Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Trajectory Tracking, Pose Regulation and Adaptive Formation Control of a Group of Nonholonomic Mobile Robots ⋆

Aurelio Lima *, **Josiel A. Gouvea** **, **Fernando Lizarralde** * **and Liu Hsu** *

* *Department of Electrical Engineering-COPPE, Federal University of Rio de Janeiro, 21941-972 RJ, Brazil, e-mail:* [aurelio,fernando,liu]@coep.ufrj.br

** *Department of Electronic Engineering, Federal Center of Technological Education, 20271-110 RJ, Brazil, e-mail:* josiel@cefet-rj.br

Abstract: In this paper the formation and trajectory tracking control problem for multi-agent systems is presented. Initially, a control strategy for a group of holonomic robots is proposed. The proposed control is extended to the nonholonomic case. The control scheme is based on potential functions which make possible the design of decentralized formation control scheme while avoiding agents collisions. The trajectory tracking is achieved defining leaders which attract the formation to a desired trajectory. Furthermore, if at least two leaders are defined, the formation orientation tends to a desired pose (for the planar case). Assuming that the communication graph is always connected, a stability analysis using Lyapunov theory ensures the minimization of the potential function and the trajectory tracking. The control strategies are verified by simulation.

Keywords: multiagent formation, adaptive control, formation orientation, tracking control

1. INTRODUCTION

Formation control of multi-agent systems has received significant attention from the control community due to its wide variety of applications. Recent works show its application in areas like naval engineering (Cui et al., 2010) and aerospace engineering (Abdessameud and Tayebi, 2011). Among several formation control strategies used, we can mention the behavior-based (Antonelli et al., 2010), consensus (Ren et al., 2007; Li et al., 2011), leader-following (Tanner et al., 2004; Chen et al., 2010), group coordination using passivity (Arcak, 2007; Franchi et al., 2011), virtual structures (Beard and Hadaegh, 1998; van den Broek et al., 2009) and potential function (Leonard and Fiorelli, 2001; Hengster-Movrić et al., 2010).

For mobile robots formation control, the main objective is to control each agent using neighbor information in a decentralized control strategy. In this framework, most of the existing results deal with holonomic mobile robots (Pereira et al., 2009; Xiao and Wang, 2008; Tanner et al., 2007). However, in practical applications, mobile robots have to satisfy nonholonomic constraints.

The control design for nonholonomic systems is quite involved, mainly due to the Brockett's condition. Therefore, for agents with nonholonomic constraints, the formation control problem becomes more challenging.

In (Tanner et al., 2004), stability properties of formation of mobile agents based on leader-following are investigated. In (Dierks and Jagannathan, 2007) a combined kinematic/torque control law is proposed for leader-follower based formation control based on backstepping. In (Mastellone et al., 2007), a decentralized control scheme which achieves dynamic formation control and collision avoidance for a group of nonholonomic robots with kinematic model is proposed. The collision avoidance strategy is based on locally defined potential functions which can take different shapes and only require each agent to detect other objects in its neighborhood. In (Dong and Farrell, 2009), a decentralized feedback control of a group of nonholonomic dynamic systems with uncertain parameters is considered. The control scheme is based on consensus, graph theory, and backstepping techniques. In (Gouvea et al., 2010), an adaptive formation control for nonholonomic mobile robots with unknown dynamic parameters is proposed. The control scheme is based on a saturated artificial potential function which allows a decentralized formation control design including collision avoidance. In (Consolini et al., 2008, 2009) a geometric approach for the stabilization of a hierarchical formation of unicycles with velocity and curvature constraints is proposed, and a leader-following strategy is suggested. However, collision avoidance is not considered. Furthermore, a drawback of leader-following strategies is that it depends heavily on the leader for achieving the goal and over-reliance on a single agent in the formation may be undesirable, especially in adverse conditions.

In this paper, we address the problem of coordinating multiple mobile robots (holonomic and nonholonomic) to

⋆ This work was supported in part by CNPq and FAPERJ, Brazil.

follow a specific trajectory while maintaining a rigid predefined geometric formation with a desired orientation. The dynamic model of each robot is assumed to be uncertain. An adaptive control law based on potential functions is proposed to achieve the desired geometric formation. Introducing virtual leaders to the communication graph and thus to the potential function, we can specify the formation position in the inertial frame. Furthermor, for an arbitrary number of virtual leaders the formation can track a predefined trajectory, and with at least two virtual leaders the formation orientation can be controlled.

2. PROBLEM FORMULATION

First, we present a team of N vehicles fully actuated and modeled by a set of Euler-Lagrange equations:

$$M_i(z_i)\ \ddot{z}_i + C_i(z_i, \dot{z}_i)\ \dot{z}_i = \tau_i \tag{1}$$

where $z_i \in \mathbb{R}^n$ is the robot configuration, $M_i(z_i) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $C_i(z_i, \dot{z}_i)\dot{z}_i \in \mathbb{R}^n$ is the centripetal/Coriolis force, $\tau_i \in \mathbb{R}^n$ is the applied torques.

The well-known properties of dynamic model (1) are used for the formation control design:

(P1) $M_i(z_i)$ is symmetric and positive definite.
(P2) $\dot{M}_i(z_i) - 2C_i(z_i, \dot{z}_i)$ is skew symmetric.
(P3) The robot dynamics is linearly parameterizable, i.e., $M_i(z_i)\dot{v}_i + C_i(z_i, \dot{z}_i)v_i = Y_i(z_i, \dot{z}_i, v_i, \dot{z}_i)\ \theta_i^*$ where $Y_i(.)$ is a regressor matrix and θ_i^* is the parameter vector.

Now, consider a team of N nonholonomic mobile robots. For $i = 1, \cdots N$, the dynamic model of each robot is described by:

$$M_i(z_i)\ \ddot{z}_i + C_i(z_i, \dot{z}_i)\ \dot{z}_i = B_i(z_i)\ \tau_i + A^T(z_i)\ \lambda_i \tag{2}$$

$$A(z_i)\ \dot{z}_i = 0 \tag{3}$$

where $A(z_i) \in \mathbb{R}^{k\times n}$ characterizes the kinematic constraints, $\lambda_i \in \mathbb{R}^k$ is the constraint multipliers, $B_i(z_i) \in \mathbb{R}^{n\times p}$ is a known input matrix and $p = n - k$.

Defining $R_i(z_i) \in \mathbb{R}^{n\times m}$ such that $A(z_i)R_i(z_i) = 0$, one can replace constraints (3) with the kinematic model

$$\dot{z}_i = R_i(z_i)\ v_i \tag{4}$$

where $v_i \in \mathbb{R}^m$ is the vector of *pseudo*-velocities. Considering that $\ddot{z}_i = R_i(z_i)\dot{v}_i + \dot{R}_i(z_i)v_i$ and substituting (4) in (2), after some algebraic manipulation, the robot dynamic model can be expressed as

$$M_{Ri}(z_i)\ \dot{v}_i + R_i^T(z_i)C_{Ri}(z_i, \dot{z}_i)\ v_i = R_i^T(z_i)B_i(z_i)\ \tau_i \tag{5}$$

where $M_{Ri}(z_i) = R_i^T(z_i)M_i(z_i)R_i(z_i)$ and $C_{Ri}(z_i, \dot{z}_i) = M_i(z_i)\dot{R}_i(z_i) + C_i(z_i, \dot{z}_i)R_i(z_i)$.

The robot dynamic model (5) also satifies properties (P1)-(P3) (Bloch et al., 1992).

The topology of information exchange among robots is described by a graph (Biggs, 1994). Then, the N mobile robots are represented as N vertices of a graph $G := \{V, E\}$, where $V := \{v_1...v_N\}$ is the set of vertices that represent the robots and $E \subseteq V \times V$ is the set of edges that define the neighborhood relationship among robots. Thus, the set of neighbors of robot i is $N_i := \{j | e_{ij} = (v_i, v_j) \in E\}$. The available information for the controller of robot i is only the states of robot i and robot j for $j \in \mathcal{N}_i$. A path of length r from robot i to robot j is a sequence of $r + 1$ distinct vertices starting with i and ending with j such that consecutive vertices are neighbors. If there is a path between any two vertices of a graph $\mathcal{G}$, then $\mathcal{G}$ is said to be connected. A graph is undirected if the edges have no orientation $((i, j) = (j, i) \in \mathcal{E})$.

In this paper, we assume that the formation graph is connected and undirected.

The control objective is to drive the N agents to a formation which minimizes

$$J = \sum_{i=1}^{N} J_i + \sum_{i}^{m} J_{ri}(z_i - z_{ri}), \tag{6}$$

with

$$J_i = \sum_{i,j\in N_i}^{N} J_{ij}(z_{ij}) \tag{7}$$

where $m \leq N$ is the number of virtual leaders, z_{ri} is the virtual leader configuration, $z_{ij} = z_i - z_j$ and J_{ij}, J_{ri} are defined as follows (Leonard and Fiorelli, 2001):

Definition 1. The potential function J_{ij} is a differentiable, nonnegative function of the distance z_{ij} between agents i and j, such that

(1) $J_{ij}(z_{ij}) \to \infty$ as $\|z_{ij}\| \to \infty$ and $\|z_{ij}\| \to 0$.
(2) J_{ij} attains its unique minimum when agents i and j are located at a desired relative position d_{ij}.

Definition 2. The tracking potential function J_{ri} is a differentiable, nonnegative function of the agent position $\|z_i\|$ such that

(1) $J_{ri}(z_i - z_{ri}) \to \infty$ as $\|z_i\| \to \infty$.
(2) J_{ri} attains its unique minimum for $z_i = z_{ri}$.

Note that J_{ij} defines the desired distances between the neighbors while J_{ri} defines the desired trajectory and the formation orientation. Furthermore, the tracking potential function J_{ri} has to satisfy the following constraint:

$$d_{ij} = \|z_{ri} - z_{rj}\| \tag{8}$$

where the chosen trajectories z_{ri} must be compatible with the desired distances between robots d_{ij} defined in J_{ij}.

In the planar case, similar to a planar rigid body, the formation orientation can be defined by the vector $z_{ri} - z_{rj}$.

3. FORMATION CONTROL OF DYNAMIC HOLONOMIC ROBOTS

In this case, consider the planar position of N robots with dynamic model (1). In order to explore the passivity property (P2) of the dynamic model, the auxiliary error function s_i and its derivative are defined:

$$s_i = \dot{z}_i - \dot{z}_{di} \quad ; \quad \dot{s}_i = \ddot{z}_i - \ddot{z}_{di} \tag{9}$$

where z_{di} is a desired kinematic state which ensures that the tracking trajectory and desired formation are reached. The control goal is $s_i \to 0$ as $t \to \infty$.

Substituting (9) in (1),

$$M_i\dot{s}_i + C_i s_i = \tau_i - M_i\ddot{z}_{di} - C_i\dot{z}_{di}. \tag{10}$$

The desired kinematic model is given by:

$$\begin{aligned} \dot{z}_{di} &= -k_{fi}\nabla_{z_i}J + \dot{z}_{ri} \qquad i = 1, \cdots, m \\ \dot{z}_{di} &= -k_{fi}\nabla_{z_i}J \qquad i = m+1, \cdots, N \end{aligned} \tag{11}$$

where k_{fi} is a positive constant.

Two formation control laws are proposed. First, a control law based on neighbors velocities and relative positions information is proposed. Later, the control law is extended to the case where the neighbor velocities information is not available.

3.1 Adaptive control using neighbour velocity information

Defining the linear parametrization (P3)

$$Y_i\theta_i^* = -M_i\ddot{z}_{di} - C_i\dot{z}_{di}, \tag{12}$$

the following control law is proposed:

$$\tau_i = Y_i\theta_i - K_{Di}s_i. \tag{13}$$

where K_{Di} is a positive constant. The closed-loop dynamic (12), (13) and (10), is given by

$$M_i\dot{s}_i + (C_i + K_{Di})s_i = Y_i\tilde{\theta}_i \tag{14}$$

where $\tilde{\theta}_i = \theta_i - \theta_i^*$ is the estimation error.

The following adaptive law based on B-MRAC (Hsu and Costa, 1990) is used:

$$\dot{\theta}_i = -\sigma\theta_i - \Gamma_i Y_i^T s_i \tag{15}$$

where $\Gamma_i \in \mathbb{R}^{n\times n}$ is the adaptive gain matrix and σ is the following projection factor:

$$\sigma = \begin{cases} 0 & ;\text{se } |\theta_i| < M_{\theta_i} \text{ or } \sigma_{eq} < 0 \\ \sigma_{eq} & ;\text{se } |\theta_i| \geq M_{\theta_i} \text{ and } \sigma_{eq} \geq 0 \end{cases} \tag{16}$$

where $\sigma_{eq} = -\theta_i\Gamma_i Y_i^T s_i/|\theta_i|^2$ and $M_{\theta_i} > |\theta_i^*|$ is a constant.

The first result of paper is stated in the following theorem.

Theorem 1. Consider a group of N robots modeled by (1), with binary adaptive control law (13) and $m \leq N$ virtual leaders. For sufficiently large K_{Di} and k_{fi} and $\|\dot{z}_r\| < l_r$ where l_r is a constant, there exists a sufficient small α such that:

- $\|z_i(t) - z_{ri}(t)\| \to O(\alpha)$ as $t \to \infty$ for $i = 1, ...m$
- all closed loop signals are uniformly bounded and the multi-agent system tends asymptotically to some constant formation corresponding to $\nabla_z J \to O(\alpha)$
- For $m \geq 2$ the formation orientation converges to the orientation defined by the m virtual leaders.

Proof: Consider the following Lyapunov function:

$$2V = \sum_{i=1}^{N}(s_i^T M_i s_i + \tilde{\theta}^T\Gamma^{-1}\tilde{\theta}) + 2\alpha J(z). \tag{17}$$

It can be shown that:

$$\dot{V} \leq -s^T K_D s + \alpha[\nabla_z J]^T\dot{z} \tag{18}$$

where $s = [s_1^T \ \cdots \ s_N^T]^T$, $z = [\,z_1^T \ ... \ z_N^T\,]^T$, $K_D = diag\{K_{Di}\}$ and $\nabla_z J = [\,\nabla_{z_1}J^T \ ... \ \nabla_{z_N}J^T\,]^T$. Rewriting (9) in term of (11)

$$\dot{z}_i = s_i - k_{fi}\nabla_{z_i}J + \dot{z}_r$$

one has that

$$\dot{V} \leq -s^T K_D s + \alpha e_f^T s - \alpha e_f^T K_f e_f + \alpha e_f^T\dot{z}_r \tag{19}$$

where $e_f = \nabla_z J$, $K_f = diag\{k_{fi}\}$ and

$$\dot{z}_r = [\dot{z}_{r1}^T \ \cdots \ \dot{z}_{rm}^T \ 0 \ \cdots \ 0]^T.$$

Then defining $e^T = [s^T e_f^T]$

$$\dot{V} \leq -e^T\begin{bmatrix} K_D & -(\alpha/2)I \\ -(\alpha/2)I & \alpha K_f \end{bmatrix} e + e^T\begin{bmatrix} 0 \\ \alpha I \end{bmatrix}\dot{z}_r \tag{20}$$

From the Schur complement of

$$K = \begin{bmatrix} K_D & -(\alpha/2)I \\ -(\alpha/2)I & \alpha K_f \end{bmatrix}$$

one can concluded that K is definite positive if

$$\alpha K_f - \frac{\alpha^2}{4}K_D^{-1} > 0$$

$$\sigma_m(K_D) > \frac{\alpha}{4\sigma_m(K_f)} \tag{21}$$

which are satisfied for sufficiently small α.

Since $\|\dot{z}_r\| < l_r$, it can be shown that $\dot{V} \leq 0$ outside a D_r domain given by:

$$D_r = \{e : \|e\| \leq l_r\alpha/\sigma_m(K)\} \tag{22}$$

For K_D and K_f large enough and $\|\dot{z}_r\| < l_r$, there exists α such that $\|e\|$ (and consequently $\nabla_z J$) tends to a residual set of order $O(\alpha)$.

Since $\|z_i - z_{ri}\|$ tends to a residual set of order $O(\alpha)$, for $m \geq 2$, the formation orientation converges to the desired orientation with a residual error of order $O(\alpha)$. ■

3.2 Adaptive control without neighbour velocity information

Here, formation and tracking control is only based on relative positions among neighbors. Similar to the previous case, define a new linear parametrization:

$$Y_i\theta_i^* = -M_i\ddot{z}_{ri} - C_i\dot{z}_{di}. \tag{23}$$

Then the following control law is proposed:

$$\tau_i = Y_i\theta_i - K_{Di}s_i. \tag{24}$$

Combining equations (23), (24) and (10) we get:

$$M_i\dot{s}_i + (C_i + K_{Di})\ s_i = Y_i\tilde{\theta}_i - M_i\ddot{z}_{fi} \tag{25}$$

where $\tilde{\theta}_i = \theta_i - \theta_i^*$ is the estimation error and $\ddot{z}_{fi} = -k_{fi}d[\nabla_{z_i}J]/dt$.

Similar to the previous section, the adaptive law (15) based on B-MRAC (Hsu and Costa, 1990) is used. The main result is stated in the following theorem.

Theorem 2. Consider a group of N robots modeled by (1), with binary adaptive control law given by (24) and $m \leq N$ virtual leaders. For sufficiently large K_D and K_f and $|\dot{z}_r| < l_r$, there exist constant α sufficiently small such that

- $|z_i(t) - z_{ri}(t)| \to O(\alpha))$ as $t \to \infty$ for $i = 1, ...m$
- all closed loop signals are uniformly bounded and the multi-agent system tends asymptotically to some constant formation corresponding to $\nabla_z J \to O(\alpha)$.
- For $m \geq 2$ the formation orientation converges to the orientation defined by the m virtual leaders.

Proof: Considering Lyapunov function (17), one has that:

$$\dot{V} \leq -s^T K_D s - s^T M\ddot{z}_f + \alpha[\nabla_z J]^T\dot{z} \tag{26}$$

where $\ddot{z}_f = [\,z_{fi} \ ... \ z_{fn}\,]$. Rewriting $\ddot{z_{fi}}$ in term of $\dot{z}_i$,

$$\ddot{z}_{fi} = -\left[\frac{\partial^2 J}{\partial z_1\partial z_i}, ..., \frac{\partial^2 J}{\partial^2 z_i}, ..., \frac{\partial^2 J}{\partial z_N\partial z_i}\right]\dot{z}_i. \tag{27}$$

and $\ddot{z}_f = H\dot{z}$ where H is the Hessian matrix, and noting from (9) that $\dot{z}_i = s_i - k_{fi}\nabla_{z_i}J + \dot{z}_r$, one has that

$$\dot{V} \leq -s^T K_D s + s^T MH\dot{z} + \alpha[\nabla_z J]^T\dot{z} \tag{28}$$

and

$$\dot{V} \leq -s^T K_D s + s^T MHs - \alpha s^T MHK_f e_f + s^T MH\dot{z}_r + \\ +\alpha e_f^T s - \alpha e_f^T K_f e_f + \alpha e_f^T \dot{z}_r \quad (29)$$

where $e_f = \nabla_z J$, $K_f = diag\{K_{fi}\}$ and $\dot{z}_r = [\dot{z}_{r1}^T \, . \, \dot{z}_{rm}^T \, 0 \, . \, 0]^T$. Furthermore,

$$\dot{V} \leq -[s^T e_f^T] \begin{bmatrix} \bar{K_D} & -1/2L(\alpha) \\ -1/2L(\alpha)^T & \alpha K_f \end{bmatrix} \begin{bmatrix} s \\ e_f \end{bmatrix} + \\ +[s^T e_f^T] \begin{bmatrix} MH \\ \alpha I \end{bmatrix} \dot{z}_r \quad (30)$$

where $\bar{K_D} = K_D - MH$, $L(\alpha) = MHK_f - \alpha I$.

Since, from property (P1), $|M| < \sigma_M$ where σ_M is a positive constant and in a compact domain defined by $V \leq c$, for a arbitrary large $c > 0$, the Hessian Matrix H is bounded by $\|H\| \leq \sigma_H$.

From Schur complement of matrix K

$$K = \begin{bmatrix} \bar{K_D} & -1/2L(\alpha) \\ -1/2L(\alpha)^T & \alpha K_f \end{bmatrix}$$

the following condition such that K is definite positive is obtained:

$$\alpha K_f - \frac{1}{4} L(\alpha)^T \bar{K_D}^{-1} L(\alpha) > 0. \quad (31)$$

which is satisfied if

$$\alpha \sigma_m(K_f) \sigma_m(\bar{K_D}) > \frac{1}{4\alpha} \sigma_M^2 L(\alpha) \quad (32)$$

For $\alpha = \sigma_M(MHK_f)$, one has that

$$\sigma_m(K_f) \sigma_m(\bar{K_D}) > 0. \quad (33)$$

Thus, the condition is satisfied.

Defining $e = [s^T e_f^T]^T$, it can be shown that $\dot{V} \leq 0$ outside domain D_r given by:

$$D_r = \{e : |e| \leq \sigma_M(MH)/\sigma_m(K) l_r\} \quad (34)$$

For K_D and K_f large enough and $\|\dot{z}_r\| < l_r$, there exists α such that $\|e\|$ (and consequently $\nabla_z J$) tends to a residual set of order $O(\alpha)$.

The residual set is given by $V \leq c_r$ where $c_r = sup_{D_r} J$. Assuming that it is possible to approximate J quadratically about an equilibrium configuration given by $\nabla_z J = 0$, it can be concluded that within the residual set, $|e|$ is of the same order.

Since $\|z_i - z_{ri}\|$ tends to a residual set of order $O(\alpha)$, for $m \geq 2$, the formation orientation converges to the desired orientation with a residual error of order $O(\alpha)$. ■

4. DYNAMIC NONHOLONOMIC ROBOTS

Now consider a team of nonholonomic mobile robot (5). For the unicycle case, one can define $z_i^T = [x_i \quad y_i \quad \psi_i]$, $v_i^T = [u_i \quad \omega_i]$ and

$$R_i = \begin{bmatrix} \cos(\psi_i) & 0 \\ \sin(\psi_i) & 0 \\ 0 & 1 \end{bmatrix} \quad (35)$$

where x_i and y_i are the cartesian coordinates, ψ_i is the robot orientation and u_i, ω_i are the linear and angular velocities respectively.

For the kinematic model (4), it can be proposed a kinematic control $v_i = v_{di}$, with $v_{di} = [u_{di} \quad \omega_{di}]$ $(i = 1, ..., m)$,

$$u_{di} = -k_{fi} e_{fi} + \bar{u}_{ri} \quad (36)$$

$$w_{di} = u_{ri} k_{ri} \bar{e}_{ri} + k_{wi} e_{\psi ri} + w_{ri} \quad (37)$$

for the robots tracking the virtual leaders and $(i = m + 1, ..., N)$

$$u_{di} = -k_{fi} e_{fi} \quad (38)$$

$$w_{di} = -k_{wi} e_{\psi fi} \quad (39)$$

for the others robots of formation, where $e_{fi} = \nabla_{z_i} J^T R_{li}$ is the projection of $\nabla_{z_i} J^T$ in nonholomic motion direction $R_{li} = [\cos(\psi_i) \sin(\psi_i)]$; $e_{\psi ri} = \sin(\psi_i - \psi_{ri})$ is the orientation error of robot i with respect to its virtual leader orientation, $\bar{e}_{ri} = (z_i - z_{ri})^T R_{ni}$ is the projection of $e_{ri} = (z_i - z_{ri})$, the relative position among robot i and it virtual leader, in $R_{ni} = [-\sin(\psi_i) \cos(\psi_i)]^T$, the space of directions orthogonal to R_{li} and $\bar{u}_{ri} = u_{ri} \cos(\psi_{ri} - \psi_i)$ is the projection of velocity of a virtual leader in direction of its follower. The configuration of each virtual leader z_{ri} is defined by

$$z_{ri}(t) = z_{r1}(t) + d_i(t) \quad (40)$$

where $z_{r1}(t) = [\, x_{r1}(t) \; y_{r1}(t) \; \psi_{r1}(t) \,]^T$ is the leader trajectory defined by

$$\dot{z}_{r1} = \begin{bmatrix} \cos(\psi_{r1}) & 0 \\ \sin(\psi_{r1}) & 0 \\ 0 & 1 \end{bmatrix} v_{r1}(t) \quad (41)$$

where $v_{r1}^T = [u_{ri} \quad w_{ri}]$ and u_{ri}, w_{ri} are the respective linear and angular velocities of leader i. $d_i(t)$ is used to define the desired orientation of formation and would be chosen to obey the geometric pattern defined by potential function.

Since, for the dynamic model, the robot velocities can be not defined instantly, it is defined the auxiliary error

$$s_i = v_i - v_{di}. \quad (42)$$

Substituting (42) in (5), one has that

$$M_{Ri} \dot{s}_i + R_i^T C_{Ri} s_i = R_i^T B_i \tau_i - M_{Ri} \dot{v}_{di} - R_i^T C_{Ri} v_{di} \quad (43)$$

The following dynamic control law is proposed:

$$\tau_i = (R_i^T B_i)^{-1} (Y_i \theta_i - K_{Di} s_i). \quad (44)$$

where $Y_i \theta_i^* = M_{Ri} \dot{v}_{di} + R_i^T C_{Ri} v_{di}$ is the linear parametrization, and $\tilde{\theta}_i = \theta_i - \theta_i^*$ is the parameter estimation error.

Replacing (44) in (43), one has that

$$M_{Ri} \dot{s}_i + (R_i^T C_{Ri} + K_{Di}) \; s_i = Y i \tilde{\theta}_i. \quad (45)$$

In order to estimate θ, the adaptive law (15) is used.

The main result is stated in the following Theorem.

Theorem 3. Consider a group of N robots modeled by (2), with binary adaptive control law given by (44) and following $m \leq N$ virtual leaders. For sufficiently large K_{Di},k_{fi},k_{wi} and k_{ri}, and assuming $|\dot{z}_r| < l_r$, for some positive constant l_r, then there exists α such that

- $|z_i(t) - z_{ri}(t)| \to O(\alpha)$ as $t \to \infty$ for $i = 1, ...m$
- all the closed loop signals are limited and a formation corresponding to residual value $\nabla_J = O(\alpha)$ is achieved asymptotically.

- For $m \geq 2$ the formation orientation converges to the orientation defined by the m virtual leaders.

Proof: Considering the Lyapunov candidate function

$$V = \sum_{i=1}^{N} \frac{1}{2}\left(s_i^T M_{Ri} s_i + \tilde{\theta}^T \Gamma^{-1}\tilde{\theta}\right) + \alpha J(z) + \alpha \sum_{i=1}^{m}\left(1 - \cos(\psi_{ri} - \psi_i) + \frac{k_{ri}}{2}\bar{e}_{ri}^2\right) + \alpha \sum_{i=m+1}^{N} \frac{1}{2} e_{\psi fi}^2, \quad (46)$$

one can concluded that:

$$\dot{V} \leq -s^T K_D s + \alpha[\nabla_z J]^T \dot{z} + \alpha e_{\psi r}^T w_r - \alpha e_{\psi r}^T \bar{w} + \alpha \bar{e_r}^T K_r \dot{\bar{e_r}} + \alpha e_{\psi f} \tilde{w} - \alpha e_{\psi f} \dot{\psi}_f \quad (47)$$

where $s = [s_1^T \cdots s_N^T]^T$, $K_D = diag\{K_{Di}\}$, $\nabla_z J = [\nabla_{z1} J^T \cdots \nabla_{zN} J^T]^T$, $z = [z_1^T \cdots z_N^T]^T$, $e_{\psi r} = [e_{\psi r1} \cdots e_{\psi rm}]^T$, $w_r = [w_{r1} \cdots w_{rm}]^T$, $\bar{w} = [w_1 \cdots w_m]^T$, $\bar{e_r} = [\bar{e_{r1}} \cdots \bar{e_{rm}}]^T$, $e_{\psi f} = [e_{\psi fm+1} \cdots e_{\psi fN}]^T$, $\tilde{w} = [w_{m+1} \cdots w_N]$, $\psi_f = [\psi_{fm+1} \cdots \psi_{fN}]$.

Replacing (42) in (47) and after some algebraic manipulation,

$$\dot{V} \leq -s^T K_D s + \alpha e_f^T s_u - \alpha e_f^T K_f e_f + \alpha e_f^T \bar{(u)}_r - \alpha e_{\psi r} s_{wr} - \alpha e_{\psi r}^T K_{wr} e_{\psi r} + \alpha e_{\psi f} s_{wf} - \alpha e_{\psi f}^T K_{wf} e_{\psi f} - \alpha e_{\psi f} L_a s_{uf}. \quad (48)$$

Defining $e = \left[\, e_f^T \; e_{\psi r}^T \; e_{\psi f}^T \,\right]^T$, $K = \begin{bmatrix} K_f & 0 & 0 \\ 0 & K_{wr} & 0 \\ 0 & 0 & K_{wf} \end{bmatrix}$, $L = \begin{bmatrix} I & 0 \\ 0 & -I \\ L_a & I \end{bmatrix} T$, $v = T^{-1}[u^T w^T]^T$ and $E = \begin{bmatrix} I & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$, one can concluded that

$$\dot{V} \leq -[s^T e^T] \begin{bmatrix} K_D & -\alpha/2L^T \\ -\alpha/2L & \alpha K \end{bmatrix} \begin{bmatrix} s \\ e \end{bmatrix} + [s^T e^T] \begin{bmatrix} 0 \\ \alpha E \end{bmatrix} \dot{v}_r \quad (49)$$

Using the Schur's complement for $A = \begin{bmatrix} K_D & -\alpha/2L^T \\ -\alpha/2L & K \end{bmatrix}$, one can concluded that $A > 0$ if

$$\alpha K - \frac{1}{4}\alpha^2 L^T K_D^{-1} L > 0. \quad (50)$$

which is satisfied for

$$\sigma_m(K)\sigma_m(K_D) > \frac{\alpha \sigma_M^2(L)}{4} \quad (51)$$

for α small enough. Defining $\bar{e} = [s^T e^T]^T$, it can be shown that $\dot{V} \leq 0$ if $\bar{e} \notin D_r$, where

$$D_r = \{\bar{e} : |\bar{e}| \geq \alpha/\sigma_m(A)|v_r|\} \quad (52)$$

Assuming that $|v_r| < l_r$ where l_r is a constant, for sufficiently large K_D and K there exists α such that $|\bar{e}|$ tends to a residual set of order $O(\alpha_r)$.

The residual set is defined by $V_c \leq c_r$, where $c_r = sup_{D_r} V_c$. Assuming that J can be approximated quadratically about an equilibrium configuration defined by $\nabla_z J = 0$, one can concluded that, in residual set, the order of $|\bar{e}|$ is the same as in D_r. Since $\|z_i - z_{ri}\|$ tends to a residual set of order $O(\alpha)$, for $m \geq 2$, the formation orientation converges to the desired orientation with a residual error of order $O(\alpha)$. ■

5. SIMULATION RESULTS

In this section, simulation results are presented to illustrate the proposed cooperative control design and performance of theoretical results. Due to lack of space, only the more difficult problem of trajectory tracking case (non-holonomic robots without using neighbor velocity information) will be discussed.

The dynamic model of each robot is a particular of (4) and (5) given by

$$M_{Ri}\, \dot{v}_i + C_{Ri}(\dot{z}_i)\, v_i = \tau_i \quad (53)$$

$$\dot{z}_i = R_i(z_i)\, v_i \quad (54)$$

where $M_{Ri} = \begin{bmatrix} 22.02 & 0.86 \\ 0.86 & 22.02 \end{bmatrix}$, $C_{Ri} = \begin{bmatrix} 0 & 7.94\dot{\psi}_i \\ -7.94\dot{\psi}_i & 0 \end{bmatrix}$.

The potential function among vehicles (J_{ij}) and among the vehicles and its respectively virtual leaders (J_{ri}) are described by

$$J_{ij} = \frac{a_{ij}}{2}\|z_{ij}\|^2 + \frac{b_{ij}c_{ij}}{2}\exp^{(-\|z_{ij}\|^2/c_{ij})}$$

$$J_{ri} = \|z_i - z_{ri}\|^2$$

where $a_{ij} = 0.01$, $b_{ij} = 10$, $c_{ij} = d_{ij}^2/log(b_{ij}/a_{ij})$ and $d_{ij} = 0.3$. The controller parameters are $K_{Di} = 100$, $k_{fi} = 1$, $k_{wi} = 0.1$ and $k_{ri} = 100$.

The simulation illustrates the case of a formation with three agents trackinkg a circular trajectory while maintaining the desired geometric pattern, which is an equilateral triangle with side lengths of $0.3m$. Figures 1, 2 and 3 show the trajectory of formation, tracking error and formation error using two virtual leaders.

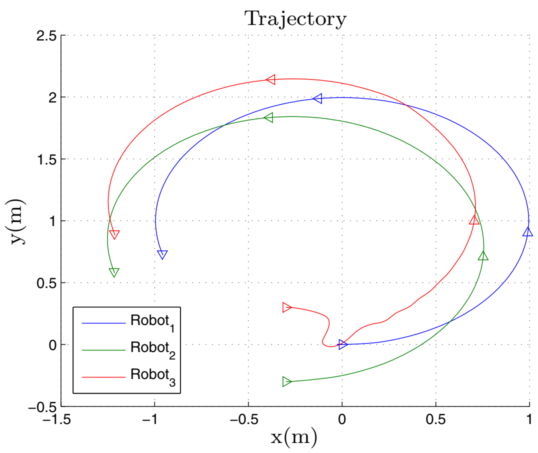

Fig. 1. Robots trajectories.

Note that, except for a residual error, the formation orientation as well as its desired geometric pattern is maintained while the desired trajectory is tracked. This result is consistent with the statement of the Theorem 3.

6. CONCLUSION

This paper presents the formation and trajectory tracking control problem for multi-agent systems. First, a control law for a group of holonomic robots was proposed. Then, the proposed control law was extended to non-holonomic

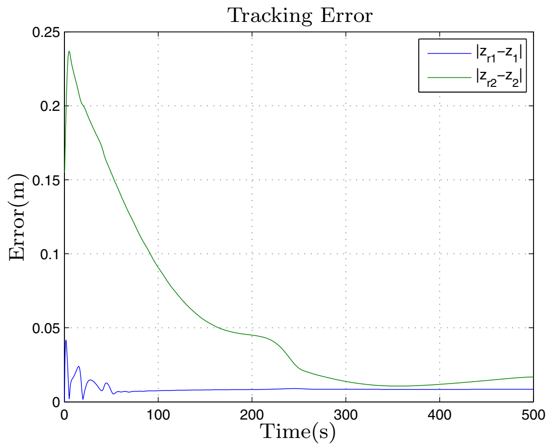

Fig. 2. Tracking error.

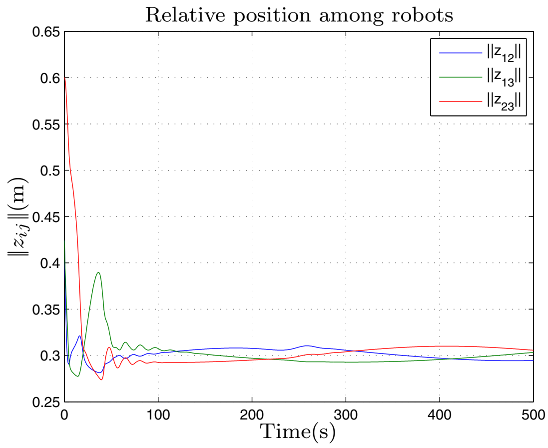

Fig. 3. Formation error: Relative position among robots.

case. The control scheme does not require neighbor velocity knowledge and it is based on a potential function, which made possible the design of decentralized formation controls scheme and avoided agents collisions. The trajectory tracking was achieved defining leaders agents which have the role of attracting the formation to desired trajectory. Assuming that the communication graph is always connected, a stability analysis using Lyapunov theory ensured that, except for a small residual error, the desired formation as well as the trajectory tracking are reached. Simulation results illustrated the performance of the control system proposed.

REFERENCES

Abdessameud, A. and Tayebi, A. (2011). Formation control of vtol unmanned aerial vehicles with communication delays. *Automatica*, 47(11), 2383–2394.

Antonelli, G., Arrichiello, F., and Chiaverini, S. (2010). Flocking for multi-robot systems via the null-space-based behavioral control. *Swarm Intelligence*, 4(1), 37–56.

Arcak, M. (2007). Passivity as a design tool for group coordination. *IEEE Trans. Automat. Contr.*, 52, 1380–1390.

Beard, R.W. and Hadaegh, F.Y. (1998). Constellation templates: An approach to autonomous formation flying. In *World Automation Congress*, 177.1–177.6. Anchorage (AK).

Biggs, N. (1994). *Algebraic Graph Theory*. Cambridge University Press.

Bloch, A.M., Reyhanoglu, M., and McClamroch, N.H. (1992). Control and stabilization of nonholonomic dynamic systems. *IEEE Trans. Automat. Contr.*, 37, 1746–1757.

Chen, J., Sun, D., Yang, J., and Chen, H. (2010). Leader-follower formation control of multiple non-holonomic mobile robots incorporating a receding-horizon scheme. *The International Journal of Robotics Research*, 29(6), 727–747.

Consolini, L., Morbidi, F., Prattichizzo, D., and Tosques, M. (2008). Leader-follower formation control of nonholonomic mobile robots with input constraints. *Automatica*, 44(5), 1343–1349.

Consolini, L., Morbidi, F., Prattichizzo, D., and Tosques, M. (2009). Stabilization of a hierarchical formation of unicycle robots with velocity and curvature constraints. *IEEE Transactions on Robotics*, 25(5), 1176–1184.

Cui, R., Sam Ge, S., Voon Ee How, B., and Sang Choo, Y. (2010). Leader–follower formation control of underactuated autonomous underwater vehicles. *Ocean Engineering*, 37(17), 1491–1502.

Dierks, T. and Jagannathan, S. (2007). Control of nonholonomic mobile robot formations: Backstepping kinematics into dynamics. In *IEEE Int. Conf. on Control Applications*, 94–99. Singapore.

Dong, W. and Farrell, J.A. (2009). Decentralized cooperative control of multiple nonholonomic dynamic systems with uncertainty. *Automatica*, 45, 706–710.

Franchi, A., Giordano, P.R., Secchi, C., Son, H.I., and Bulthoff, H.H. (2011). A passivity-based decentralized approach for the bilateral teleoperation of a group of uavs with switching topology. In *Robotics and Automation (ICRA), 2011 IEEE International Conference on*, 898–905. IEEE.

Gouvea, J.A., Pereira, A.R., Hsu, L., and Lizarralde, F. (2010). Adaptive formation control of dynamic nonholonomic systems using potential functions. In *American Control Conference*, 1865–1870. Baltimore (MD).

Hengster-Movrić, K., Bogdan, S., and Draganjac, I. (2010). Multi-agent formation control based on bell-shaped potential functions. *Journal of Intelligent and Robotic Systems*, 58(2), 165–189.

Hsu, L. and Costa, R.R. (1990). Binary control approach to design globally exponential stable systems. In *7th International Conf. on Systems Engineering, Las Vegas, USA*.

Leonard, N.E. and Fiorelli, E. (2001). Virtual leaders, artificial potentials and coordinated control of groups. In *IEEE Conf. on Decision and Control*, 2968–2873. Orlando (FL).

Li, S., Du, H., and Lin, X. (2011). Finite-time consensus algorithm for multi-agent systems with double-integrator dynamics. *Automatica*, 47(8), 1706–1712.

Mastellone, S., Stipanovic, D.M., and Spong, M.W. (2007). Remote formation control and collision avoidance for multi-agent nonholonomic systems. In *IEEE Int. Conf. on Robotics and Automation*, 1062–1067. Roma (Italy).

Pereira, A.R., Hsu, L., and Ortega, R. (2009). Globally stable adaptive formation control of euler-lagrange agents via potential functions. In *American Control Conference*, 2606–2611. St. Louis (MO).

Ren, W., Beard, R.W., and Atkins, E.M. (2007). Information consensus in multivehicle cooperative control. *IEEE Control Systems Magazine*, 27, 71–82.

Tanner, H.G., Jadbabaie, A., and Pappas, G.J. (2007). Flocking in fixed and switching networks. *IEEE Trans. Automat. Contr.*, 52(5), 863–868.

Tanner, H.G., Pappas, G.J., and Kumar, V. (2004). Leader-to-formation stability. *IEEE Trans. on Robotics and Automation*, 20, 443–455.

van den Broek, T.H., van de Wouw, N., and Nijmeijer, H. (2009). Formation control of unicycle mobile robots: a virtual structure approach. In *48th IEEE Conf. on Decision and Control*, 8328–8333. IEEE.

Xiao, F. and Wang, L. (2008). Asynchronous consensus in continuous-time multi-agent systems with switching topology and time-varying delays. *IEEE Trans. Automat. Contr.*, 53(8), 1804–1816.