Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

A Hybrid MPC Strategy Applied to Flotation Process *

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Abstract: Several dynamic models are able to represent the phenomenological behaviour of a flotation process. In addition, the interest in developing control strategies for large-scale processes has led to formulate novel methodologies which allow to consider the global performance of a plant, facilitating the design, validation and evaluation of more complex optimizing control strategies. In this work, we first develop a dynamic hybrid model for flotation which is calibrated with industrial data. Subsequently, a hybrid prediction model is obtained by applying identification techniques to the different scenarios and it is used to formulate a hybrid model predictive control (HMPC) strategy for flotation. Our simulation results show that the proposed methodology is suitable for modelling the behaviour of a flotation process and its control stage. This is achieved minimizing the tail grade of a flotation line considering several operating modes and constrains, providing a robust hybrid model predictive control strategy.

Keywords: Mineral processing, Flotation process, Nonlinear identification, Hybrid systems, Hybrid model predictive control.

1. INTRODUCTION

One of the most common processes of a mineral processing plant is flotation, whose aim is to increase the concentration of valuable mineral with the highest possible selectivity. This process takes place in cells which receive pulp from the grinding stage. Actually, the interest in developing control methodologies for large scale processes has led to propose new approaches for flotation facilitating the design, validation and evaluation of more complex control strategies.

To date, several dynamic models have been developed to represent a flotation cell or a set of them. The simplest are hydraulic models used to evaluate control strategies of pulp levels (Stenlund and Medvedev, 2002), (Kämpjärvi and Jämsä-Jounela, 2003). A second group are the mass balance models based on mineralurgical conditions of one phase (Casali et al., 2002), (Sbarbaro et al., 2008) and two phases (Pérez-Correa et al., 1998), (Rojas and Cipriano, 2010). However, the mentioned models only consider the dynamic evolution of a cell under a single operating mode. Recently, new efforts have been applied to solve this problem developing models for training operators (Ortiz and Toro, 2013), plant simulators (Bergh et al., 2013) and mineralurgical predictions (Putz and Cipriano, 2013) which consider different operating modes of a flotation cell.

Moreover, multivariable predictive control methodologies have been applied to flotation process. Cortés et al. (2008) proposed the application of a control strategy for stabilizing the process, generating increases in the copper recovery. Similarly, Foroush et al. (2009) proposed froth thickness control allowing improvement of concentrate mineralurgical characteristics and Bergh et al. (2013) developed supervisory control strategies.

In this paper, we first develop a dynamic hybrid model for a flotation cell to obtain a rougher simulator for control design validated with industrial data. Then, a hybrid model predictive control (HMPC) strategy for the plant is proposed obtaining hybrid prediction models through identification techniques with data generated by the simulator. This methodology is easily applicable to other flotation stages (regrinding, scavenger and cleaner) and considering different operation lines.

1.1 Plant description

Flotation is a physicochemical separation process widely used in the mining industry. It is based on the characteristics of hidrophobicity of mineral particles that form part of the pulp. These characteristics are generated by the use of reagents (collectors, depressants and foaming) and injection of air into the cell (Wills and Napier-Munn, 2006).

Figure 1 shows the typical configuration of a flotation process. This consists in a set of cells in series grouped into banks connected by controlled valves. Each line is actuated by a discrete valve, which controls the admission of pulp from the grinding stage. The process delivers concentrate and tail where the first is accumulated in a collector tank and sent to a regrinding stage. The second is discarded in tailing dams.

The current control strategies for rougher flotation have focused on the use of PID controllers for pulp levels, configured in local operator stations or in Distributed Control

* This study was funded by the FONDECYT N°1120047, “Distributed Hybrid Model Predictive Control for Mineral Processing”.

Systems (DCS). Additionally, many variables influence this process such as levels of pulp, air flow, rate of froth and bubble size. In the feed flow the critical variables are the mineral law, particle size and percent of solids (Cortés et al., 2008).

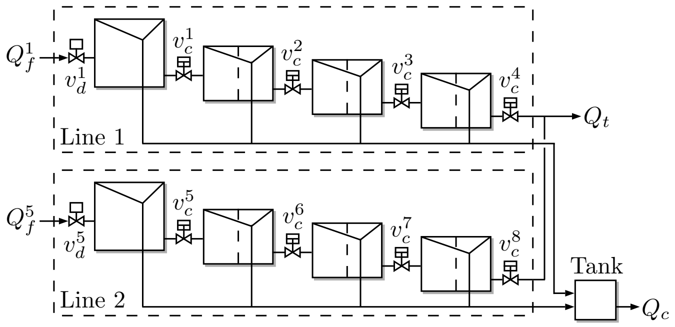

Fig. 1. Plant scheme

2. DYNAMIC HYBRID MODEL

A hybrid system is a dynamic representation whose behaviour is determined by the interaction between continuous and discrete dynamics governed by logical rules (Paoletti et al., 2007). These type of systems allow to represent complex dynamics and its evolution over time involves transitions between different operating modes which is very common in large mineral processing plants.

2.1 Operating modes

The plant evolution through the different operating modes is achieved by using auxiliary logical variables (Bemporad and Morari, 1999). These allow to generate instant changes in the continuous components of the system, represented by differential equations. According to the first approach developed by Putz and Cipriano (2013) which explains a hybrid dynamic model available only for three, each flotation cell is modelled considering a set of auxiliary logical variables and its evolution is determined by the following logical rules:

$$\left[\delta_1^i = 1\right] \Leftrightarrow \left[h_p^i \leq h_c^i\right] \tag{1}$$

$$\left[\delta_2^i = 1\right] \Leftrightarrow \left[h_p^{i+1} \geq \Delta h^i\right] \tag{2}$$

$$\left[\delta_3^i = 1\right] \Leftrightarrow \left[h_p^i + h_e^i \geq h_c^i\right] \tag{3}$$

Where δ_1^i, δ_2^i and δ_3^i are auxiliary logical variables with $i \in \{1, 2, \ldots, n\}$ the cell number, h_p^i the pulp level, h_e^i the froth height, h_c^i the maximum height of the cell i and Δh^i the height difference between the cells i and $i+1$. Figure 2 shows the evolution of the four operating modes of a flotation cell which are: (1) Cell operation without concentrate overflow and without pressure influence of next cell, (2) Cell operation without concentrate overflow but with pressure influence of next cell, (3) Normal cell operation with concentrate overflow and (4) Presence of pulp in the concentrate overflow.

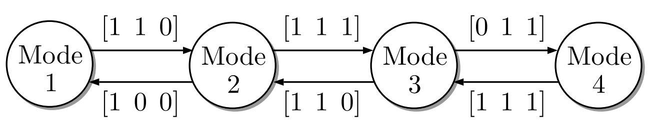

Fig. 2. Cell operating modes where $\left[\delta_1^i \;\; \delta_2^i \;\; \delta_3^i\right]$

2.2 Cell flotation model

The mineralurgical model of Pérez-Correa et al. (1998) represents the behaviour of a flotation cell by the interaction between two phases: pulp and froth. Additionally, it also considers the following mass flows: feed, tail and concentrate, as shown in figure 3. Our hybrid dynamic model is based on the following assumptions: (1) Each phase of the cell is perfectly mixed, (2) There is transfer between both phases depending on the collection rate of pulp phase and the drainage rate of froth phase, (3) m different granulometries are considered and (4) Flotation cell has constant area.

By performing a mass balance for each phase and considering all operating modes set in figure 2, we obtain the following mineralurgical balance inside a cell:

$$\dot{\mathbf{m}}_p^i = \mathbf{M}_f^i + \boldsymbol{\alpha}_e^i \mathbf{m}_e^i - \left[\boldsymbol{\alpha}_p^i + \frac{Q_t^i}{V_p^i\left(1-\epsilon_p^i\right)}\right]\mathbf{m}_p^i \tag{4}$$

$$\dot{\mathbf{m}}_e^i = \boldsymbol{\alpha}_p^i \mathbf{m}_p^i - \left[\boldsymbol{\alpha}_e^i + \frac{Q_c^i}{V_e^i\left(1-\epsilon_e^i\right)}\right]\mathbf{m}_e^i \tag{5}$$

Where $\mathbf{m}_p^i$ and $\mathbf{m}_e^i$ are vectors which contain the mineral mass for pulp and froth phases of cell i. These are defined by (6) and (7) where m_p^{ij} and m_e^{ij} are the mineral mass of a granulometry $j \in \{1, 2, \ldots, m\}$ for each phase, respectively.

$$\mathbf{m}_p^i = \left[\, m_p^{i1} \;\; m_p^{i2} \;\cdots\; m_p^{im} \,\right]^\top \tag{6}$$

$$\mathbf{m}_e^i = \left[\, m_e^{i1} \;\; m_e^{i2} \;\cdots\; m_e^{im} \,\right]^\top \tag{7}$$

Similarly, $\boldsymbol{\alpha}_p^i$ and $\boldsymbol{\alpha}_e^i$ are matrices with the collection and drainage rates for pulp and froth phases of cell i. These are defined by (8) and (9) where α_p^{ij} and α_e^{ij} are the collection and drainage rates of a granulometry j for each phase.

$$\boldsymbol{\alpha}_p^i = \begin{bmatrix} \alpha_p^{i1} & 0 & \cdots & 0 \\ 0 & \alpha_p^{i2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha_p^{im} \end{bmatrix} \tag{8}$$

$$\boldsymbol{\alpha}_e^i = \begin{bmatrix} \alpha_e^{i1} & 0 & \cdots & 0 \\ 0 & \alpha_e^{i2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha_e^{im} \end{bmatrix} \tag{9}$$

Furthermore, $\mathbf{M}_f^i$ is a vector which contains the mineral mass feed of cell i. It can be obtained with $\mathbf{c}_f^i Q_f$, where $\mathbf{c}_f^i$ is a vector that contains the mineral concentrations in the feed pulp. Q_c^i and Q_t^i are the concentrate and tail flows of cell i. V_p^i is the pulp phase volume and V_e is the froth phase volume, ϵ_p^i and ϵ_e^i are the air holdup for each phase of cell i.

The collector and drainage mineral rates of matrices (8) and (9) for j different granulometries are defined according to the model of Pérez-Correa et al. (1998) and the four modes set in figure 2. These rates depend of the volumetric flow of collector Q_{col}, foaming Q_{esp} and the air holdup inside the froth phase ϵ_e^i.

The collection and drainage constants are defined as:

$$\alpha_p^{ij} = a^{ij}Q_{col}^3 + b^{ij}Q_{col}^2 + c^{ij}Q_{col} + \alpha_{p0}^i \tag{10}$$

$$\alpha_e^{ij} = \left(d^iQ_{col} + e^iQ_{esp} + \alpha_{e0}^{ij}\right)\epsilon_e^i \tag{11}$$

Where a^{ij}, b^{ij}, c^{ij}, d^i, e^i, α_{p0}^i and α_{e0}^{ij} of cell i and granulometry j are tuned with industrial data.

The hydraulic model of Kämpjärvi and Jämsä-Jounela (2003) allows to represent the pulp level behaviour inside of a flotation cell. Taking into account that the froth height is negligible compared to pulp level (Sbarbaro et al., 2008), the volumetric balance of a cell valid for all operating modes is:

$$\dot{h}_p^i = \frac{1}{A_c^i}\left(Q_f^i - Q_t^i - Q_c^i\right)\delta_1^i \tag{12}$$

Where Q_f^i is the pulp feed flow and A_c^i the cell's area. The pulp feed flow of a line is $Q_f^i = v_d^iQ_f$, where Q_f is known and v_d^i is a binary valve of cell i which represents the state of the discrete valve. According to the hydraulic model of Stenlund and Medvedev (2002), the tail flow is derived from a physical laws based in Torricelli's principle. Considering a linear model of the cell output valve v_c^i, the tail flow of cell i is:

$$Q_t^i = \alpha_t^iv_c^i\sqrt{h_p^i + \left(\Delta h^i - h_p^{i+1}\right)\delta_2^i} \tag{13}$$

The pulp feed flow of the next cell is the tail flow of the previous cell, it other words $Q_f^{i+1} = Q_t^i$. Additionally, the tail flow of the last cell is modelled by equation (13) but setting $h_p^{i+1} = 0$. The concentrate volumetric flow Q_c^i of cell i is defined as a non-linear function of pulp level, froth and cell height (Bascur, 2010):

$$Q_c^i = \alpha_c^i\left(h_p^i + h_e^i - h_c^i\right)^{1.5}\delta_3^i \tag{14}$$

Where α_c^i is a tuning constant and the froth height is modelled as a linear function of the total mineral mass of the froth phase. Defining a sum vector $\mathbf{m}_s = [1\ 1\ \cdots\ 1]$ (with dimension $1 \times j$) and α_f^i another tuning constant, the froth height is:

$$h_e^i = \alpha_f^i\mathbf{m}_s\mathbf{m}_e^i \tag{15}$$

The total volumes of pulp and froth phases of cell i are defined as:

$$V_p^i = A_c^ih_p^i \tag{16}$$

$$V_e^i = A_c^ih_e^i \tag{17}$$

Considering the approach proposed by Ortiz and Toro (2013) an air volumetric balance for each phase of cell is done. This allows to defined the air holdup for pulp and froth phases of cell i as:

$$\dot{\epsilon}_p^i = \frac{1}{V_p^i}\left(Q_a^i - v_b^iA_c\epsilon_p^i\right) \tag{18}$$

$$\dot{\epsilon}_e^i = \frac{1}{V_e^i}\left(v_b^iA_c\epsilon_p^i - \alpha_a^i\epsilon_e^i\right) \tag{19}$$

Where Q_a^i is the air flow into pulp phase of cell i and v_b^i is the bubble terminal velocity in this phase that is defined as (Bozzano and Dente, 2001):

$$v_b^i = \sqrt{\frac{gd_b}{3\alpha_d}} \tag{20}$$

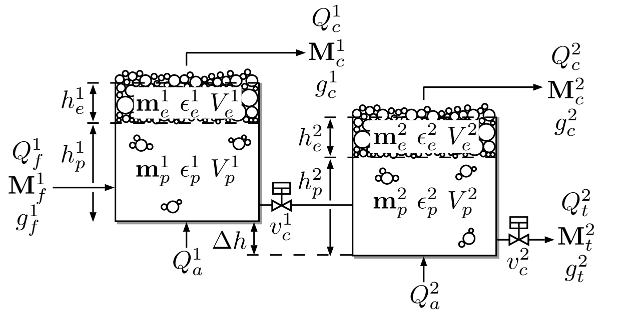

Fig. 3. Flotation cell model diagram

Where d_b is the bubble diameter, g is the gravitational acceleration and α_d is the drag coefficient of a bubble whose dimensionless value is 0.47 (perfectly spherical bubble).

The tail and concentrate mineral mass flows of cell i are defined by:

$$\mathbf{M}_t^i = \frac{Q_t^i}{V_p^i\left(1 - \epsilon_p^i\right)}\mathbf{m}_p^i \tag{21}$$

$$\mathbf{M}_c^i = \frac{Q_c^i}{V_e^i\left(1 - \epsilon_e^i\right)}\mathbf{m}_e^i \tag{22}$$

Where $\mathbf{M}_t^i$ and $\mathbf{M}_c^i$ are vectors with dimension $j \times 1$. Additionally, the tail and concentrate mineral grades valid for all operating modes are defined dividing the sum of the total fine by the total mineral, both contained in the tail and concentrate flow, respectively:

$$g_t^i = \frac{\boldsymbol{\beta}\mathbf{M}_t^i}{\mathbf{m}_s\mathbf{M}_t^i}g_{cu} \tag{23}$$

$$g_c^i = \frac{\boldsymbol{\beta}\mathbf{M}_c^i}{\mathbf{m}_s\mathbf{M}_c^i}g_{cu} \tag{24}$$

Where $\boldsymbol{\beta}$ are grade tuning matrices for tail and concentrate flows of cell i. This is defined by (25) where it is calibrated with industrial data and set constant for each granulometry j of cell i. The variable g_{cu} is the copper grade in a molecule of chalcopyrite (approximately 34.6%).

$$\boldsymbol{\beta} = \begin{bmatrix} \beta^1 & 0 & \cdots & 0 \\ 0 & \beta^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \beta^m \end{bmatrix} \tag{25}$$

Finally, the recovery of mineral in cell i is obtained by dividing the total fine in the concentrate by the total fine in the feed.

3. HYBRID SYSTEM IDENTIFICATION

3.1 Hybrid identification

Given a set of input-output data which is measured from a process with hybrid dynamic, a PieceWise AutoRegresive eXogenous (PWARX) model is defined as (Ferrari-Trecate et al., 2003):

$$y(k) = \begin{cases} \theta_1\begin{bmatrix}\mathbf{x}(k)\\1\end{bmatrix} + e(k) & \text{if } \mathbf{x}(k) \in \mathcal{X}_1 \\ \vdots & \\ \theta_s\begin{bmatrix}\mathbf{x}(k)\\1\end{bmatrix} + e(k) & \text{if } \mathbf{x}(k) \in \mathcal{X}_s \end{cases} \tag{26}$$

The regressor vector $\mathbf{x}(k) \in \mathbb{R}^n$ contains only past n_a outputs and n_b inputs measurements:

$$\mathbf{x}(k) = \left[\, y(k-1) \cdots y(k-n_a)\ \mathbf{u}^\top(k-1) \cdots \mathbf{u}^\top(k-n_b) \,\right]^\top \quad (27)$$

Where $\mathbf{u}(k) \in \mathbb{R}^p$ is the input, $y(k) \in \mathbb{R}$ is the output and $e(k) \in \mathbb{R}$ is a noise term. Furthermore, s is the number of submodels, $\theta_{\sigma(k)}$ are the parameter vectors of each affine ARX submodel with $\sigma(k) \in \{1,\ldots,s\}$ the discrete state (or mode). The regressors lie in a bounded polyhedron $\mathcal{X} \in \mathbb{R}^n$ hereafter referred to as regressor set where $n = n_a + p \cdot n_b$. The switching mechanism is determined by a polyhedral partition $\{\mathcal{X}\}_{i=1}^s$ of $\mathcal{X}$ and the discrete state $\sigma(k)$ is given by the rule:

$$[\sigma(k) = i] \Leftrightarrow [\mathbf{x}(k) \in \mathcal{X}_i] \quad (28)$$

With $i \in \{1,\ldots,s\}$ and $\{\mathcal{X}\}_{i=1}^s$ is a complete partition of $\mathcal{X}$. For PWARX models defined by (26) and (28), the identification problem is formulated as follows: Given a collection of N input-output pairs $(y(k), \mathbf{u}^\top(k))$ with $k \in \{1,\ldots,N\}$ estimate the model with orders n_a and n_b, the number of submodels s, the parameter vector θ_i and the regions $\mathcal{X}_i$ with $i \in \{1,\ldots,s\}$ (Paoletti et al., 2007).

The identification problem can be approached by several methodologies (Paoletti et al., 2007). We have used an identification algorithm called clustering-based procedure. This method was developed by Ferrari-Trecate et al. (2003) and it works for fixed parameters n_a, n_b and s.

3.2 Predictive models for HMPC

Our principal aim is to develop predictive models from input-output data in order to be used in HMPC strategies. Besides, one of the most interesting features of PWARX model is its equivalence with Mixed Logical Dynamical (MLD) models. According to the works developed by Bemporad and Morari (1999) a MLD model can be generalized through the following linear relations:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}_1\mathbf{u}(k) + \mathbf{B}_2\boldsymbol{\delta}(k) + \mathbf{B}_3\mathbf{z}(k) \quad (29)$$

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}_1\mathbf{u}(k) + \mathbf{D}_2\boldsymbol{\delta}(k) + \mathbf{D}_3\mathbf{z}(k) \quad (30)$$

$$\mathbf{E}_2\boldsymbol{\delta}(k) + \mathbf{E}_3\mathbf{z}(k) \leq \mathbf{E}_1\mathbf{u}(k) + \mathbf{E}_4\mathbf{x}(k) + \mathbf{E}_5 \quad (31)$$

Where $\mathbf{x}(k) \in \mathbb{R}^{n_c} \times \{0,1\}^{n_l}$ is a vector of continuous and binary states, $\mathbf{u}(k) \in \mathbb{R}^{m_c} \times \{0,1\}^{m_l}$ is a vector of continuous and binary inputs and $\mathbf{y}(k) \in \mathbb{R}^{p_c} \times \{0,1\}^{p_l}$ is a vector of continuous and binary outputs. Also, $\boldsymbol{\delta}(k) \in \{0,1\}^{r_l}$ and $\mathbf{z}(k) \in \mathbb{R}^{r_c}$ represent auxiliary binary and continuous variables, respectively. These are introduced when transforming logic relations into mixed-integer inequalities. Additionally, $\mathbf{A}$, $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{B}_3$, $\mathbf{C}$, $\mathbf{D}_1$, $\mathbf{D}_2$, $\mathbf{D}_3$, $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{E}_3$, $\mathbf{E}_4$ and $\mathbf{E}_5$ are matrices of suitable dimensions.

Additionally, Torrisi and Bemporad (2004) developed a language called HYSDEL available for Matlab® which fully automates the process of generating the matrices associated to a MLD model defined by (29), (30) and (31). The code 1 shows our HYSDEL strategy to transform PWARX models of the form (26) in a MLD form suitable to be used in HMPC strategies. In this example we have considered the version 2.0.6 of HYSDEL, only one input, state and output, two modes and $n_a = n_b = 1$.

Code 1: HYSDEL example for PWARX to MLD

```
System pwarx2mld {
Interface {
    State     {real x [x_min, x_max]; ... }
    Input     {real u [u_min, u_max]; ... }
    Output    {real y; ... }
    Parameter {real θ11, θ12, θ13;
               real θ21, θ22, θ23;
               real a11, a12, b11; ... }}
Implementation {
    Aux {real z; ...
         bool d; ... }
    AD { d = (a11 · x + a12 · u) ≤ b11; ... }
    DA { z = { if d then θ11 · x + θ12 · u + θ13
               else θ21 · x + θ22 · u + θ23 }; ... }
    Continuous { x = z; ... }
    Output { y = x; ... }
    Must {  u ≤  u_max;
           −u ≤ −u_min; ...  }}}
```

3.3 Simulation results

Simulation results of the hybrid simulator were used for the model identification procedure. It was considered a sample time of 9 seconds over eleven hours of simulation and 4400 different data samples. The first half for identification and the next for validation. The identification was developed using the Hybrid Identification Toolbox (HIT) (Ferrari-Trecate, 2005) and generating variations in pulp feed flow of line one from 1500 to 2500 m^3/h. A MISO PWARX model was identified for each cell's pulp level whose inputs were the line's pulp feed flow and the position of its control valve. By using the HYSDEL Toolbox (Torrisi and Bemporad, 2004) and the code 1 a MLD model was obtained. Figure 4 shows the results of MLD pulp level for the first cell of line one.

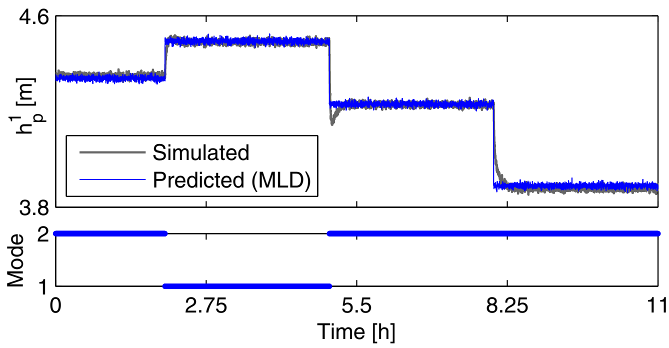

Fig. 4. MLD model for cell's pulp level

In addition, a model for line's tail grade was obtained whose inputs were the line's pulp feed flow and the pulp level of the last cell. The same procedure detailed previously is used to obtain the MLD model and results can be seen in figure 5. In both cases the best performance was achieved with $n_a = n_b = 1$ orders, and with a submodel number of $s = 2$. Besides, the results shows that the identified models allow to predict the behaviour of the interest variables and our work considers the instrumentation generally available in any mineral flotation plant. Therefore, these models are valid for use in hybrid predictive control strategies.

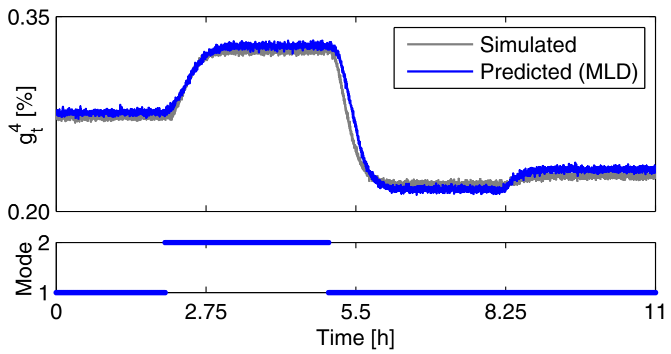

Fig. 5. MLD model for line's tail grade

4. HYBRID MPC STRATEGY

4.1 HMPC for flotation process

Hybrid model predictive control (HMPC) uses models characterized by the interaction of dynamic behaviours, logical rules and operating constraints (Bemporad and Morari, 1999). In this work our models are obtained through hybrid identification techniques according to section 3. Figure 6 shows the model predictive control strategy proposed. The advantage is that by considering a single controller for each cell the formulation process is simplified, allowing to obtain an accurate and simple controller valid for different operating modes of the plant. Moreover, this structure allows validating new optimizing control strategies, such as ones based on distributed controllers. The main difficulty in developing a hybrid predictive controller is the identification of a suitable model. The flotation process is highly complex, where in our work each cell has four possible modes of operation. When considering a large line, the number of possible combinations of operation modes for the cells in the line increase. In this situation is needed a highly representative data obtained in the identification process, in order to use highly reliable models.

Our hybrid model predictive control strategy applied to flotation process is set as follows:

$$\min_{\{\Delta u,\delta,z\}} \sum_{i=0}^{N-1} \|g_t^4(k+i+1|k) - r(k+i|k)\|_Q^2 \cdots \\ \cdots + \|\Delta u(k+i|k)\|_R^2 \quad (32)$$

Subject to:

$$\mathbf{x}(k+1|k) = \mathbf{A}\mathbf{x}(k|k) + \mathbf{B}_3\mathbf{z}(k|k) \quad (33)$$

$$\mathbf{y}(k|k) = \mathbf{C}\mathbf{x}(k|k) \quad (34)$$

$$\mathbf{E}_2\boldsymbol{\delta}(k|k) + \mathbf{E}_3\mathbf{z}(k|k) \leq \mathbf{E}_1\mathbf{u}(k|k) + \mathbf{E}_4\mathbf{x}(k|k) + \mathbf{E}_5 \quad (35)$$

$$\underline{h_p} \leq h_p^i(k|k) \leq \overline{h_p} \quad (36)$$

$$\underline{h_p^c} \leq h_p^{c\,i}(k|k) \leq \overline{h_p^c} \quad (37)$$

Where g_t^4 is the tail final grade of a line, $\Delta u(k)$ corresponds to the changes in the output control valve position v_c and $r(k)$ is the reference which can be set as zero. If the line has more than one output control valve, the changes in this control variable must be included in the objective function of the HMPC controller. The princi

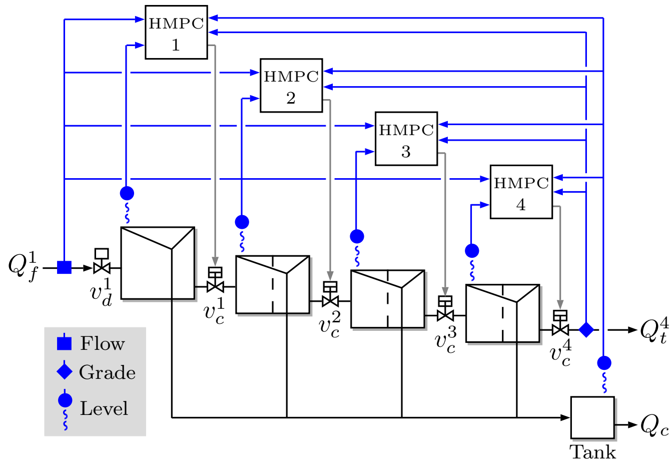

Fig. 6. Flotation's Hybrid MPC system

N is the prediction horizon and equations (33), (34) and (35) corresponds to the MLD identified model. Additionally, equations (36) and (37) are operating constraints, $\|g_t^4\|_Q^2 = {g_t^4}^\top Q g_t^4$, $\|\Delta u\|_R^2 = \Delta u^\top R \Delta u$ with $Q = Q^\top \geq 0$ and $R = R^\top \geq 0$. This problem can be solved defining the following vectors:

$$\boldsymbol{\Omega} = [\,\Delta u(k|k) \; \cdots \; \Delta u(k+N-1|k)\,]^\top \quad (38)$$

$$\boldsymbol{\Xi} = [\,\delta(k|k) \; \cdots \; \delta(k+N-1|k)\,]^\top \quad (39)$$

$$\boldsymbol{\Gamma} = [\,z(k|k) \; \cdots \; z(k+N-1|k)\,]^\top \quad (40)$$

And the general vector:

$$\boldsymbol{\Lambda} = [\,\boldsymbol{\Omega} \; \boldsymbol{\Xi} \; \boldsymbol{\Gamma}\,]^\top \quad (41)$$

The problem defined by equations (32) through (37) can be formulated like as mixed integer quadratic programming (MIQP) which can be solved as:

$$\min_{\{\boldsymbol{\Lambda}\}} \frac{1}{2}\boldsymbol{\Lambda}^\top \mathbf{F}_1 \boldsymbol{\Lambda} + \mathbf{F}_2 \boldsymbol{\Lambda} \quad (42)$$

Subject to:

$$\mathbf{F}_3 \boldsymbol{\Lambda} \leq \mathbf{F}_4 \quad (43)$$

Where $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$ and $\mathbf{F}_4$ are matrices with suitable dimensions. According to the receding horizon theory only the control action corresponding to the current sampling time $v_c = \mathbf{u}(0)$ is applied and hence the optimization problem is repeatedly solved at next sampling time (Bemporad and Morari, 1999).

4.2 Simulation results

Figure 7 shows the results of the HMPC strategy applied to the first flotation cell of the line with a horizon time of $N = 4$ hours and a sampling time of 18 seconds. These values were established by trial and error due to calibration developed simulator availability. The initial reference to the tail grade was established in 0.11% and later it is changed to 0.19%. It is seen that the opening percentage of the control valve changes according to the reference and the level of pulp cell remains within the defined upper and lower limits. The same design can be replicated for each of the following cells of the line.

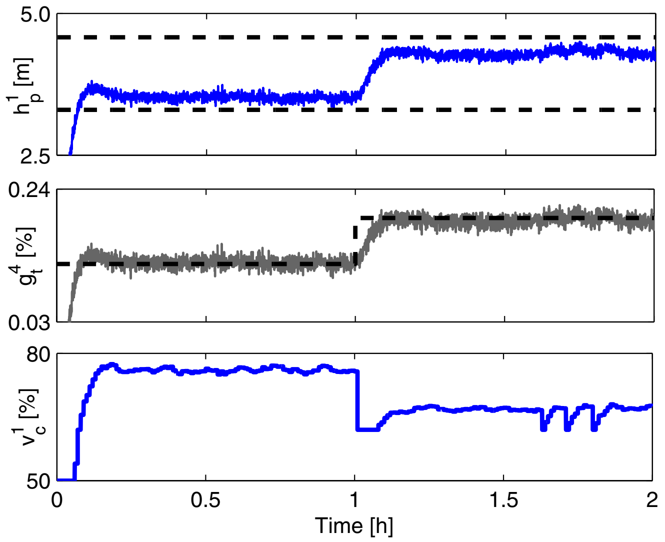

Fig. 7. HMPC simulation results

5. CONCLUSION

This paper presented the development of a hybrid dynamic model for a flotation cell. It was used successfully in the development of a simulator for flotation calibrated with industrial data. Subsequently, a robust hybrid model predictive control (HMPC) strategy was formulated using models generated by hybrid identification techniques and with data generated by the simulator. This strategy showed satisfactory the tail grade control of a line, keeping the variables of interest within operating ranges. In addition, our strategy allows to control the plant in different operating modes which can be set by the operator, respecting the constraints of each cell and without needing to use different controllers for each mode of a flotation line.

Currently, we are working on the development and validation of distributed control strategies applied to the plant studied.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the flotation plant Las Tórtolas of AngloAmerican Copper, especially Fernando Romero for the industrial data provided, and Conicyt for the scholarship given to Eduardo Putz.

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