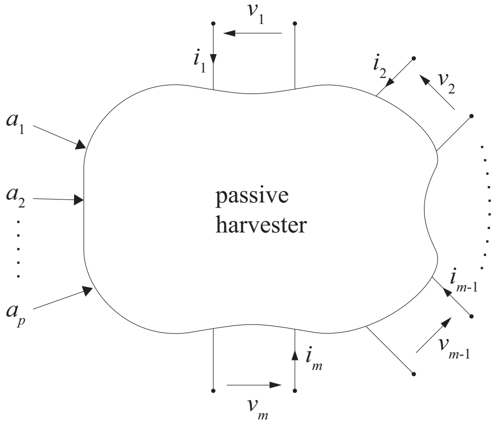

Fig. 1. Diagram of a general energy harvesting system

system (i.e., the transfer function from current to voltage), and which identifies a stochastic disturbance model in real time, from response data. This approach falls into a class of control theory which is sometimes called *adaptive regulation*, to imply the situation in which the plant model is assumed to be known, but the controller must be made to adapt to unknown or variable disturbance characteristics [Landau et al., 2011]. The approach taken in this paper accomplishes adaptation indirectly; i.e., by identifying a stochastic model for the disturbance, and then re-optimizing the feedback law under the assumption of certainty-equivalence.

For the majority of the paper, we develop the theory for general energy harvesting systems, as shown in Figure 1. In this diagram, i and $v \in \mathbb{R}^m$ are the colocated current and voltage vectors for each transducer port of a generalized harvester system, and $a \in \mathbb{R}^p$ is the vector of exogenous disturbances, presumed to be stationary stochastic processes. The control algorithm determines the current vector i to apply at each time, based on present and past measurements for v. Although the theory can be extended to accommodate other feedback measurements besides v, we will not pursue this here. It is also worth noting that for some technologies, such as hydraulic power take-off systems, it makes more sense to think about control of mechanical colocated quantities instead of electrical quantities. In such circumstances the theory here may still be applied; in this case the control variable would become the force (or torque) of the power take-off device, which would be determined as a function of the linear (or rotational) velocity over which it acts.

The notation used throughout the paper is mostly standard. However, here we note some terms that may not be entirely commonplace. The sets $\mathbb{R}_{\geq 0}$ and $\mathbb{Z}_{\geq 0}$ refer to the sets of nonnegative real numbers and integers, respectively. We refer to a square matrix A as Hurwitz if each of its eigenvalues has negative real component. We refer to a square matrix A as contractive if each of its eigenvalues has modulus less than 1. The notation $\mathrm{tr}\{A\}$ refers to the trace of A. For a Hermitian matrix Q, we use notation $Q > 0$ and $Q \geqslant 0$ to denote positive definiteness and positive semidefiniteness, respectively. Analogous notation is used for negative definiteness and semidefiniteness. For a matrix A, we denote the transpose and complex-conjugate-transpose as A^T and A^H, respectively. For a vector x and (Hermitian) matrix $Q > 0$ of compatible dimension, $\|x\|_Q^2 = x^H Q x$. For a random variable u, we denote its expectation as $\mathcal{E}\{u\}$. Finally the set ℓ_2 denotes the set of all infinite, discrete-time sequences $\{..., x_{-1}, x_0, x_1, ...\}$ that are square-summable; i.e., $\sum_{k=-\infty}^{\infty} \|x_k\|_2^2 < \infty$.

2. THE DISCRETE-TIME ENERGY HARVESTING PROBLEM

2.1 Modeling assumptions

We assume the energy harvesting system can be modeled as the linear dynamical system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} i + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} w \tag{1a}$$

$$\begin{bmatrix} v \\ a \end{bmatrix} = \begin{bmatrix} C_1 & C_2 \\ 0 & E_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} R \\ 0 \end{bmatrix} i \tag{1b}$$

where $x \in \mathbb{R}^n$ is the state vector, and $w \in \mathbb{R}^{n_w}$ is the white noise sequence that generates a, through the disturbance model. Note that state vector x has been partitioned such that $x_2 \in \mathbb{R}^{n_a}$ is the largest subspace which is uncontrollable from i. We refer to this subspace as comprised of the *disturbance states* of the system. The remaining states $x_1 \in \mathbb{R}^{n_h}$ are referred to as the *harvester states* of the system [1]. We assume w has unit spectral intensity; i.e., that $\mathcal{E}w(t)w^T(\tau) = I\delta(t-\tau)$. The above model implicitly assumes that the mapping $w \mapsto a$ is strictly proper, implying that a has finite variance. The above model also makes the assumption that $w \mapsto v$ is strictly proper. To simplify the presentation, we will assume a single harvesting transducer; i.e., $m = 1$. However, all the techniques discussed in this paper extend easily to multi-transducer systems, merely at the expense of more elaborate notation.

The transfer function $Z_1 : i \to v$, equal to

$$Z_1(s) \triangleq C_1 \left[sI - A_{11}\right]^{-1} B_1 + R \tag{2}$$

is called the *input impedance* of the harvester; i.e., it is the driving point impedance across the terminals of the transducer. Assuming the harvester is dissipative (i.e., is stable, contains no internal energy sources and no undamped modes) then we may presume $Z(s)$ to be positive-real in the weakly-strict sense (WSPR) [Brogliato et al., 2007]. This condition implies any of the three equivalent conditions:

(1) $\exists\ W \in \mathbb{R}^{n_h \times n_h}$ with $W = W^T \geqslant 0$ such that $(A_{11}, \sqrt{W})$ is observable and
$$\begin{bmatrix} A_{11}^T W + W A_{11} & W B_1 - C_1^T \\ B_1^T W - C_1 & -2R \end{bmatrix} \leqslant 0 \tag{3}$$
(2) A_{11} is Hurwitz, and $\int_0^\infty i(t)v(t)dt \geqslant 0$, $\forall i \in \mathcal{L}_2$.
(3) A_{11} is Hurwitz, and $Z(s) + Z^H(s) \geqslant 0$, $\forall \Re\{s\} \geqslant 0$.

As shown by Scruggs [2010], the WSPR condition is necessary in order for the optimal energy harvesting control problem to be well-posed. If it does not hold, then the

[1] Note that it is possible to achieve further specificity in the above dynamical model by choosing a basis resulting in $G_1 = 0$; i.e., a basis in which a is determined solely from x_2, which then partitions x into orthogonal subspaces corresponding to physical and disturbance models. However, such a partitioning is unnecessary.

optimal feedback law to maximize harvested energy is destabilizing.

We presume that the current $i(t)$ is controlled in discrete-time, and mapped to continuous-time via a zero-order-hold D/A conversion; i.e.,

$$i(t) = i_k \quad , \quad t \in \mathcal{T}_k \tag{4}$$

where $\mathcal{T}_k = [kT, (k+1)T)$ and where T is the sample time. There then exists a discrete-time sampled system with sample times $\mathcal{T}_s = \{..., -T, 0, T, ...\}$, of the form

$$\begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ 0 & \Phi_{22} \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix} + \begin{bmatrix} \Upsilon_1 \\ 0 \end{bmatrix} i_k + \varpi_k \tag{5a}$$

$$v_k = [C_1 \ C_2] \begin{bmatrix} x_{1,k} \\ x_{2,k} \end{bmatrix} + R i_k \tag{5b}$$

where $x_k = x(kT)$ and $v_k = v(kT)$, $\varpi_k \in \mathbb{R}^n$ is a discrete-time white noise sequence with zero mean and $\mathcal{E}\varpi_k \varpi_k^T = \Omega$. The discrete-time parameter matrices are

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} \\ 0 & \Phi_{22} \end{bmatrix} = \exp\left\{ \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix} T \right\} \tag{6}$$

$$\Upsilon_1 = A_{11}^{-1} \left(\exp\{A_{11} T\} - I \right) B_1 \tag{7}$$

To determine the appropriate covariance matrix for ϖ_k, first define $x_o(t)$ as the open-circuit state response; i.e., the component of the response of (1) due to w. Then the stationary covariance matrix $\mathcal{E}\{x_o(t) x_o^T(T)\} \triangleq X$ is the solution to the continuous-time Lyapunov equation

$$0 = AX + XA^T + GG^T \tag{8}$$

and $G = \left[G_1^T \ G_2^T \right]^T$. For the discrete-time system, we choose a noise covariance matrix $\mathcal{E}\{\varpi\varpi^T\} \triangleq \Omega$ that results in the same state covariance matrix for the discrete-time open-circuit state; i.e., $\mathcal{E}\{x_{o,k} x_{o,k}^T\} = X$. This implies that Ω is

$$\Omega = X - \Phi X \Phi^T \tag{9}$$

The instantaneous continuous-time power is

$$P_{gen}(t) = -i(t)v(t) \tag{10}$$

In stationary stochastic response, we denote the mean power generated by the harvester by an overbar; i.e.,

$$\bar{P}_{gen} = -\mathcal{E}\{iv\} \tag{11}$$

In discrete time, the feedback system maps $\mathcal{V}_k \mapsto i_k$, where

$$\mathcal{V}_k = \{v_{k-1}, v_{k-2}, ...\}. \tag{12}$$

Consequently, i_k is uncorrelated with the residual $r_{k|k-1} = x_k - \mathcal{E}\{x_k|\mathcal{V}_k\}$. For $t \in \mathcal{T}_k$, the unbiased estimate for $P_{gen}(t)$ over $t \in \mathcal{T}_k$, given $\mathcal{V}_k$, is therefore

$$\begin{aligned} &\hat{P}_{gen}(t|\mathcal{V}_k) && (13)\\ &\triangleq \mathcal{E}\{P_{gen}(t)|\mathcal{V}_k\} && (14)\\ &= -i_k \mathcal{E}\{v(t)|\mathcal{V}_k\} && (15)\\ &= -i_k C \exp\{A(t-kT)\} \mathcal{E}\{x_k|\mathcal{V}_k\} \\ &\quad - i_k \left(R i_k + CA^{-1} \left(\exp\{A(t-kT)\} - I \right) B i_k \right) && (16) \end{aligned}$$

Consequently, the expected mean power generation over interval $t \in \mathcal{T}_k$, given $\mathcal{V}_k$, is

$$\begin{aligned} \hat{\bar{P}}_{gen}(k|\mathcal{V}_k) &\triangleq \frac{1}{T} \int_{kT}^{(k+1)T} \hat{P}_{gen}(t|\mathcal{V}_k) dt && (17)\\ &= -i_k F \mathcal{E}\{x_k|\mathcal{V}_k\} - D i_k^2 && (18) \end{aligned}$$

where F is

$$F = \frac{1}{T} C \int_0^T \exp\{At\} dt \triangleq [F_1 \ F_2] \tag{19}$$

where

$$\begin{aligned} F_1 &= C_1 A_{11}^{-1} \tfrac{1}{T} (\Phi_{11} - I) && (20)\\ F_2 &= C_1 A_{11}^{-1} \tfrac{1}{T} \Phi_{12} + (C_2 - C_1 A_{11}^{-1} A_{12}) A_{22}^{-1} \tfrac{1}{T} (\Phi_{22} - I) && (21) \end{aligned}$$

and

$$\begin{aligned} D &= R + \frac{1}{T} C \int_0^T A^{-1} \left(\exp\{At\} - I \right) B dt && (22)\\ &= R + C_1 A_{11}^{-1} \left(\tfrac{1}{T} \Upsilon_1 - B_1 \right) && (23) \end{aligned}$$

Finally, $\bar{P}_{gen}$ is then obtained as the expectation of $\hat{\bar{P}}_{gen}(k|\mathcal{V}_k)$ over $\mathcal{V}_k$

$$\begin{aligned} \bar{P}_{gen} &= \mathcal{E}\left\{ -i_k F \mathcal{E}\{x_k|\mathcal{V}_k\} - D i_k^2 \right\} && (24)\\ &= \mathcal{E}\left\{ -i_k F x_k - D i_k^2 \right\} && (25) \end{aligned}$$

It will be advantageous to use an approximation of F_2 which does not require A_{12} and A_{22} to be solved from Φ. We note that if T is small, the integral in (19) can be approximated as the average of the integrand at the boundary values; i.e.,

$$\frac{1}{T} \int_0^T \exp\{At\} dt \approx \frac{1}{2} (\Phi + I) \tag{26}$$

As such, F_2 can be approximated to high accuracy as

$$F_2 \approx \tfrac{1}{2} C_1 \Phi_{12} + \tfrac{1}{2} C_2 (\Phi_{22} + I) \tag{27}$$

We finish this section on modeling assumptions by noting that the continuous- to discrete-time conversion above preserves the WSPR property, as explained in the theorem below. This property is essential to the well-posedness of the optimal discrete-time energy harvesting problem.

Lemma 1. Let $G_1(z)$ be defined as

$$G_1(z) = F_1 [zI - \Phi_{11}]^{-1} \Upsilon_1 + D \tag{28}$$

If $Z_1(s)$ is WSPR in continuous time, then $G_1(z)$ is WSPR in discrete time, which is defined by any the equivalent conditions:

(1) $\exists W = W^T \geqslant 0$ such that $(\Phi_{11}, \sqrt{W})$ is observable and

$$\begin{bmatrix} -W & -F_1^T \\ -F_1 & -2D \end{bmatrix} + \begin{bmatrix} \Phi_{11}^T \\ \Upsilon_1^T \end{bmatrix} W \begin{bmatrix} \Phi_{11}^T \\ \Upsilon_1^T \end{bmatrix}^T \leqslant 0 \tag{29}$$

(2) Φ_{11} is contractive, and $\sum_{k=0}^{\infty} i_k v_k \geqslant 0, \quad \forall i_k \in \ell_2$

(3) Φ_{11} is contractive, and $G_1(z) + G_1^H(z) \geqslant 0, \forall |z| \geqslant 1$.

2.2 Optimal discrete-time energy harvesting

We seek a discrete-time feedback law $\mathcal{V}_k \mapsto i_k$ which maximizes $\bar{P}_{gen}$. Toward this end, we have the following theorem. In the interest of brevity, this theorem is presented without proof. However, it is the direct analogy for discrete-time, of the theory presented by Scruggs [2010] for continuous time.

Theorem 1. Let $\phi : \mathcal{V}_k \to i_k$ be any stabilizing feedback law, and assume the $G_1(z)$ is WSPR. Then in stationary discrete-time response,

$$\bar{P}_{gen} = \bar{P}_{gen}^{max} - \mathcal{E}\|i - Kx\|_\Delta^2 \tag{30}$$

where $P_{gen}^{max} = -\mathrm{tr}\{P\Omega\}$, $\Delta = D + \Upsilon^T P \Upsilon$, $K = -\Delta^{-1} \left[\frac{1}{2} F + \Upsilon^T P \Phi \right]$, and P is the unique stabilizing solution to the discrete-time Riccati equation

$$P = \Phi^T P \Phi - K^T \Delta K \tag{31}$$

A few remarks are important regarding this theorem, and its implications:

Certainty-equivalence controllers A class of sub-optimal energy harvesting controllers can be generated via the certainty-equivalence principle, through the use of a stabilizing Luenberger observer; i.e.,

$$\hat{x}_{k+1} = \Phi\hat{x}_k + \Upsilon i_k + L\left(C\hat{x}_k - v_k\right) \tag{32a}$$

$$i_k = K\hat{x}_k \tag{32b}$$

where $\Phi_L = \Phi + LC$ is contractive. Observer matrix L can be designed by any standard technique (e.g., pole placement, etc.). The resultant performance is then

$$\bar{P}_{gen} = \bar{P}_{gen}^{max} - KSK^T\Delta \tag{33}$$

where S is the stationary estimation error covariance; i.e.,

$$S = \Phi_L S \Phi_L^T + \Omega \tag{34}$$

Performance optimization via Kalman filter $\bar{P}_{gen}$ is optimized by minimizing the expectation in (30); i.e., by imposing the feedback law

$$i_k = K\hat{x}_k \tag{35}$$

with prior estimates $\hat{x}_k = \mathcal{E}\{x_k|\mathcal{V}_k\}$ reconstructed via a discrete-time Kalman filter, i.e., by a Luenberger observer with $L = L_0$, the Kalman gain, equal to

$$L_0 = -\Phi S_0 C^T\left(CS_0C^T\right)^{-1} \tag{36}$$

where S_0 is the solution to the discrete-time Riccati equation

$$S_0 = \Phi S_0 \Phi^T + \Omega - \Phi S_0 C^T\left(CS_0C^T\right)^{-1} CS_0\Phi^T \tag{37}$$

The physical limit on generated power (i.e., the value of (30) attained by (35)) is then (33) evaluated with $S = S_0$.

Partitioned solutions to state feedback gains In the above theorem, we note that due to the special structure of Φ and Υ, the solutions to P and K also have special structure. Specifically, we have that if we similarly partition these matrices as $P = \left[\begin{smallmatrix} P_{11} & P_{12} \\ P_{21} & P_{22}\end{smallmatrix}\right]$ and $K = [K_1\ K_2]$ then the parameters $\{P_{11}, K_{11}, \Delta\}$ can be solved independently of P_{12} and P_{22}, and depend only on the parameters of $G_1(z)$; i.e.,

$$P_{11} = \Phi_{11}^T P_{11}\Phi_{11} - K_1^T\Delta K_1 \tag{38}$$

$$K_1 = -\Delta^{-1}\left[\tfrac{1}{2}F_1 + \Upsilon_1^T P_{11}\Phi_{11}\right] \tag{39}$$

$$\Delta = D + \Upsilon_1^T P_{11}\Upsilon_1 \tag{40}$$

With these terms solved, P_{12} is solved via a (linear) Sylvester equation:

$$P_{12} - \bar{\Phi}_{11}^T P_{12}\Phi_{22} = \bar{\Phi}_{11}^T P_{11}\Phi_{12} + \tfrac{1}{2}K_1^T F_2 \tag{41}$$

where $\bar{\Phi}_{11} = \Phi_{11} + \Upsilon_1 K_1$. By the conditions of Theorem 1, we know that this Sylvester equation has a unique solution. (Indeed, this is ensured merely by the fact that $\bar{\Phi}_{11}$ is guaranteed to be contractive.) With P_{12} found, K_2 is a linear function of it, as

$$K_2 = -\Delta^{-1}\left[\tfrac{1}{2}F_2 + \Upsilon_1^T\left(P_{11}\Phi_{12} + P_{12}\Phi_{22}\right)\right] \tag{42}$$

Note that to determine K_1 and K_2, it is not necessary to find P_{22} explicitly.

2.3 Isolating disturbance-dependent feedback terms

For convenience define

$$\mathcal{D} = \{\Phi_{11}, C_1, \Upsilon_1, R\} \tag{43}$$

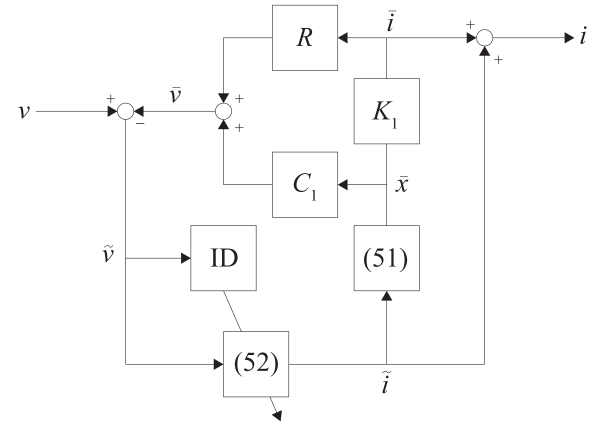

Fig. 2. Block diagram of feedback partitioning

as the state space parameters that depend only on the discrete-time harvester impedance model. In this paper, we consider $\mathcal{D}$ to be known with certainty, and expressed in a convenient (and fixed) state space basis. Now, we define the deterministic feedback system $\bar{H}$ as

$$\bar{H}: \begin{cases} \bar{x}_{k+1} = \Phi_{11}\bar{x}_k + \Upsilon_1 i_k \\ \begin{bmatrix}\bar{v}_k \\ \bar{i}_k\end{bmatrix} = \begin{bmatrix}C_1 \\ K_1\end{bmatrix}\bar{x}_k + \begin{bmatrix}R \\ 0\end{bmatrix} i_k \end{cases} \tag{44}$$

We note that the evolution of $\{\bar{v}, \bar{i}\}$ is deterministic in real-time, because the system parameters are all contained in $\mathcal{D}$ (or, in the case of K_1, derived from $\mathcal{D}$) and its input i_k is known precisely at time k.

We then consider the subtraction of the deterministic system outputs from $\{v_k, i_k\}$, resulting in the perturbations

$$\tilde{v}_k = v_k - \bar{v}_k \qquad \tilde{i}_k = i_k - \bar{i}_k \tag{45}$$

$$\tilde{x}_k = x_k - \begin{bmatrix}\bar{x}_k \\ 0\end{bmatrix} \qquad \hat{\tilde{x}}_k = \hat{x}_k - \begin{bmatrix}\bar{x}_k \\ 0\end{bmatrix} \tag{46}$$

As illustrated in the block diagram in Figure 2, the perturbed system $\tilde{H}$ as

$$\tilde{H}: \begin{cases} \hat{\tilde{x}}_{k+1} = \left(\Phi + LC\right)\hat{\tilde{x}}_k - L\tilde{v}_k \\ \tilde{i}_k = K\hat{\tilde{x}}_k \end{cases} \tag{47}$$

In addition to $\mathcal{D}$, knowledge of $\tilde{H}$ requires the parameter set

$$\mathcal{N} = \{\Phi_{12}, \Phi_{22}, C_2, \Omega\} \tag{48}$$

as well as parameters $\{K_2, L\}$, which is derived from $\{\mathcal{D}, \mathcal{N}\}$. (Depending on the chosen design technique for L, Ω may not need to be known.) In this paper, we presume $\mathcal{N}$ to be uncertain and unmodeled. As such, its contents must be estimated from the system response, to indirectly adapt $\tilde{H}$. We note that in the estimation of $\mathcal{N}$, any basis which preserves the known block structure of Φ and C (i.e, the fact that their first n_h columns are known) is equivalent. Thus, identified parameter sets are equivalent under any similarity transformation T with the block structure

$$T = \begin{bmatrix} I & T_{12} \\ 0 & T_{22}\end{bmatrix} \tag{49}$$

for any T_{12} and any invertible T_{22}.

To eliminate the ambiguity related to the basis of the identified $\mathcal{N}$, it will be useful to refer to the transfer function representation of the mapping $\tilde{H} : \tilde{v} \to \tilde{i}$ directly in terms of its input/output coefficients. Define $\{a_0, ...a_{n-1}\}$

as the coefficients of the characterisitc polynomial for the observer system in (47); i.e.,

$$z^n + \sum_{\ell=1}^{n} a_{n-\ell} z^{n-\ell} = \det \{zI - \Phi - LC\} \quad (50)$$

Then

$$\tilde{H}(z) = \frac{\sum_{\ell=0}^{n-1} b_\ell z^\ell}{z^n + \sum_{\ell=0}^{n-1} a_\ell z^\ell} \quad (51)$$

where the numerator coefficients $\{b_0...b_{n-1}\}$ can be found easily by exciting the state space (47) with the sequence

$$\tilde{v}_k = \begin{cases} 0 & : k < 0 \\ 1 & : k = 0 \\ a_{n-k} & : k \in \{1..n\} \end{cases} \quad (52)$$

which produces the numerator coefficients as the transient output of the state space simulation; i.e.,

$$\tilde{i}_k = \begin{cases} 0 & : k < 1 \\ b_{n-k} & : k \in \{1..n\} \end{cases} \quad (53)$$

Combining $\bar{H}$ and $\tilde{H}$ as in Figure 2 gives the energy harvesting feedback controller in (32) as

$$\bar{x}_{k+1} = (\Phi_{11} + \Upsilon_1 K_1)\,\bar{x}_k + \Upsilon_1 \tilde{i}_k \quad (54)$$

$$\tilde{i}_{k+1} = \sum_{\ell=1}^{n} \left(b_{n-\ell} \tilde{v}_{k-\ell+1} - a_{n-\ell} \tilde{i}_{k-\ell+1} \right) \quad (55)$$

$$\tilde{v}_k = v_k - (C_1 + RK_1)\,\bar{x}_k - R\tilde{i}_k \quad (56)$$

$$i_k = K_1 \bar{x}_k + \tilde{i}_k \quad (57)$$

Although this representation is not minimal for time-invariant $\tilde{H}$, it isolates the disturbance dependent parameters $\{a_\ell, b_\ell,\ \ell = 1..n\}$ and permits them to be adaptively re-evaluated at each time step in response to an update in $\mathcal{N}$.

3. OPTIMAL ADAPTIVE ENERGY HARVESTING

3.1 Covariance realization algorithm for $\tilde{H}$

This subsection overviews the determination of $\mathcal{N}$, assuming certainty of $\mathcal{D}$, and assuming adquate response data to accurately evaluate expectations from time averages. Subspace-based system identification techniques are used [Katayama, 2005]. The primary justification for this is that subspace-based techniques scale well to large system models, and to systems with many transducers. Within the subspace-based paradigm, our objective can be accomplished by any of several related algorithms, most of which are variants of the methods of Faurre [1976] or Akaike [1975]. Although Akaike-based methods offer certain advantages for reliable estimation when the data size is finite, we opt for a Faurre-based technique here. The reason for this is that the technique is computationally more efficient, because it is straight-forward to implement recursively.

The linear stochastic state space characterizing the dynamic response of $\tilde{v}$ due to ϖ is

$$\tilde{x}_{k+1} = \Phi \tilde{x}_k + \varpi_k \quad (58a)$$

$$\tilde{v}_k = C \tilde{x}_k \quad (58b)$$

In stationarity, the state covariance matrix $X = \mathcal{E}\tilde{x}\tilde{x}^T$ is the solution to (9). Define the stationary autocorrelation function R_ℓ, $\ell \in \mathbb{Z}_{\geq 0}$ as

$$R_\ell = \lim_{k\to\infty} \mathcal{E}\tilde{v}_{k+\ell}\tilde{v}_k = C\Phi^\ell X C^T \quad (59)$$

Following the standard Faurre algorithm for stochastic realization, define Hankel matrix $H_{\ell,m}$ as

$$H_{\ell,m} = \begin{bmatrix} R_1 & R_2 & \cdots & R_m \\ R_2 & R_3 & \cdots & R_{m+1} \\ \vdots & \vdots & \ddots & \vdots \\ R_\ell & R_{\ell+1} & \cdots & R_{\ell+m-1} \end{bmatrix} \quad (60)$$

For $\ell, m > n$, define the extended observability and controllability matrices as

$$\mathcal{O}_\ell = \begin{bmatrix} C \\ C\Phi \\ \vdots \\ C\Phi^{\ell-1} \end{bmatrix} \qquad \mathcal{C}_m = \begin{bmatrix} \bar{C} \\ \bar{C}\Phi^T \\ \vdots \\ \bar{C}(\Phi^T)^{m-1} \end{bmatrix}^T \quad (61)$$

where $\bar{C} = CX\Phi^T$. We have that $H_{\ell,m} = \mathcal{O}_\ell \mathcal{C}_m$. It then follows that the singular value decomposition for $H_{\ell,m}$ is

$$H_{\ell,m} = U_\ell \Sigma V_m^T \quad (62)$$

where $U_\ell \in \mathbb{R}^{\ell\times n}$ and $V_m \in \mathbb{R}^{m\times n}$ are orthonormal matrices (i.e., $U_\ell^T U_\ell = V_\ell^T V_\ell = I$) and $\Sigma \in \mathbb{R}^{n\times n}$ is diagonal and positive-definite, its main diagonal comprised of the (nonzero) singular values of $H_{\ell,m}$ in descending order. It then follows that there exists a similarity transformation matrix T such that

$$\mathcal{O}_\ell = U_\ell T \qquad \mathcal{C}_m = T^{-1}\Sigma V_m^T \quad (63)$$

It is then useful to partition $\mathcal{O}_\ell$ as

$$\mathcal{O}_\ell = [\mathcal{O}_{\ell 1}\ \mathcal{O}_{\ell 2}] \quad (64)$$

where $\mathcal{O}_{\ell 1}$ contains the leading n_h columns of $\mathcal{O}_\ell$. Similarly, we partition T as $T = \left[\begin{smallmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{smallmatrix}\right]$, where $T_{11} \in \mathbb{R}^{n_h\times n_h}$. Then because the first n_h columns of Φ and C are $\left[\begin{smallmatrix} \Phi_{11} \\ 0 \end{smallmatrix}\right]$ and C_1, both of which are available *a priori*, $\mathcal{O}_{\ell 1}$ is known *a priori* as well, as

$$\mathcal{O}_{\ell 1} = \begin{bmatrix} C_1 \\ C_1\Phi_{11} \\ \vdots \\ C_1\Phi_{11}^{k-1} \end{bmatrix} \quad (65)$$

Any similarity transformation T that renders $\mathcal{O}_{\ell 1}$ as its *a priori* value will produce a state space realization with the desired partitioning for x_1 and x_2. To assure this, we have equations for T_{11} and T_{21}:

$$U_\ell \begin{bmatrix} T_{11} \\ T_{21} \end{bmatrix} = \begin{bmatrix} C_1 \\ \underline{\mathcal{O}}_{\ell 1} \end{bmatrix} \quad (66)$$

If $H_{\ell,m}$ is known precisely then these equations should permit unique solutions for T_{11} and T_{21}. If $H_{\ell,m}$ is perturbed, then approximate solutions are obtained through the orthogonal projection

$$\begin{bmatrix} T_{11} \\ T_{21} \end{bmatrix} = \left[U_\ell^T U_\ell\right]^{-1} U_\ell^T \mathcal{O}_{\ell 1} = U_\ell^T \mathcal{O}_{\ell 1} \quad (67)$$

Meanwhile, any T_{21} and any invertible T_{22} represent a valid choice for the identified model. As such, we choose $T_{21} = 0$, $T_{22} = I$. This gives us the following equations for C_2, Φ_{12}, and Φ_{22}:

$$\mathcal{O}_{\ell 2} = \begin{bmatrix} C_2 \\ \underline{\mathcal{O}}_{\ell 1}\Phi_{21} + \underline{\mathcal{O}}_{\ell 2}\Phi_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \underline{\mathcal{O}}_{\ell 1} & \underline{\mathcal{O}}_{\ell 2} \end{bmatrix} \begin{bmatrix} C_2 \\ \Phi_{21} \\ \Phi_{22} \end{bmatrix} \quad (68)$$

where $\underline{\mathcal{O}}_\ell$ denotes the truncation of last row of $\mathcal{O}_\ell$. If $H_{\ell,m}$ is known precisely then this equation gives a unique solution for C_2, Φ_{21}, and Φ_{22}. In the presence of perturbations to $H_{\ell,m}$ we can find an approximate solution through the orthogonal projection

$$\begin{bmatrix} C_2 \\ \Phi_{21} \\ \Phi_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \left[\underline{\mathcal{O}}_\ell^T \underline{\mathcal{O}}_\ell\right]^{-1} \underline{\mathcal{O}}_\ell^T \end{bmatrix} \mathcal{O}_{\ell 2} \tag{69}$$

Using the solution above for T, $\mathcal{C}_m$ can then be determined, and from it, $\bar{C}$, as

$$\bar{C} = [1 \;\; 0]\, V_m \Sigma \begin{bmatrix} T_{11} & 0 \\ T_{21} & I \end{bmatrix}^{-T} \tag{70}$$

Technically, to complete the identification of $\mathcal{N}$, Ω must be found. However, for some control design techniques, it may not be necessary to find Ω. This is advantageous because the determination of Ω (or equivalently, the determination of the Kalman gain associated with the innovations model for $\tilde{x}$) is numerically problematic when $H_{\ell,m}$ is not known precisely. Because such methods are unnecessary to the problem at hand, we will not discuss them here.

3.2 Observer gain adaptation

There many ways to adapt the observer gain L, based on the identified $\mathcal{N}$ parameters. One obvious technique would be to set L equal to the Kalman gain, L_0, associated with the identified system. However, this approach is not robust to perturbations in $H_{\ell,m}$, making it unreliable in practice. Although there may be techniques to enhance its reliability, we take a simpler course of action in this paper for adapting L.

It turns out that the following observer gain

$$L = -\Phi\Theta C^T \left(C\Theta C^T + R_0\right)^{-1} \tag{71}$$

where Θ is the solution to Riccati equation

$$\Theta = \Phi\Theta\Phi^T + \bar{C}^T R_0^{-1} \bar{C} - L(C\Theta C^T + R_0)L^T \tag{72}$$

performs extremely well in all case studies examined by the authors in the context of this work, usually resulting in a stationary performance which is within 10 or 15% of the theoretical optimal performance achievable with the Kalman gain. Although the approach is an ad-hoc solution to the determination of a suitable observer gain, the technique does have a few compelling justifications:

- It is realization-independent.
- It can be calculated directly from estimations for Φ, C, and $\bar{C}$, and a solution will not fail to exist as a consequence of perturbations in estimations for these parameters.
- It is straight-forward to show that the above L results in a stable estimation error residual for the optimal $\tilde{i}$, irrespective of the accuracy of Φ, C, and $\bar{C}$.

For the purposes of the present paper, we motivate the use of this technique based on the above justifications. Further theoretical justification of this choice of L will be published in a forthcoming journal paper.

3.3 Recursive identification of $\mathcal{N}$ from data

For time k, define the vectors

$$y_k^+ = [\tilde{v}_{k-\ell+1} \;\; \tilde{v}_{k-\ell+2} \;\; \cdots \;\; \tilde{v}_k]^T \tag{73}$$

$$y_k^- = [\tilde{v}_{k-\ell} \;\; \tilde{v}_{k-\ell-1} \;\; \cdots \;\; \tilde{v}_{k-\ell-m+1}]^T \tag{74}$$

Then in stationary response, we have that $H_{\ell,m} = \mathcal{E} y_k^+ (y_k^-)^T$. For the purposes of using the above-described technique to identify $\mathcal{N}$ at time k from response data $\{\tilde{v}_k, \tilde{v}_{k-1}, ...\}$, we use the sampling approximation

$$H_{\ell,m} \approx \hat{H}_{\ell,m}(k) \tag{75}$$

$$= (1-\beta) \sum_{j=-\infty}^{k} \beta^{k-j} y_j^+ (y_j^-)^T \tag{76}$$

where $\beta \in (0,1)$ is a forgetting factor. The above can be computed recursively as

$$\hat{H}_{\ell,m}(k) = \beta \hat{H}_{\ell,m}(k-1) + (1-\beta) y_k^+ (y_k^-)^T \tag{77}$$

Similarly, R_0 can be estimated as

$$R_0 \approx \hat{R}_0(k) = (1-\beta) \sum_{j=-\infty}^{k} \beta^{k-j} \tilde{v}_j^2 \tag{78}$$

$$= \beta \hat{R}_0(k-1) + (1-\beta)\tilde{v}_k^2 \tag{79}$$

Using these sampled estimations, one can then consider the re-identification of $\mathcal{N}$ via the procedure in Section 3.1 at each time step k, each time using the latest estimates $\hat{H}_{\ell,m}(k)$ and $\hat{R}_0(k)$. Such an approach produces a time-indexed sequence of parameter identifications; i.e., $\hat{\mathcal{N}}(k)$.

Although conceptually straight-forward, this described approach requires some modifications to make it numerically reliable and efficient:

- For stochastic disturbances with high quality factor, estimation errors in $\hat{H}_{\ell,m}(k)$ and $\hat{R}_0(k)$ may result in an estimation $\hat{\Phi}_{22}(k)$ which is not asymptotically stable. To guard against this, we resort to the common ad-hoc practice of reflecting these poles into the unit circle. To make the derivations consistent, this pole reflection is imposed on Θ directly following its evaluation from $\hat{U}_\ell(k)$.
- The requirement to solve singular value decomposition (SVD) in (62) at each time step becomes computationally intensive as the Hankel dimensions $\{\ell, m\}$ are made large. However, larger $\{\ell, m\}$ can also lead to more accurate identifications. In order to make it numerically practical to repeatedly re-solve the SVD, we make use of the iterative subspace-tracking algorithm proposed by Goethals et al. [2004]. Let the true SVD for $\hat{H}_{\ell,m}(k)$ be denoted

$$\hat{H}_{\ell,m}(k) = \hat{U}_\ell(k) \hat{\Sigma}(k) \hat{V}_m^T(k) \tag{80}$$

Then $\hat{U}_\ell(k)$ satisfies

$$\hat{U}_\ell(k) = \underset{W \in \mathbb{R}^{\ell \times n}}{\operatorname{argmin}} \left\| \left(I - WW^T\right) \hat{H}_{\ell,m}(k) \right\|_F^2 \tag{81}$$

Assuming the forgetting factor β to be sufficiently close to 1 such that the changes in $\hat{U}_\ell(k)$ from one iteration to the next are small, the above is approximately equivalent to

$$\hat{U}_\ell(k) = \underset{W \in \mathbb{R}^{\ell \times n}}{\operatorname{argmin}} \left\| \left(I - W\hat{U}_\ell^T(k-1)\right) \hat{H}_{\ell,m}(k) \right\|_F^2 \tag{82}$$

which has the solution

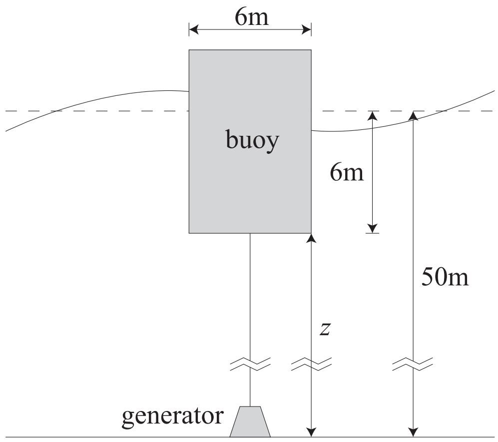

Fig. 3. Diagram of example WEC

$$\hat{U}_\ell(k) \approx \hat{H}_{\ell,m}(k)\left(\hat{U}_\ell^T(k-1)\hat{H}_{\ell,m}(k)\right)^\dagger \tag{83}$$

Implementing this approximation successively produces a sequence $W_\ell(k)$, as

$$W_\ell(k) = \hat{H}_{\ell,m}(k)\left(W_\ell^T(k-1)\hat{H}_{\ell,m}(k)\right)^\dagger \tag{84}$$

for which $\hat{U}_\ell(k) \approx W_\ell(k)$. Additionally, we may approximate $\hat{V}_m(k)\hat{\Sigma}(k) \approx \hat{H}_{\ell,m}^T(k)W_\ell(k)$.

- Ideally, an estimate $\hat{\mathcal{N}}(k)$ would be used immediately to update the controller parameters at time $k+1$. However, in reality this was found not to be viable. The reason is that the values of $\hat{\mathcal{N}}(k)$ are strongly correlated with the inputs to $\tilde{H}$; i.e., $\{\tilde{v}_{k-1}, ...\tilde{v}_{k-n}\}$. This correlation can introduce "hidden" positive feedback into the dynamics of $\tilde{H}$, causing the value of $\tilde{i}$ to destabilize. To remedy this, it was necessary to introduce a delay in the parameter updating, such that the controller at time k is evaluated using $\hat{\mathcal{N}}(k-\delta)$, for $\delta > 0$ being the delay shift. For the example described in the next section, it was found that δ had to be quite large (i.e., around 10^4) in order to avoid instabilities. The reason for this is that, due to extremely low damping in the closed-loop system, the autocorrelation function for $\tilde{v}_k$ decays very slowly.

4. EXAMPLE

Consider the simple single-degree-of-freedom WEC shown in Figure 3, comprised of a cylindrical buoy coupled to a floor-mounted generator through a pre-tensioned tether. We assume that the buoy's response is predominately in heave. For the model parameters chosen, the heave response of the buoy has a natural period of approximately 5.5s.

We presume electromechanical power conversion through the reciprocal relationships

$$f = \kappa i \qquad\qquad e = \kappa \dot{z} \tag{85}$$

where κ is the effective back-EMF constant of the generator, and $v = e + Ri$. The most useful parameter to characterize the generator capability is actually the *short circuit viscosity*, equal to

$$c_e = \kappa^2/R \tag{86}$$

i.e., the effective linear viscous damping imposed on the buoy as a consequence of setting $v = 0$. For our example, we assume the hardware is designed to yield a value of $c_e = 10^5$kg/s. Theoretically, it can be shown that any combination of κ and R producing a given c_e value will perform identically. Thus, we arbitrarily choose $R = 1\Omega$.

To model the mechanical dynamics of the buoy, identical techniques to those reported in [Scruggs et al., 2013] were used. The infinite-dimensional transfer functions mapping $\{a, i\}$ into v were solved, accounting for all linearized fluid-structure interactions. Model reduction techniques were then used to determine high-fidelity finite-dimensional models for these transfer functions. For the simulation, a Pierson-Moskowitz spectrum was assumed, with a mean wave period of 7s, and a significant wave height of 1m. (Because the response of the system will be homogeneous, the particular value of the significant wave height is immaterial to the analysis.) For simulation purposes, this spectrum was approximated by a rational spectrum, and modeled as filtered white noise. The buoy and wave dynamic state space models were then augmented, and a balanced truncation was performed to eliminate common dynamics. The resultant reduced system is six-dimensional, with $n_h = 2$ and $n_a = 4$.

For the adaptive controller, a sample time of $T = 0.5$s was used. For this sample time, the true value of $\bar{P}_{gen}^{max} =$ 2.33kW (for a significant wave height of 1m.) We assumed $n_a = 4$. The forgetting factor β was taken to be such that the system memory had a half-life of 5×10^4 samples; i.e., $\beta = 0.5^{1/(5\times10^4)}$.

Figure 4 shows a transient plot of the output power for this system, over a time duration of 5×10^4s. For the first 10^4s, the identification algorithm is operational but the generator is not (i.e., $i_k = 0$). During this time the algorithm develops a preliminary model for the stochastic disturbance, which is later refined after power generation comes online. It is perhaps undesirable that the algorithm requires such a long duration for initial identification. However, we note that during this duration, $\hat{\mathcal{N}}$ was required to be estimated with no prior information. This stands in contrast to the steady-state adaptation of the controller due to shifting sea state conditions, in which the parameters in $\hat{N}$ are recursively updated.

The true value of $\bar{P}_{gen}$ for the control design (with the sub-optimal observer) is 2.02kW. The actual mean power generation over the interval during which the control loop is closed is 2.09kW, which is well within the estimation error for the simulation data size. Thus we conclude that the adaptive controller converges to its theoretical performance, as desired.

5. CONCLUSIONS

It has only been very recently that the connection between stochastic energy harvesting and optimal stochastic control has been well understood. The fact that the optimal causal feedback law turns out to be the solution to an associated LQG problem, motivates the use of the many existing techniques from this theory in energy harvesting problems. The aim of this paper has been to investigate the implications of indirect adaptive LQG control techniques,

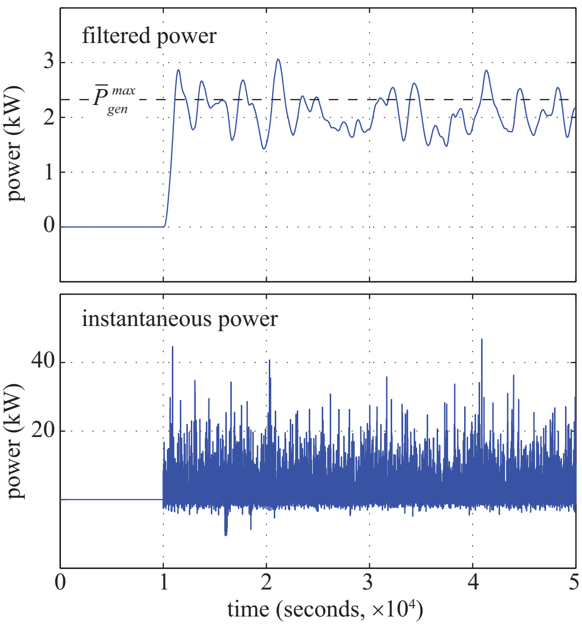

Fig. 4. Simulation of power generation output for example scenario. In the top plot, the power output is lowpass filtered with a cutoff frequency of 1mHz

in this new context. The scope of the paper has been somewhat modest, in that we have focused only on the adaptive *regulation* problem, in which our knowledge of the plant is precise, while the knowledge of the disturbance is limited. However, even for these assumptions, the development of controllers that can be shown to perform close to the theoretical stochastic power generation limit, proved nontrivial. Clearly, the next step is to examine the case in which the plant (i.e., input impedance G_1) is also unknown *a priori*. This case seems likely to be a significantly greater challenge.

We have focused the application of this paper on an ocean wave energy application, for a few reasons. Firstly, ocean energy applications provide strong incentive for the development of optimal control algorithms, as a means of maximizing the available resource. Secondly, it constitutes an application for which the levels of generated power are large enough to offset the parasitic losses associated with the implementation of sophisticated control adaptation algorithms. This places WEC control in contrast to many smaller-scale energy harvesting applications, such as energy scavengers for embedded sensing, which generate only a few milliwatts of power. Nonetheless, the control algorithms discussed in this paper may be of some use in some smaller-scale applications.

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