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Guaranteed Set-based Controller Parameter Estimation for Nonlinear Systems – Magnetic Levitation Platform as a Case Study

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Abstract: The design of a suitable controller often consists of two steps: first, the choice of a specific controller structure; second, the choice of suitable controller parameters achieving the desired performance. In case of uncertainties and nonlinearities, choosing suitable controller parameters that lead to a satisfying performance is challenging. Simulation of single parameter values does often not provide enough information and insight, especially in the case when the robustness with respect to noise or model-plant mismatch have to be taken into account. We propose a set-based method for deriving guaranteed outer bounds on the admissible controller parameter values, such that the system satisfies performance constraints with respect to a set of initial conditions and desired terminal conditions – set-points. In terms of robustness we consider unknown-but-bounded parameter values, as well as bounded actuator and sensor noise. The outlined approach is illustrated considering the design of a set-point change controller of a magnetic levitation platform, and comparing the theoretically guaranteed estimation results to the performance of the actual experiment.

Keywords: nonlinear control, bounded disturbances, controller parametrization, set-based approach

1. INTRODUCTION

Tuning a controller, i.e. finding a controller parametrization such that the performance specifications of the closed loop are satisfied, is a challenging task. This is further complicated if uncertain nonlinear systems, as they typically appear for industrial plants, are considered.

However, the majority of the controllers used in industrial applications belong to the class of proportional-integral-derivative (PID) controllers (Åström and Hägglund, 2010), or model-based controllers exploiting linear models. A number of tuning rules exists for such controllers, yet finding suitable parameters is often achieved in an ad hoc trial-and-error method exploiting look-up tables or frequency-response based methods (Gao, 2006).

At the same time, tuning methods for nonlinear systems are limited, especially when measurement noise and parametric uncertainty have to be taken into account. Even more challenging, in some instances (e.g. to ensure safe operation) the controller has to perform according to specified qualitative or quantitative constraints.

A possibility to tackle these problems are set-based approaches for nonlinear systems. Set-based methods are an alternative way to analyze system properties subject to uncertainties, such as robustness or stability (Streif et al., 2013b). They rely on the notion of unknown-but-bounded constraints, allowing the variables to attain any value within a defined set. Such a formulation results in a guaranteed region of admissible solutions, that satisfy system constraints. For a general overview of the set-based methods we refer to (Milanese et al., 1996; Blanchini and Miani, 2008; Rumschinski et al., 2010a). Application of the methods to controller design was illustrated i.a. in (Jaulin and Walter, 1996; Kieffer et al., 2002), where interval analysis methods are employed, and (Korda et al., 2013), where the set approximations were obtained via occupation measures.

This work outlines how set-based methods can be used for guaranteed controller parametrization subject to uncertainties. For this purpose, the methodology proposed in (Rumschinski et al., 2010b; Savchenko et al., 2011) is employed, constructing a so-called feasibility problem of a general nonlinear model of the system in Section 2, that incorporates model dynamics with set-based constraints on the involved variables. These constraints allow to include uncertainties of both model and controller parameters, possible (bounded) measurement noise, desired qualitative or quantitative behavior of the controlled plant as well as global bounds on the states of the system. This formulation is then relaxed, leading to an approximation of the set of feasible solutions. It guarantees that the resulted set contains the controller parameter values, that satisfy all such constraints.

We illustrate the proposed method for a magnetic levitation platform in Sections 3 and 4. The platform is set up in a configuration that is open-loop unstable, and we estimate the admissible values of its parameter against a specified reference value.

Section 4 contains the experimental results, additionally assuming model-plant mismatch, measurement noise and uncertain knowledge about the parameters of the system. We compare the simulation results to the actual plant experiment, illustrating that for the estimated feasible controller parameter set the plant performs according to the desired specifications.

2. SETUP AND METHODS

Consider an implicit discrete-time model of the plant as described by

$$\begin{cases} g(x(k+1), x(k), u(k), p) = 0, \\ h(y(k), x(k), p) = 0. \end{cases} \tag{1}$$

Here $x(k) \in \mathbb{R}^{n_x}$ denotes the system states, $p \in \mathbb{R}^{n_p}$ the plant parameters, $u(k) \in \mathbb{R}^{n_u}$ the inputs and $y(k) \in \mathbb{R}^{n_y}$ the outputs of the system. The time index is denoted by $k \in \mathbb{N}$.

2.1 Controller Structure

We consider a memoryless output-feedback controller, which is given in the general implicit form

$$l(u(k), y(k), p, r, c) = 0, \tag{2}$$

where the variables $u(k)$, $y(k)$ and p are as defined before, $r \in \mathbb{R}^{n_r}$ stands for the desired reference values and $c \in \mathbb{R}^{n_c}$ represents the controller parameters. For simplicity we refer to both r and c as controller parameters.

With respect to the controller parametrization we consider the following problem: for given initial conditions find the controller parameters (r, c), that allow for the desired (qualitative or quantitative) behavior/performance of the system. Such behavior/performance criteria can be represented in form of boundary conditions, set-point changes, global state constraints (e.g. to avoid overshooting or obstacle collisions), desired trajectories, etc.

Solving this problem in a structured way is challenging, especially in the case of nonlinear dynamics, possible sensor and actuator noise, disturbances, as well as parameter uncertainties. Furthermore, to guarantee that results obtained for the model also hold true for the actual plant, model-plant mismatch from simplified or neglected dynamics of the process needs to be taken into account. Rather than finding a single value of the controller parameters (r^*, c^*) that achieves the desired behavior, it is often advantageous to find all possible parameters that would allow constraint satisfaction. This allows to pick the value of the controller parameters that provide a degree of robustness, or the value that satisfies additional optimality conditions even during operation.

Next we present a framework for estimating the admissible region of controller parameters for the system subject to unknown-but-bounded system constraints and parameters using a set-based problem formulation.

2.2 Set-based Controller Parameter Estimation

We assume the functions g, h and l to be polynomial or rational. Note that other nonlinearities can be approximated by such functions to an arbitrary precision, see e.g. (Hasenauer et al., 2010) and references therein.

To account for process uncertainty, we assume the parameters p to be unknown-but-bounded within a semi-algebraic set, i.e. $p \in \mathcal{P} \subseteq \mathbb{R}^{n_p}$. In similar form we can describe the initial search space of the desired controller parameter values $(r, c) \in \mathcal{C} \subseteq \mathbb{R}^{n_r} \times \mathbb{R}^{n_c}$. Both sets can be derived from the physical meaning of the parameters, limitations of the controller or bounds set by the operator.

The desired behavior of the system is expressed in form of unknown-but-bounded constraints on $x(k) \in \mathcal{X}_k$ (resp. $y(k) \in \mathcal{Y}_k$, $w(k) \in \mathcal{W}_k$). If no explicit constraints for states, inputs or outputs are provided, we assume that their values are still bounded to (possibly) large semialgebraic sets for technical reasons. We collect the constraints on all variables in terms of bounding sets

$$\begin{aligned} \mathcal{X} &:= \{ \mathcal{X}_k \subset \mathbb{R}^{n_x}, k \in \mathcal{T} \}, \\ \mathcal{Y} &:= \{ \mathcal{Y}_k \subset \mathbb{R}^{n_y}, k \in \mathcal{T} \}, \\ \mathcal{U} &:= \{ \mathcal{U}_k \subset \mathbb{R}^{n_u}, k \in \mathcal{T} \}, \end{aligned}$$

for the set of time indices

$$\mathcal{T} := \{k_0, k_1, \ldots, k_e\} \subseteq \mathbb{N},$$

that define the time horizon of interest. We denote with $\mathcal{T}^-$ the set of all time instances except the last one, i. e. $\mathcal{T}^- = \mathcal{T} \setminus \{k_e\}$.

For short notation, we write $x \in \mathcal{X}$ (resp. $y \in \mathcal{Y}$, $u \in \mathcal{U}$) meaning $x(k) \in \mathcal{X}_k$ (resp. $y(k) \in \mathcal{Y}_k$, $u(k) \in \mathcal{U}_k$) for each $k \in \mathcal{T}$.

The framework for estimating admissible control parameters is based on the following definition:

Definition 1. (Consistent controller parameters).
The controller parameter $(r, c) \in \mathcal{C}$ is said to be consistent with the desired qualitative/quantitative behavior, if there exist initial conditions $x(k_0) \in \mathcal{X}_{k_0}$ and plant parameters $p \in \mathcal{P}$, for which the closed-loop system model (1)-(2) satisfies $x \in \mathcal{X}$, $u \in \mathcal{U}$ and $y \in \mathcal{Y}$ within the time horizon $\mathcal{T}$. ■

Using Definition 1 we can formulate the problem of estimating the set of admissible controller parameters as

Problem 1. (Admissible controller parameter set).
Find the controller parameter set $\mathcal{C}_F$ such that the closed-loop system (1)-(2) is consistent with the desired quantitative behavior of the system for each $(r, c) \in \mathcal{C}_F$. ■

We address Problem 1, i.e. finding the set of controller parameters (r, c), by constructing a feasibility problem of the system model, following (Rumschinski et al., 2010b; Savchenko et al., 2011). The feasibility problem based on the system model (1)-(2) can be formally described as follows.

$$FC : \begin{cases} g(x(k+1), x(k), u(k), p) = 0, & k \in \mathcal{T}^-, \\ h(y(k), x(k), p) = 0, & k \in \mathcal{T}, \\ l(u(k), y(k), p, r, c) = 0, & k \in \mathcal{T}, \\ (x, y, u) \in (\mathcal{X}, \mathcal{Y}, \mathcal{U}), \\ p \in \mathcal{P}, \ (r, c) \in \mathcal{C}. \end{cases}$$

The solution set of FC consists of all values of the involved variables that satisfy the constraints, i.e. performance specifications. Extracting the set of consistent controller parameter values is straightforward, since the following holds

$$proj_{(r,c)}(FC) = \mathcal{C}_F,$$

where $proj_{(r,c)}$ is the projection of the feasible set on the subspace of the controller parameters (r, c).

However, exactly determining the solution set FC is generally difficult for nonlinear system dynamics. As in (Savchenko et al., 2011), we employ a linear relaxation. Using available efficient linear solvers (i.a. *Gurobi*, `http://www.gurobi.com`) allows to consider reasonably large problems. We refer to (Savchenko et al., 2011; Streif et al., 2013a) for the detailed overview of the necessary relaxation steps.

2.3 Outer Approximation of Feasible Sets

The relaxation procedure leads to a so-called outer approximation of the feasible set, guaranteeing that every solution of the original problem is present in the solution set of the relaxed problem. A detailed description of algorithms to obtain outer approximations of feasible sets can be found in (Rumschinski et al., 2010a), next we only briefly outline the key underlying ideas.

We denote with $LP(\mathbf{C})$ the relaxed formulation of the original problem, where the search space for the controller parameter values is reduced to $(r, c) \in \mathbf{C} \subseteq \mathcal{C}$. With $dLP(\mathbf{C})$ we denote the Lagrangian-dual of $LP(\mathbf{C})$ and employ the following observation: weak duality implies, that if the problem $dLP(\mathbf{C})$ is unbounded, the corresponding problem $LP(\mathbf{C})$ is infeasible. In turn, it implies that there are no points $(r, c) \in \mathbf{C}$ that belong to the feasible set $\mathcal{C}_F$.

Hence, the outer approximation of the feasible set $\mathcal{C}_F$ can be obtained by eliminating the subsets of $\mathcal{C}$ for which the above observation is satisfied, namely

$$\mathcal{C}_F \subseteq \mathcal{C}_O = \mathcal{C} \setminus \bigcup_{i \in \mathcal{I}, LP(\mathbf{C}_i) \to \infty} \mathbf{C}_i,$$

where $\mathbf{C}_i$ represent a partition of the set $\mathcal{C}$ for some finite index set $i \in \mathcal{I}$.

Theoretically, the introduced method provides guaranteed results for the model of the closed-loop system, i.e. the estimated feasible set will always include the set of consistent controller parameter values $\mathcal{C}_F$.

As mentioned, the considered model has to account for model-plant mismatch, discretization error etc. to successfully apply these results on an actual plant. Employing the described set-based method allows to incorporate these phenomena considering the system variables to be unknown-but-bounded.

In the following sections we employ the outlined approach to an actual plant, solving Problem 1 for the parametrization of a set-point change nonlinear controller in simulation as well as actual experiments.

3. CASE STUDY

We consider the control of a magnetic levitation platform (MagLev 730, Educational Control Products, ECP, Bell Canyon, USA, `www.ecpsystems.com`) illustrated in Figure 1.

Fig. 1. Considered magnetic levitation platform.

In this section we evaluate the approach by simulations, considering a set-point change controller and fixing the initial, terminal and performance constraints on some of the system states. Specifically, we request an absence of overshoot by specifying suitable state constraints at all times.

3.1 System Description

The setup consists of a high field density rare earth magnet and a drive coil. A laser sensor measures the permanent magnet position. A turntable incorporates a conductive spin platter that interacts with the permanent magnet and causes it to levitate. Position control is accomplished through spin speed changes in the platter. We consider an open-loop unstable setup, when only the upper coil (see Figure 1) is used to control the position of the permanent magnet.

The discrete time model of the considered plant takes the form

$$\begin{aligned} x_1(k+1) &= x_1(k) + \Delta T x_2(k), \\ x_2(k+1) &= x_2(k) + \\ &\quad \Delta T \left(g - \alpha x_2(k) - \frac{u(k)}{ma(x_1(k)+b)^2} \right), \\ y(k) &= x_1(k). \end{aligned} \tag{3}$$

Here x_1 represents the observed position of the permanent magnet, and x_2 is its velocity. The variable u represents the input in form of the coil voltage, ΔT denotes the sampling time. Details on the parameter values can be found in Table 1. The values of parameters a, b, m and α were estimated from the actual plant using least-squares methods.

Table 1. Reference Parameter Values

Parameter	Symbol	Bounds	Unit
Standard gravity	g	981	cm s^{-2}
Magnet mass	m	0.125	kg
Aggregated parameter	a	$5.4976 \cdot 10^{-5}$	V s^{2}kg^{-1}cm^{-3}
Offset parameter	b	2.4512	cm
Friction coefficient	α	$[15.68, 15.70]$	s^{-1}

We choose a memoryless state-feedback linearizing controller (Isidori, 1999) of the form

$$u(k) = ma(y(k) + b)^2(g + c(y(k) - r)), \qquad (4)$$

where c is the gain, and r is the reference value.

Our objective is to find suitable values for the parameters c and r such that the performance specifications are met.

3.2 Simulation Results

For determining the set of consistent controller parameters we choose the discretization of $\Delta T = 0.03$ seconds.

Our goal is to determine controller parameter values that allow for a set-point change in finite time (within 0.5 seconds) without significant overshoot (requiring appropriate global bounds on the state x_2), as specified next.

The terminal time for the transition is set to 0.48 seconds which leads to

$$\mathcal{T} := \{k_0 = 0, \ldots, k_e = 16\}.$$

The bounds on the initial conditions (for k_0) as well as terminal conditions (for k_e) are set allowing for small errors, see Table 2.

Table 2. Desired quantitative model behavior

State	Bounds
$x_1(k_0)$	[2.99, 3.01]
$x_2(k_0)$	[−0.01, 0.01]
$x_1(k_e)$	[$r - 0.05$, $r + 0.05$]
$x_2(k_{e-1})$	[−0.1, 0.1]

Note that the terminal condition on the state depends on the reference value we want to estimate.

We restrict the operating regime for the system and the controller by setting the bounds on $x_1(k)$, $x_2(k)$ and $u(k)$ as shown in Table 3.

Table 3. State and controller bounds

Variable	Bounds
$x_1(k)$	[1, 5]
$x_2(k)$	[−10, 0.1]
$u(k)$	[−0.3, 0.3]

The initial bounds of the controller parameter values are provided in Table 4.

We perform an outer approximation of the search space, using the algorithm illustrated in Section 2.3 and implemented in the *Analysis, Design and Model Invalidation Toolbox* (ADMIT) (Streif et al., 2012). We iteratively partition the search space to estimate first the one-dimensional bounds (see Table 4) and then to approximate the shape of the feasible set via a collection of two-dimensional orthogonal sets. The corresponding two-dimensional results are shown in Figure 2. The blue area represents the region, where feasible parameter values can be found, i.e. the outer approximated feasible set.

As the algorithm results in an outer approximation, the obtained feasible set can contain so-called spurious solutions, that only satisfy the constraints of the relaxed problem. To illustrate the approximation quality of the obtained set we run a Monte-Carlo sampling procedure within the initial search space of controller parameter values using initial condition values from Table 2. Controller parameter values for the obtained solutions that satisfy the constraints in Tables 2 and 3 are illustrated in Figure 2 as red dots.

The results show that the search range of the parameter c was significantly reduced. Besides, the bisectioning procedure resulted in a tight approximation of the actual feasible set despite its nonconvex shape and a linear relaxation of the nonlinear problem FC.

Table 4. Controller parameter bounds

Parameter	Initial Bounds	Estimated Bounds
c	[0, 1000]	[81.63, 99.18]
r	[1.5, 2.5]	[1.5, 2.5]

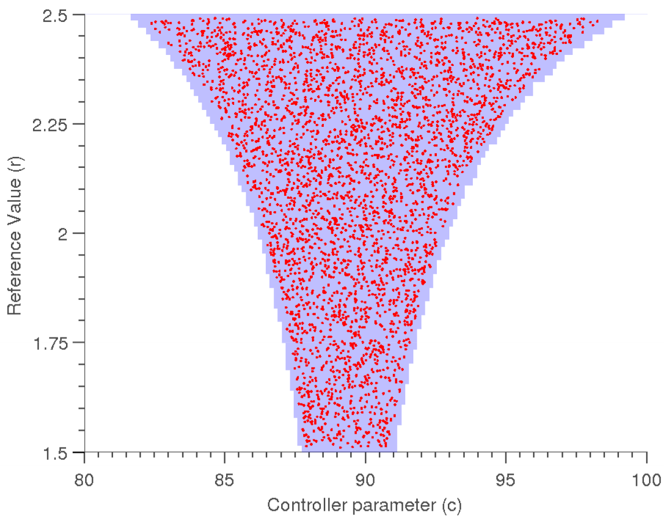

Fig. 2. Feasible set of the admissible controller parameters.

While the information derived from this analysis is helpful in understanding relations between controller parameters, for an actual experiment we can significantly reduce the workload. Usually for an actual plant the reference value is known a priori, hence we can simplify the formulation of the problem and only estimate the controller parameter c.

As mentioned in Section 2, for the simulation result to provide guarantees on the performance of the actual plant, several phenomena have to be taken into account. Limited knowledge about the plant parameter values, measurement noise, the discretized time setting and approximated dynamics of the plant should be compensated via appropriate setting of the system variable bounds. In the next section we introduce a modified setup and compare the estimated bounds with the plant performance through an actual experiment.

4. EXPERIMENTAL VALIDATION

To account for possible disturbances in the actual plant, we modify the system model (3) inserting additive noise $e(k)$ to the permanent magnet position dynamics equation:

$$\begin{aligned} x_1(k+1) &= x_1(k) + \Delta T(x_2(k) + e(k)), \\ x_2(k+1) &= x_2(k) + \\ &\quad \Delta T \left(g - \alpha x_2(k) - \frac{u(k)}{ma(x_1(k)+b)^2} \right), \\ y(k) &= x_1(k). \end{aligned} \quad (5)$$

The structure of the controller (4) is left unchanged.

To reduce the discretization error, we consider a smaller time step of $\Delta T = 0.02$ seconds and hence for the terminal time of 0.5 seconds the set of time indices is given by

$$\mathcal{T} = \{k_0 = 0, \ldots, k_e = 25\}.$$

The controller implemented on the actual plant is set with the values from Table 1 (and friction coefficient $\alpha = 15.69$). However, for the simulated estimation procedure we account for measurement uncertainties, as well as model-plant mismatch, allowing for parameter variations of up to ± 5 percent compared to the values of the applied controller. The search space for the controller parameter c is given in Table 4, and the new parameter bounds are provided in Table 5.

Table 5. Plant Parameter Bounds

Parameter	Bounds	Unit
a	$[\ 5.25 \cdot 10^{-5},\ 5.75 \cdot 10^{-5}\ ]$	$\text{V s}^2\text{kg}^{-1}\text{cm}^{-3}$
b	$[\ 2.33,\ 2.57\ ]$	cm
α	$[\ 14.9,\ 16.5\ ]$	s^{-1}
$e(k)$	$[\ -0.5,\ 0.5\ ]$	cm s^{-1}

Fixing the reference value to a single point

$$r = 1.5,$$

allows the terminal state to take values in the range

$$x_1(k_e) \in [1.45, 1.55],$$

according to Table 3.

4.1 Results and Discussion

The feasible set of parameter c was obtained using ADMIT, leading to

$$c \in [67.6391, 136.587].$$

To be able to compare the simulation results with those obtained by an actual experiment, it is required to compute admissible bounds on the states and the inputs of the system. It is done in two steps. First, we propagate the estimated region for the controller parameter c through (4). Second, the obtained estimates on the input values are employed as unknown-but-bounded input set in the formulation (5).

The comparison between the estimated data and the experimental results are shown in Figures 3 and 4. The blue lines illustrate measured outputs and inputs of the magnetic levitation platform, the red bars show the admissible bounds on the corresponding variable values, derived using ADMIT. The black dashed line in Figure 3 corresponds to the reference value and the thick black marks define the bounds on the initial and terminal states of the system. We can conclude that the actual plant with the designed controller has performed the set-point change within the required time horizon of 0.5 seconds, and the measured data agrees with the simulation results.

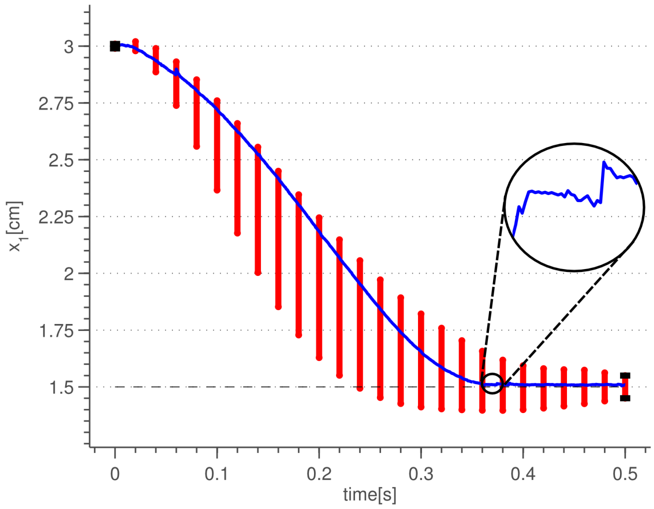

Fig. 3. Real measured outputs compared to the set-based model estimates. The magnified area illustrates the noise of the real measurement data.

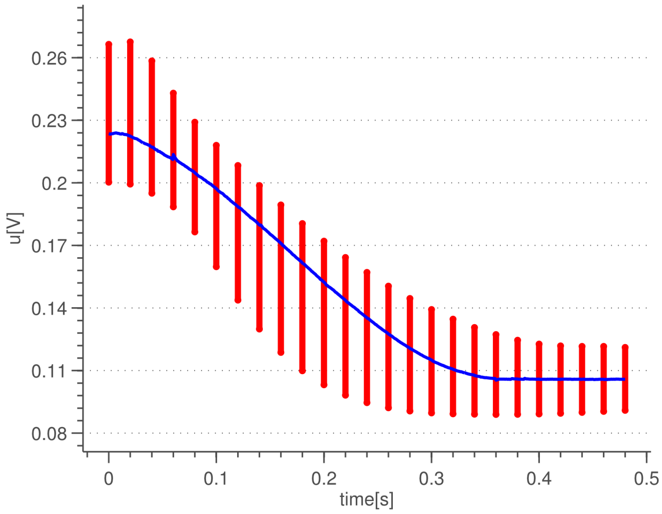

Fig. 4. Real applied input compared to the set-based model estimates.

Compared to the model in Section 3, the estimated region for the values of controller parameter c is significantly more conservative. It is a direct consequence of relaxing system parameter bounds and the injection of an additive error term into the model (5). The decreased sampling time furthermore resulted in a significantly larger problem formulation, that in turn affected the relaxation error of the estimation method.

However, even in this scenario the estimated region was significantly improved compared to the initial bounds on the controller parameter c . Furthermore, the admissible state values presented in Figure 3 are rather narrow considering the nonlinear dynamics of the plant and allowed parameter variations.

5. CONCLUSIONS

We employed a method based on a relaxed set-based feasibility formulation for controller parameterization. The method allows for unknown-but-bounded parameter values, as well as various constraints of the desired behavior of the system given in form of (possibly nonlinear) inequalities on system states and measurement data.

It was shown that the method is able to provide an outer approximation of the feasible controller parameterization. Furthermore, the estimated closed-loop controller performed adequately in simulation and an actual experiment considering a magnetic levitation platform.

The presented method, however, has certain drawbacks. In order to closely approximate the feasible set of a complex shape (for example, when controller parameters are not sensitive or depend nonlinearly on posed constraints) we require a computationally challenging procedure, that partitions the search space into a large number of subregions. Effective ways to reduce the computational burden of the approach are the reduction of the amount of variables to be estimated and the simplification of the underlying feasibility problem formulation (Savchenko et al., 2013).

Outer approximation of the feasible region always introduces a set of spurious solutions, that do not satisfy the constraints of the original model, hence the estimated controller parameter bounds provide information on the set of values that *do not* satisfy desired constraints. We view the application of the presented method as a first step in reducing the search region of suitable or optimal control parameters, that can be followed by e.g. a Monte-Carlo search or set-based methods providing inner approximations (Garloff, 2000; Streif et al., 2013c). Indeed, for the proposed example we were able to significantly reduce the search area even for highly relaxed constraints and added noise on the system dynamics. When multiple parameters have to be estimated at the same time, such a reduction can immensely speed up sampling-based estimation methods.

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