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Structural Control of Probabilistic Boolean Networks and Its Application to Design of Real-Time Pricing Systems

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Abstract: In this paper, a new control method for a probabilistic Boolean network (PBN) is proposed. A PBN is widely used as a model of complex systems such as gene regulatory networks. For a PBN, the structural control problem is newly formulated. In this problem, a discrete probability distribution appeared in a PBN is controlled by the continuous-valued input. In the proposed solution method, using a matrix-based representation for a PBN, the problem is approximated by a linear programming problem. Furthermore, design of real-time pricing systems of electricity is considered as an application. By appropriately designing real-time pricing systems, electricity conservation is achieved. The effectiveness of the proposed method is presented by a numerical example on real-time pricing systems.

Keywords: probabilistic Boolean networks, real-time pricing systems, structural control.

1. INTRODUCTION

Analysis and control of complex systems such as power systems and gene regulatory networks are one of the fundamental problems in control theory of large-scale systems. In order to deal with such a complex system, it is one of the appropriate methods to approximate a complex system by a discrete abstract model (see, e.g., Tabuada (2009)). On the other hand, human decision making is also complex, and is modeled by a discrete model (see, e.g., Adomi et al. (2010)). Thus, in analysis and control of complex systems and those with human decision making, a discrete model plays the important role.

Several discrete models such as Petri nets, Bayesian networks, automata-based models, Boolean networks have been proposed so far (see, e.g., Jong (2002)). In this paper, we focus on a Boolean network (BN) (Kauffman (1969)). In a BN, the state is given by a binary value (0 or 1), and the dynamics are expressed by a set of Boolean functions. In addition, since the behavior of complex systems is frequently stochastic by the effects of noise, it is appropriate that a Boolean function is randomly decided at each time among the candidates of Boolean functions. Thus, a probabilistic BN (PBN) has been proposed in Shmulevich et al. (2002a). In this paper, we adopt a probabilistic Boolean network (PBN) as a discrete model.

For a given PBN, we consider the structural control problem (see, e.g., Kobayashi and Hiraishi (2013); Shmulevich et al. (2002b); Xiao and Dougherty (2007)). In this problem, a discrete probability distribution is controlled. For example, in Kobayashi and Hiraishi (2013), a discrete probability distribution at each time is selected among a given set. In this paper, we consider fine control of a discrete probability distribution by using the continuous-valued input. For a newly formulated problem, we propose an approximate solution method. First, a matrix-based representation of BNs proposed in Kobayashi and Hiraishi (2014) is extended to that of PBNs. Next, using the obtained representation, the original problem is approximated by a linear programming (LP) problem.

Furthermore, as one of the applications, we consider a design method of real-time pricing systems (see, e.g., Borenstein et al. (2002); Roozbehani et al. (2010); Samadi et al. (2010); Vivekananthan et al. (2013)). A real-time pricing system of electricity is a system that charges different electricity prices for different hours of the day and for different days, and is effective for reducing the peak and flattening the load curve. In the existing methods, the price at each time is given by a simple function with respect to power consumptions and voltage deviations and so on (see, e.g., Vivekananthan et al. (2013)). To the best of our knowledge, decision making of customers has not been explicitly considered so far. Thus, decision making of customers is modeled by a PBN, and the problem of finding the price at each time is formulated as a structural control problem. The price corresponds to the continuous-valued input. By a numerical example, the effectiveness of the proposed method is presented.

Notation: For the n-dimensional vector $x = [\ x_1 \ \ x_2 \ \ \cdots \ \ x_n\ ]^\top$ and the index set $\mathcal{I} = \{i_1, i_2, \ldots, i_m\} \subseteq \{1, 2, \ldots, n\}$, define $[x_i]_{i \in \mathcal{I}} := [\ x_{i_1} \ \ x_{i_2} \ \ \cdots \ \ x_{i_m}\ ]^\top$. For two matrices A and B, let $A \otimes B$ denote the Kronecker product of A and B. In addition, for q vectors $y_1, y_2, \ldots, y_q$ and the index set $\mathcal{J} = \{j_1, j_2, \ldots, j_p\} \subseteq \{1, 2, \ldots, q\}$, define $\bigotimes_{j \in \mathcal{J}} y_j := y_{j_1} \otimes y_{j_2} \otimes \cdots \otimes y_{j_p}$.

2. PROBABILISTIC BOOLEAN NETWORK

First, we explain a (deterministic) Boolean network (BN). A BN is defined by

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$$\begin{cases} x_1(k+1) = f^{(1)}([x_j(k)]_{j\in\mathcal{N}^{(1)}}), \\ x_2(k+1) = f^{(2)}([x_j(k)]_{j\in\mathcal{N}^{(2)}}), \\ \vdots \\ x_n(k+1) = f^{(n)}([x_j(k)]_{j\in\mathcal{N}^{(n)}}), \end{cases} \tag{1}$$

where $x := [\ x_1 \quad x_2 \quad \cdots \quad x_n\ ]^\top \in \{0,1\}^n$ is the state, and $k = 0, 1, 2, \ldots$ is the discrete time. The set $\mathcal{N}^{(i)} \subseteq \{1, 2, \ldots, n\}$ is a given index set, and the function $f_i : \{0,1\}^{|\mathcal{N}^{(i)}|} \to \{0,1\}^1$ is a given Boolean function consisting of logical operators such as AND ($\wedge$), OR ($\vee$), and NOT ($\neg$). If $\mathcal{N}^{(i)} = \emptyset$ holds, then $x_i(k+1)$ is uniquely determined as 0 or 1.

Next, we explain a probabilistic Boolean network (PBN) (see Shmulevich et al. (2002a) for further details). In a PBN, the candidates of $f^{(i)}$ are given, and for each x_i, selecting one Boolean function is probabilistically independent at each time. Let

$$f_l^{(i)}\left([x_j(k)]_{j\in\mathcal{N}_l^{(i)}}\right), \quad l = 1, 2, \ldots, q(i)$$

denote the candidates of $f^{(i)}$. The probability that $f_l^{(i)}$ is selected is defined by

$$c_l^{(i)} := \text{Prob}\left(f^{(i)} = f_l^{(i)}\right). \tag{2}$$

Then, the following relation

$$\sum_{l=1}^{q(i)} c_l^{(i)} = 1 \tag{3}$$

must be satisfied. Probabilistic distributions are derived from experimental results. Finally, $\mathcal{N}_i$, $i = 1, 2, \ldots, n$ are defined by

$$\mathcal{N}_i := \bigcup_{l=1}^{q(i)} \mathcal{N}_l^{(i)}.$$

We show a simple example.

Example 1. Consider the PBN in which Boolean functions and probabilities are given by

$$f^{(1)} = \begin{cases} f_1^{(1)} = x_3(k), & c_1^{(1)} = 0.8, \\ f_2^{(1)} = \neg x_3(k), & c_2^{(1)} = 0.2, \end{cases}$$
$$f^{(2)} = f_1^{(2)} = x_1(k) \wedge \neg x_3(k), \quad c_1^{(2)} = 1.0,$$
$$f^{(3)} = \begin{cases} f_1^{(3)} = x_1(k) \wedge \neg x_2(k), & c_1^{(3)} = 0.7, \\ f_2^{(3)} = x_2(k), & c_2^{(3)} = 0.3, \end{cases}$$

where $q(1) = 2$, $q(2) = 1$ and $q(3) = 2$ hold, $\mathcal{N}_1 = \{3\}$, $\mathcal{N}_2 = \{1, 3\}$, and $\mathcal{N}_3 = \{1, 2\}$ hold, and we see that the relation (3) is satisfied. Next, consider the state trajectory. Then, for $x(0) = [\ 0\ \ 0\ \ 0\ ]^\top$, we obtain

$$\text{Prob}\left(x(1) = [\ 0\ \ 0\ \ 0\ ]^\top \mid x(0) = [\ 0\ \ 0\ \ 0\ ]^\top\right) = 0.8,$$
$$\text{Prob}\left(x(1) = [\ 1\ \ 0\ \ 0\ ]^\top \mid x(0) = [\ 0\ \ 0\ \ 0\ ]^\top\right) = 0.2.$$

In this example, the cardinality of the finite state set $\{0,1\}^3$ is given by $2^3 = 8$, and we can obtain the discrete-time Markov chain by computing the transition from each state. □

3. PROBLEM FORMULATION

In this section, we formulate the control problem studied in this paper. In the standard control problem, the control input is added to a given Boolean function. For example, the control input is added as follows: $f^{(i)}([x_j(k)]_{j\in\mathcal{N}^{(i)}}, u(k))$, $u(k) \in \{0,1\}^1$. In general, we assume that the value of the control input can be arbitrarily given. However, there is a possibility that there exists no control input satisfying this assumption. In control of gene regulatory networks, a structural control (or structural intervention) method for PBNs has been proposed so far (see, e.g., Kobayashi and Hiraishi (2013); Shmulevich et al. (2002b)). For example, in Kobayashi and Hiraishi (2013), the discrete probabilistic distribution is switched at each time. In other words, the discrete probabilistic distribution is selected among the set of candidates. On the other hand, in complex systems such as gene regulatory networks, power systems, and social systems, it will be desirable to consider a weaker control method. Thus, in this paper, we consider fine control of probabilities in a discrete probabilistic distribution. This control method can be regarded as a kind of structural control methods.

In the structural control problem formulated here, we assume that the probability $c_l^{(i)}$ in (2) is given by

$$c_l^{(i)}(k) = a_l^{(i)} + b_l^{(i)} u_i(k), \tag{4}$$

where $u := [\ u_1 \quad u_2 \quad \cdots \quad u_n\ ]^\top \in [\underline{u}_1, \overline{u}_1] \times [\underline{u}_2, \overline{u}_2] \times \cdots \times [\underline{u}_n, \overline{u}_n] \subseteq \mathcal{R}^n$ is the control input. The set $[\underline{u}_i, \overline{u}_i]$ expresses the input constraint, and $\underline{u}_i, \overline{u}_i \in \mathcal{R}^1$ are given in advance. Of course, we must find $u(k)$ such that $c_l^{(i)}(k)$ satisfies (3). In addition, the dimension of the control input may be less than the dimension n of the state.

Under the above preparation, we consider the following problem.

Problem 1. Suppose that for the PBN with (4), the lower and upper bounds of input constraints $\underline{u}_i, \overline{u}_i$, and the initial state $x(0) = x_0$ are given. Then, find a control input sequence $u(0), u(1), \ldots, u(N-1) \in [\underline{u}_1, \overline{u}_1] \times [\underline{u}_2, \overline{u}_2] \times \cdots \times [\underline{u}_n, \overline{u}_n]$ minimizing the following cost function

$$J = E\left[\sum_{k=0}^{N-1} \{Qx(k) + Ru(k)\} + Q_f x(N) \;\middle|\; x(0) = x_0\right], \tag{5}$$

where $Q, Q_f \in \mathcal{R}^{1\times n}$, $R \in \mathcal{R}^{1\times m}$ are weighting vectors whose element is a non-negative real number, and $E[\cdot|\cdot]$ denotes a conditional expected value.

The linear cost function (5) is appropriate from the following reason: For a binary variable $\delta \in \{0,1\}$, the relation $\delta^2 = \delta$ holds. That is, in the cost function, the quadratic term such as $x_i^2(k)$ is not necessary.

According to the result in Kobayashi and Hiraishi (2012), Problem 1 can be rewritten as a polynomial optimization problem. However, in the case of large-scale PBNs, it will be difficult to solve a polynomial optimization problem. In this paper, an approximate solution method for Problem 1 is proposed.

Hereafter, the condition $x(0) = x_0$ in the conditional expected value is omitted.

4. DERIVATION OF APPROXIMATE SOLUTION METHOD

In this section, we derive an approximate solution method for Problem 1. First, a matrix-based representation for PBNs is derived. The obtained representation is an extension of a matrix-based representation for BNs proposed in Kobayashi and Hiraishi (2014). Next, using the matrix-based representation, an approximate solution method for Problem 1 is derived.

4.1 Matrix-Based Representation for PBN

As preparations, some notations are defined. Binary variables $x_i^0(k)$ and $x_i^1(k)$ are introduced. If $x_i(k) = 0$ holds, then $x_i^0(k) = 1$ holds, otherwise $x_i^0(k) = 0$ holds. If $x_i(k) = 1$ holds, then $x_i^1(k) = 1$ holds, otherwise $x_i^1(k) = 0$ holds. Then, the equality $x_i^0(k) + x_i^1(k) = 1$ is satisfied. Using $x_i^0(k)$ and $x_i^1(k)$, Consider transforming the BN (1) into a matrix-based representation.

First, we explain the outline of a matrix-based representation by using a simple example.

Example 2. Consider the following BN:

$$\begin{cases} x_1(k+1) = \neg x_2(k), \\ x_2(k+1) = x_1(k), \\ x_3(k+1) = x_1(k) \wedge \neg x_2(k), \end{cases}$$

where $\mathcal{N}^{(1)} = \{2\}$, $\mathcal{N}^{(2)} = \{1\}$, and $\mathcal{N}^{(3)} = \{1, 2\}$. Then, we can obtain the truth table for each $x_i(k+1)$. See Table 1 and Table 2. From these truth tables, we can obtain the following matrix-based representation:

$$\begin{bmatrix} x_1^0(k+1) \\ x_1^1(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{A^{(1)}} \begin{bmatrix} x_2^0(k) \\ x_2^1(k) \end{bmatrix},$$

$$\begin{bmatrix} x_2^0(k+1) \\ x_2^1(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{A^{(2)}} \begin{bmatrix} x_1^0(k) \\ x_1^1(k) \end{bmatrix},$$

$$\begin{bmatrix} x_3^0(k+1) \\ x_3^1(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{A^{(3)}} \begin{bmatrix} x_1^0(k)x_2^0(k) \\ x_1^0(k)x_2^1(k) \\ x_1^1(k)x_2^0(k) \\ x_1^1(k)x_2^1(k) \end{bmatrix},$$

where each element of $A^{(i)}$, $i = 1, 2, 3$ is given by a binary value (0 or 1), and a sum of all elements in each column of $A^{(i)}$ is equal to 1. □

Such a matrix-based representation has been proposed in also Cheng and Qi (2009); Cheng et al. (2011). However, in the representation proposed in Cheng and Qi (2009); Cheng et al. (2011), the matrix with the size of $2^n \times 2^n$ must be manipulated (n is the dimension of the state). In the matrix-based representation proposed in Kobayashi and Hiraishi (2014), the matrix with the size of $2 \times 2^{|\mathcal{N}_i|}$ are manipulated for each x_i. Thus, the proposed representation enables us to model a BN using matrices with the smaller size.

Table 1. Truth tables for $x_i(k+1)$, $i = 1, 2$.

$x_2(k)$	$x_1(k+1)$
0	1
1	0

$x_1(k)$	$x_2(k+1)$
0	0
1	1

Table 2. Truth table for $x_3(k+1)$.

$x_1(k)$	$x_2(k)$	$x_3(k+1)$
0	0	0
0	1	0
1	0	1
1	1	0

Consider a general case. Define

$$\overline{x}_i(k) := \begin{bmatrix} x_i^0(k) \\ x_i^1(k) \end{bmatrix} \left(= \begin{bmatrix} 1 - x_i(k) \\ x_i(k) \end{bmatrix} \right).$$

Then, the matrix-based representation for $x_i(k+1)$ is given by

$$\overline{x}_i(k+1) = A^{(i)} \bigotimes_{j \in \mathcal{N}_i} \overline{x}_j(k), \tag{6}$$

where $A^{(i)} \in \{0, 1\}^{2 \times 2^{|\mathcal{N}_i|}}$ and $\bigotimes_{j \in \mathcal{N}_i} \overline{x}_j(k) \in \{0, 1\}^{2^{|\mathcal{N}_i|}}$. The matrix $A^{(i)}$ can be derived from the following procedure.

Procedure for deriving $A^{(i)}$ in (6):

Step 1: Derive a truth table for $x_i(k+1)$.

Step 2: Based on the obtained truth table, assign $x_i(k+1) = 0$ or $x_i(k+1) = 1$ for each element of $\bigotimes_{j \in \mathcal{N}_i} \overline{x}_j(k)$.

Step 3: Express the assignment obtained in Step 2 by a row vector. Denote the obtained row vector by $\overline{A}^{(i)} \in \{0, 1\}^{1 \times 2^{|\mathcal{N}_i|}}$.

Step 4: Derive $A^{(i)}$ as

$$A^{(i)} = \begin{bmatrix} 1_{1 \times 2^{|\mathcal{N}_i|}} - \overline{A}^{(i)} \\ \overline{A}^{(i)} \end{bmatrix}.$$

Next, consider extending the matrix-based representation of BNs to that of PBNs. First, using a simple example, we explain the outline.

Example 3. Consider the PBN in Example 1. Using the matrix-based representation, the expected value of $\overline{x}_i(k+1)$ can be obtained as

$$E[\overline{x}_1(k+1)] = \left(0.8 \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{A_1^{(1)}} + 0.2 \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{A_2^{(1)}} \right) \begin{bmatrix} E[x_2^0(k)] \\ E[x_2^1(k)] \end{bmatrix},$$

$$E[\overline{x}_2(k+1)] = \underbrace{\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{A_1^{(2)}} \begin{bmatrix} E[x_1^0(k)x_3^0(k)] \\ E[x_1^0(k)x_3^1(k)] \\ E[x_1^1(k)x_3^0(k)] \\ E[x_1^1(k)x_3^1(k)] \end{bmatrix}, \tag{7}$$

$$E[\overline{x}_3(k+1)] = \left(0.7 \underbrace{\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{A_1^{(3)}} + 0.3 \underbrace{\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}}_{A_2^{(3)}}\right) \times \begin{bmatrix} E[x_1^0(k)x_2^0(k)] \\ E[x_1^0(k)x_2^1(k)] \\ E[x_1^1(k)x_2^0(k)] \\ E[x_1^1(k)x_2^1(k)] \end{bmatrix},$$

where the condition $x(0) = x_0$ is omitted. In this representation, the matrices $A_1^{(1)}$ and $A_2^{(1)}$ correspond to the Boolean functions $f_1^{(1)}$ and $f_2^{(1)}$, respectively. In a similar way, $A_1^{(2)}$, $A_1^{(3)}$, and $A_1^{(3)}$ correspond to the Boolean functions $f_1^{(2)}$, $f_1^{(3)}$, and $f_2^{(3)}$, respectively. □

In general, using the matrix-based representation, the expected value of $\overline{x}_i(k+1)$ can be obtained as

$$E[\overline{x}_i(k+1)] = \left(\sum_{l=1}^{q(i)} c_l^{(i)}(k) A_l^{(i)}\right) \bigotimes_{j\in\mathcal{N}_i} E[\overline{x}_j(k)], \quad (8)$$

where $A_l^{(i)} \in \{0,1\}^{2\times 2^{|\mathcal{N}_i|}}$ and $\bigotimes_{j\in\mathcal{N}_i} \overline{x}_j(k) \in \{0,1\}^{2^{|\mathcal{N}_i|}}$. The matrix $A_l^{(i)}$ can be derived from the above procedure.

4.2 Reduction to a Linear Programming Problem

Using the matrix-based representation (8), consider transforming Problem 1. First, Problem 1 can be rewritten as the following problem:

Problem 2. Suppose that for the PBN with (4), the lower and upper bounds of input constraints $\underline{u}_i, \overline{u}_i$, and the initial state $x(0) = x_0$ are given. Then, find $u(0), u(1), \ldots, u(N-1)$ minimizing the cost function (5) subject to (3), (8), and the input constraint.

In a similar way to Problem 1, Problem 2 is rewritten as a polynomial optimization problem. In this paper, we focus on the structure of $\bigotimes_{j\in\mathcal{N}_i} E[\overline{x}_j(k)]$, and derive the relaxed problem for Problem 2. The relaxed problem is a linear programming (LP) problem, and can be solved faster than a polynomial optimization problem.

First, we show an example.

Example 4. Consider the matrix-based representation obtained in Example 3. We remark that the discrete probabilistic distribution for each x_i is independent. Then, in (7), we can obtain

$$\begin{aligned} &E[x_1^0(k)x_3^0(k)] + E[x_1^0(k)x_3^1(k)] \\ &= E[x_1^0(k)](E[x_3^0(k)] + E[x_3^1(k)]) \\ &= E[x_1^0(k)]. \end{aligned}$$

In a similar way, we can obtain

$$\begin{aligned} E[x_1^1(k)x_3^0(k)] + E[x_1^1(k)x_3^1(k)] &= E[x_1^1(k)], \\ E[x_1^0(k)x_3^0(k)] + E[x_1^1(k)x_3^0(k)] &= E[x_3^0(k)], \\ E[x_1^0(k)x_3^1(k)] + E[x_1^1(k)x_3^1(k)] &= E[x_3^1(k)]. \end{aligned}$$

In addition,

$$\begin{aligned} &E[x_1^0(k)x_3^0(k)] + E[x_1^0(k)x_3^1(k)] + E[x_1^1(k)x_3^0(k)] \\ &+ E[x_1^1(k)x_3^1(k)] = 1 \end{aligned}$$

holds. The obtained equalities can be used as constraints in the relaxed problem. □

Next, consider a general case. Define

$$z_i(k) := \bigotimes_{j\in\mathcal{N}_i} E[\overline{x}_j(k)] \in [0,1]^{2^{|\mathcal{N}_i|}}.$$

Then, (8) can be rewritten as

$$E[\overline{x}_i(k+1)] = \left(\sum_{l=1}^{q(i)} a_l^{(i)} A_l^{(i)}\right) z_i(k) + \left(\sum_{l=1}^{q(i)} b_l^{(i)} A_l^{(i)}\right) w_i(k) \quad (9)$$

where $w_i(k) := u_i(k) z_i(k) \in [0,1]^{2^{|\mathcal{N}_i|}}$. The relation between $E[\overline{x}_i(k)]$ and $z_i(k)$ is given by

$$E[\overline{x}_j(k)] = C_j z_i(k), \quad j \in \mathcal{N}_i, \quad (10)$$

where the matrix $C_j \in \{0,1\}^{2\times 2^{|\mathcal{N}_i|}}$ can be derived in a similar to Example 4. Let $z_i^{(j)}(k)$ and $w_i^{(j)}(k)$ denote the j-th element of $z_i(k)$ and $w_i(k)$, respectively. Then, we can obtain

$$\sum_{j=1}^{2^{|\mathcal{N}_i|}} z_i^{(j)}(k) = 1. \quad (11)$$

From $w_i(k) := u_i(k) z_i(k)$, we can obtain

$$\sum_{j=1}^{2^{|\mathcal{N}_i|}} w_i^{(j)}(k) = u_i(k). \quad (12)$$

In addition, we introduce the following constraints:

$$\underline{u}_i z_i(k) \le w_i(k) \le \overline{u}_i z_i(k), \quad (13)$$

$$0 \le z_i(k) \le 1, \quad (14)$$

$$0 \le w_i(k) \le 1. \quad (15)$$

For probabilities, we introduce the following constraint:

$$0 \le a_l^{(i)} + b_l^{(i)} u_i(k) \le 1, \quad l = 1, 2, \ldots, q(i). \quad (16)$$

Thus, we can obtain the following problem as a relaxed problem of Problem 2.

Problem 3. Suppose that for the PBN with (4), the lower and upper bounds of input constraints $\underline{u}_i, \overline{u}_i$, and the initial state $x(0) = x_0$ are given. Then, find $u(0), u(1), \ldots, u(N-1)$ minimizing the cost function (5) subject to (3), (9)–(16).

By a simple calculation, Problem 3 is reduced to an LP problem. By solving Problem 3, we can evaluate the lower bound of the optimal value of the cost function in Problem 3. In this paper, only an approximate solution method is provided. However, since the control input is obtained by solving an LP problem, the proposed solution method using the matrix-based representation enables us to solve the structural control problem for several classes of PBNs.

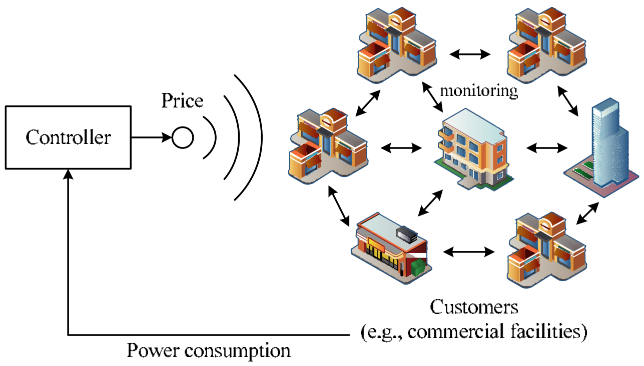

Fig. 1. Illustration of real-time pricing systems.

5. APPLICATION TO DESIGN OF REAL-TIME PRICING SYSTEMS

In this section, we consider a design method of real-time pricing systems as an application of structural control of PBNs. First, the outline of real-time pricing systems of electricity is explained. Next, the PBN-based model of real-time pricing systems is derived. Finally, a numerical example is presented.

5.1 Outline

Fig. 1 shows an illustration of real-time pricing systems studied in this paper. This system consists of one controller and multiple electric customers such as commercial facilities and homes. For an electric customer, we suppose that each customer can monitor the status of electricity conservation of other customers. In other words, the status of some customer affects that of other customers. For example, in commercial facilities, we suppose that the status of rival commercial facilities can be checked by lighting, Blog, Twitter, and so on. Depending on power consumption, i.e., the status of electricity conservation, the controller determines the price. If electricity conservation is needed, then the price is set to a high value. Since the economic load becomes high, customers conserve electricity. Thus, electricity conservation is achieved. The price does not depend on each customer, and is uniquely determined.

5.2 Model

Consider modeling the set of customers as a PBN. The number of customers is given by n. We assume that the state of customer $i \in \{1, 2, \ldots, n\}$ is binary, and is denoted by x_i. The state implies

$$x_i = \begin{cases} 0 & \text{customer } i \text{ conserves electricity,} \\ 1 & \text{customer } i \text{ normally uses electricity.} \end{cases}$$

The binary value of x_i is determined by power consumption of customer i. Let $\mathcal{D}_i \subseteq \{1, 2, \ldots, n\}$, $i = 1, 2, \ldots, n$ denote the set of customers, which affect to customer i. In addition, we assume that there exists one leader in the local area. The state of a leader is given by x_1. Then, for customer i, we consider the following PBN:

$$x_i(k+1) = \begin{cases} f_1^{(i)} = 1, \quad c_1^{(i)} = a_1^{(i)} + b_1^{(i)} u(k), \\ f_2^{(i)} = 0, \quad c_2^{(i)} = a_2^{(i)} + b_2^{(i)} u(k), \\ f_3^{(i)} = x_i(k), \quad c_3^{(i)} = a_3^{(i)} + b_3^{(i)} u(k), \\ f_4^{(i)} = g^{(i)}([x_j(k)]_{j \in \mathcal{D}_i}), \\ \qquad\qquad c_4^{(i)} = a_4^{(i)} + b_4^{(i)} u(k), \\ f_5^{(i)} = x_1(k), \quad c_5^{(i)} = a_5^{(i)} + b_5^{(i)} u(k), \end{cases}$$

where $u(k) \in [\underline{u}, \overline{u}] \subseteq \mathcal{R}^1$ is the control input corresponding to the price. The Boolean functions $f_1^{(i)}$ and $f_2^{(i)}$ imply that customer i forcibly conserves (or does not conserve) electricity. In these cases, time evolution of the state does not depend on the past state. The Boolean function $f_3^{(i)}$ implies that the state is not changed. The Boolean function $f_4^{(i)}$ implies that the state of customer i is changed depending on the other customers. The Boolean function $f_5^{(i)}$ implies that the state of customer i is changed depending on the leader. Thus, decision making of customers can be modeled by a PBN. The above Boolean functions are an example of models for decision making. Depending on real situations, we may use other Boolean functions.

For the PBN-based model obtained, we consider the following problem:

- find a time sequence of the price such that customers conserve electricity as much as possible. However, it is not desirable that the price is too high.

The condition that customers conserve electricity as much as possible can be characterized by $E[x_i]$. In other words, power consumption is expressed by $E[x_i]$. Hence, this problem can be formulated as Problem 1 by appropriately setting the weights Q and R.

5.3 Numerical Example

We present a numerical example. Parameters in the system are given as follows: $n = 8$, $\mathcal{D}_1 = \{2, 8\}$, $\mathcal{D}_i = \{i-1, i+1\}$, $i = 2, 3, \ldots, 7$, $\mathcal{D}_8 = \{1, 7\}$, $a_1^{(i)} = 0.1$, $b_1^{(i)} = 0$, $a_2^{(i)} = 0$, $b_2^{(i)} = 0.25$, $a_3^{(i)} = 0.9$, $b_3^{(i)} = -1$, $a_4^{(i)} = 0$, $b_4^{(i)} = 0.5$, $a_5^{(i)} = 0$, $b_5^{(i)} = 0.25$, $\underline{u} = 0.3$, and $\overline{u} = 0.7$. We remark that under the input constraint $u(k) \in [\underline{u}, \overline{u}]$, (3) and (16) hold. The Boolean function $g^{(i)}$ is given by

$$g^{(i)}([x_j(k)]_{j \in \mathcal{D}_i}) = x_{j_1}(k) \wedge x_{j_2}(k) \wedge \cdots \wedge x_{j_{|\mathcal{D}_i|}}(k), \\ \{j_1, j_2, \ldots, j_{|\mathcal{D}_i|}\} = \mathcal{D}_i.$$

Parameters in Problem 1 are given as follows: $x(0) = [\, 0 \;\; 1 \;\; \cdots \;\; 1 \,]^\top$, $N = 15$, $Q = [\, 1 \;\; \cdots \;\; 1 \,]$, and $R = 5$.

Next, we present the computation result. Fig. 2 shows trajectories of $E[x_i(k)]$. Fig. 3 shows trajectories of the control input (the price). From these figures, we see that $E[x_i]$ becomes small by fine adjustment of the control input. In this example, the expected value of each state converges to 0.32.

In addition, when the obtained control input is applied to the system, the value of the cost function in Problem 1 was 79.4311. In the case of $u(k) = 0.3$ (i.e., the constant

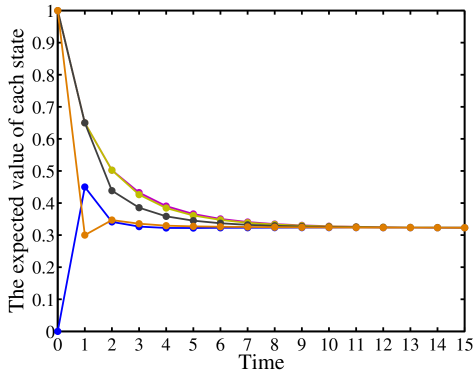

Fig. 2. The expected value of the state. Some states are indistinguishable.

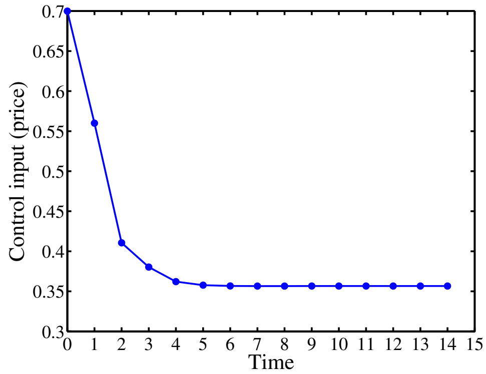

Fig. 3. The obtained control input (price).

input), the value of the cost function in Problem 1 was 85.3581. In the case of $u(k) = 0.7$, the value of the cost function in Problem 1 was 87.0247. From these values, we see that the obtained control input is effective than trivial control inputs. Of course, there is a possibility that the obtained control input is not optimal for Problem 1.

Finally, we discuss the computation time for solving Problem 3. The computation time was 0.6 [sec], where we used IBM ILOG CPLEX 11.0 as the LP solver. Thus, although Problem 3 is an approximation of the original problem, Problem 3 can be solved fast. It will be difficult to solve the original problem (i.e., the polynomial optimization problem) for this case by using MATLAB 32bit.

6. CONCLUSION

In this paper, we discussed the structural control problem for a probabilistic Boolean network (PBN). First, the structural control problem was formulated. Next, an approximate solution method was proposed. Finally, as an application, we considered design of real-time pricing systems of electricity. The proposed method provides us a new control method for complex dynamical systems.

In future work, it is significant to evaluate the accuracy of an approximation from the theoretical viewpoint.

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