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Performance portrait - a 3D approach ⋆

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Abstract: The paper deals with a comparison of two possible approaches to the performance portrait method application for an optimal nominal tuning of an integral plus dead time (IPDT) plant. The first one relies on a more compact 2D approach used by Huba et al. [2009] inspired by the triple real dominant pole method [Víteček and Vítečková, 2008]. The second approach is a simpler but, at the same time, in some aspects a computationally more demanding 3D method. Pros and cons of both approaches are analyzed and discussed.

1. INTRODUCTION

Tuning of the PI controller for the integral plus dead time (IPDT) plant described by transfer function $F(s)$ with a gain K_s, dead time T_d, an input u and output y

$$F(s) = \frac{Y(s)}{U(s)} = \frac{K_s}{s} e^{-T_d s} \tag{1}$$

is frequently treated in control areas, because with appropriate model reduction techniques it enables us to approximate a broad range of processes (Skogestad [2003], Åström and Hägglund [2006]). Recently, two papers dealing with an optimal controller tuning for this plant by the performance method appeared. The performance portrait (PP) method is a relatively new approach for designing an optimal nominal or robust controller tuning. It has been described e.g. in Huba et al. [2009], Huba [2013a,b] and was recently shown to be also able to provide closed-loop plant identification [Soós and Huba, 2014]. The method searches for an optimal point by sweeping a matrix of output related data based on user defined parameters. It can take into account output monotonicity, actuator wear (represented by a measure for integral deviations from an ideal plant input shape) and speed of transients. It complies to the requirements given in Skogestad [2006] by achieving a user defined trade-off between:

- fast speed of responses and disturbance rejection
- stability and robustness, less input usage
- less sensitivity to measurement noise.

Whereas the paper Huba [2013b] deals with an optimal disturbance observer based PI control in a 2D parameter space, Huba [2013a] considers an equivalent problem of an optimal two-degree-of-freedom (2DOF) PI control in a much more demanding 3D parameter space. What is behind this difference? A 2D approach to the 2DOF PI control has been already applied in Huba et al. [2009], where the PP had two normalized parameters κ and τ. The 2DOF control law corresponds to a PI controller with setpoint weighing b resulting in the control algorithm

$$U(s) = K_c[bW(s) - Y(s)] + \frac{K_c}{sT_i}[W(s) - Y(s)] \tag{2}$$

that can be shown to be equivalent to using prefilter F_p and controller C

$$F_p(s) = \frac{bT_i s + 1}{T_i s + 1}; \;\; C(s) = K_c \frac{1 + T_i s}{T_i s} \tag{3}$$

with T_i being the integral time constant and K_c the controller gain. The parameter b was calculated to cancel one of the real roots of the characteristic equation.

This approach was inspired by the triple real dominant pole method (TRDPM) proposed in Víteček and Vítečková [2008]. Calculating the roots of the transcendent characteristic equation is not so trivial. Especially because the existence of a triple real pole does not allow the use of methods based on the first and second order derivatives. With respect to this they used a fourth order Householder's method with quadratic convergence [Huba et al., 2009]. However, this method only finds one root, which is the "slowest" one—the first root found when searching from 0. The equation, however, can have up to three different real roots, which causes the question to arise, which one of them to cancel with parameter b.

The other concern is, that when looking for a robust controller, we can only assign a single value to the parameter b. But the uncertainty curve segment (UCS) defined for an interval of $K_s \in [K_{s,min}, K_{s,max}]$ or $T_d \in [T_{d,min}, T_{d,max}]$ passes through several points, all with a different calculated value for b. Huba et al. [2009] proposed to choose the smallest value given by these points as the optimal one, but the PP might change if we set b to a single value, possibly causing some points to no longer respect the defined shape constraints. To determine the effect of b on the PP this paper also considers a 3D approach for both nominal and robust controller tuning with b as the new parameter dimension. It is shown that although this approach might be computationally more complex, the achieved results are better than those of the 2D approach for both tunings.

2. DESCRIPTION OF THE PP

First we introduce normalized parameters

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$$\kappa = K_c K_s T_d; \;\; \tau = \frac{T_i}{T_d}; \;\; \sigma = s T_d \tag{4}$$

so that the characteristic polynomial can be written as

$$A(\sigma) = \sigma^2 e^{\sigma} + \kappa\sigma + \frac{\kappa}{\tau}. \tag{5}$$

This way we can move a given system to a specific point of the PP by choosing the appropriate control values K_c and T_i. Theorem 2 in Huba [2013a] states that it is sufficient to generate the PP for the IPDT plant given by $K_s = 1, T_d = 1$ over a chosen grid of points K_c, T_i, b by simulating setpoint responses with $w = 1$ and input disturbance responses with $w = 0, d_i = 1$. The normalized time and shape related quantitative measures can then be used to calculate the real values of these measures for a given IPDT plant.

The PP first creates a data matrix consisting of output and input shape and speed related values corresponding to normalized parameters κ and τ. Then we search for a point (or in case of a robust controller a segment or a more dimensional subspace) that provides the fastest transient while satisfying the user defined constraints on the output and/or input deviation from a preferred transient shape (in our case given by monotonic output and one-pulse input). The controller values are gained after substituting the plant parameters K_s and T_d into (4).

2.1 Time and shape related performance measures

The speed of transients will be characterized by the Integral of Absolute Error (IAE) performance index

$$IAE = \int_0^{\infty} |w - y(t)| dt \approx \sum_{i=0}^{\infty} |w - y(i)| * t_s \tag{6}$$

with sampling time t_s.

To evaluate required control effort and to express the smoothness of a given signal, one can use the total variance (TV) measure from Skogestad [2003] defined as

$$TV = \int_0^{\infty} \left| \frac{du}{dt} \right| dt \approx \sum_{i=0}^{\infty} |u_{i+1} - u_i|. \tag{7}$$

The plant output is frequently required to have a smooth monotonic (MO) transient preserving the direction of change. To evaluate the deviation of the plant output y having an initial value y_0 and a final value y_∞ from a MO shape we can use the $TV_0(y)$ criterion defined in Huba [2010] as

$$TV_0(y) = \sum_{i=0}^{\infty} |y_{i+1} - y_i| - |y_\infty - y_0| \tag{8}$$

where $|y_\infty - y_0|$ represents the minimal required output change. $TV_0(y)$ equals zero only for strictly MO responses, otherwise $TV_0(y) > 0$.

When dealing with the disturbance response, it is to note that each feedback loop needs some time to identify the acting disturbance. This delay causes an output error before the (ideally MO) compensation starts. Thus the output will have an extreme point separating two monotonic intervals - an increasing and a decreasing one, denoted in Huba [2010] as a one-pulse (1P) transient. Therefore to evaluate the shape properties of the output y corresponding to the disturbance step we should use the $TV_1(y)$ criterion that describes deviation from a 1P transient. It is calculated as

$$TV_1(y) = \sum_{i=0}^{\infty} |y_{i+1} - y_i| - |2y_m - y_\infty - y_0| \tag{9}$$

with $y_m \notin (y_0, y_\infty)$ being the dominant extreme value of $y(t)$. $TV_1(y)$ equals zero only for strictly 1P responses, otherwise $TV_1(y) > 0$.

MO transients at the output of the IPDT plant generally correspond to a 1P input, since the number of significant control pulses cannot decrease below the number of unstable poles Huba [2013a]. Therefore, to evaluate the shape properties of the plant input u we use the TV_1 criterion for both the setpoint and disturbance steps. All these values are computed by simulation after appropriate discretization with sampling period as small as possible.

To find an optimal tuning corresponding to a 1P input and MO/1P output one may search the PP for areas where $TV_1(u) = 0$ and $TV_0(y_s) = 0$ or $TV_1(y_d) = 0$. However, to respect the always limited precision of both control and computer simulation, it is meaningful to define some tolerable deviations from ideal input and output shapes. These represent a trade-off between practical usability and computational effort. They may be given as

$$\begin{aligned} TV_0(y_s) \le \epsilon_{ys}; TV_1(u_s) \le \epsilon_{us}; \\ TV_1(y_d) \le \epsilon_{yd}; TV_1(u_d) \le \epsilon_{ud}. \end{aligned} \tag{10}$$

These parameters are adjustable and greatly influence the resulting transients. In this paper we chose for both the 2D and 3D approach the simplest setup

$$\epsilon = \epsilon_{ys} = \epsilon_{yd} = \epsilon_{us} = \epsilon_{ud} = 0.001 \tag{11}$$

which represents a 0.1% deviation from the desired value change.

2.2 Servo-regulatory trade-off

The defined tolerable shape deviations (10) restrict the search area of the PP, but they do not change the IAE values. Even if we set all ϵ to the same value, we still have to deal with the servo-regulatory trade-off. The minimal setpoint IAE values usually do not correspond to the minimal disturbance IAE values. Thus we can find an optimal controller for a setpoint step and a different optimal controller for a disturbance step. To find a controller tuning that is appropriate for both setpoint and disturbance steps, the following cost function was proposed in Grimholt and Skogestad [2012]

$$J = \frac{w_s IAE_s}{IAE_{s,min}} + \frac{w_d IAE_d}{IAE_{d,min}} \tag{12}$$

with $w_s = w_d = 0.5$ representing weighing between the setpoint and disturbance responses, (IAE_s, IAE_d) representing the IAE values of a given point in the PP, and $(IAE_{s,min}, IAE_{d,min})$ representing the optimal setpoint and disturbance IAE values. The optimal controller tuning with a given setpoint and disturbance weighing is gained by minimizing the cost function J.

3. PP: 2D APPROACH

A 2D approach to PP calculation was described in Huba et al. [2009]. The PP is created in two dimensions κ and τ. Inspired by the TRDPM proposed in Víteček and Vítečková [2008] the setpoint weighing (prefilter) coefficient b can be calculated as

$$b = \frac{1}{|s_0|T_i} \tag{13}$$

where s_0 is a real pole of the characteristic equation. Thus the prefilter nominator cancels a closed loop pole, which accelerates the transient responses. However, the root of the characteristic equation (5) changes for every point of the PP. Furthermore, up to three different real roots may exist. The method described in Huba et al. [2009] always finds the same root, which turns out to be the slowest one. The values of b corresponding to such roots are displayed in Fig. 1. The PP is shown in Figs 2- 3. The cross indicates the found optimal point for the minimal cost function J. The PP was calculated for 131x131 points providing a reasonably high precision.

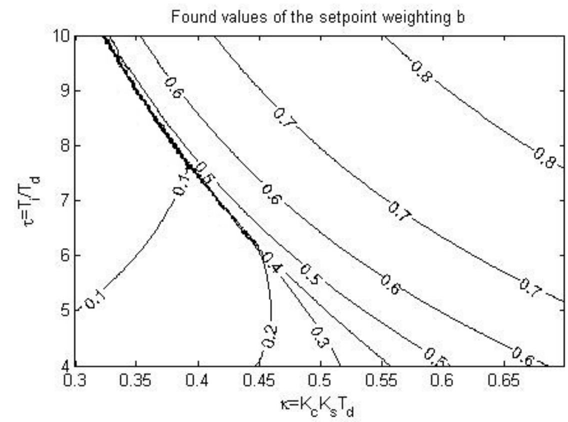

Fig. 1. Values of b calculated by the 2D approach

3.1 *The second and third real pole*

If starting to evaluate step by step from initial conditions chosen from 0 towards $-\infty$, we could obviously find all real roots of the characteristic equation. This would be, however, computationally not efficient, so we should simplify the task. The equation can have three different real poles only at a local maximum and minimum. Whether there are any local extremes present we can find out from the derivative of the characteristic polynomial

$$\dot{A}(\sigma) = (\sigma^2 + 2\sigma)e^{\sigma} + \kappa \tag{14}$$

For local extremes to exist, this must take zero values. This is only possible if

$$\kappa \leq 2(\sqrt{2} - 1)e^{\sqrt{2}-2} \approx 0.4611 \tag{15}$$

which corresponds to $A(\sigma) = 0; \dot{A}(\sigma) = 0$. Furthermore, the poles of (14) must lie within the interval $< -2, 0 >$ so it is rather easy to find them. These poles correspond

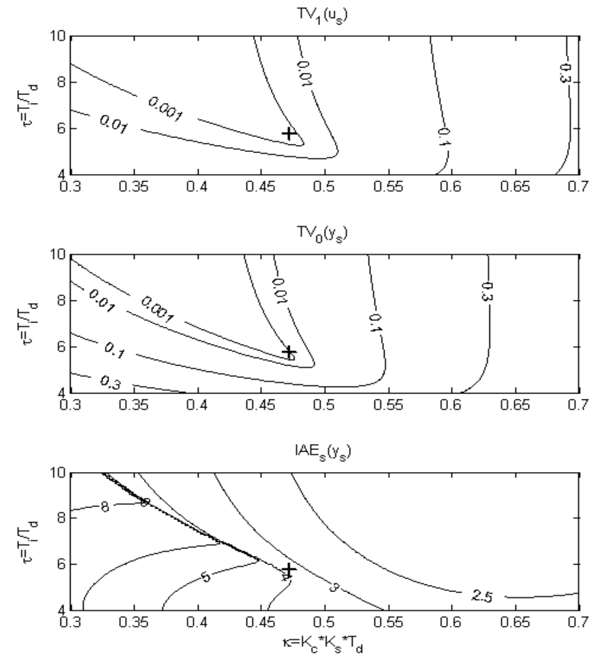

Fig. 2. Setpoint step PP values with found optimal parameters (cross)

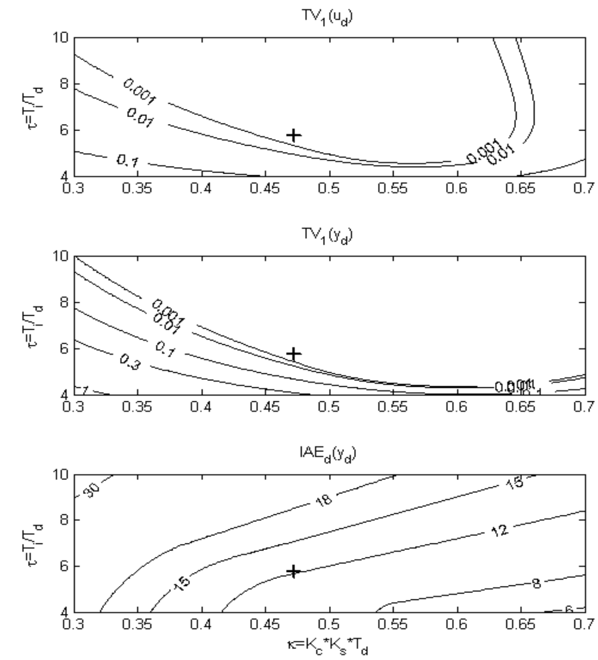

Fig. 3. Disturbance step PP values with found optimal parameters (cross)

to the local extremes of the characteristic equation. If we evaluate the characteristic equation in these points we get the values of the local maximum and minimum. For three different real poles to appear, the local maximum must have a positive value (> 0) and the local minimum must have a negative value (< 0). If any of them equals zero, we only have two different poles. These mathematical constraints considerably reduce computations. Several different real poles actually appear only in a relatively small part of the PP. The computed setpoint weighing and respective IAE_s values for the second and third real pole are plotted in Figs 4-5.

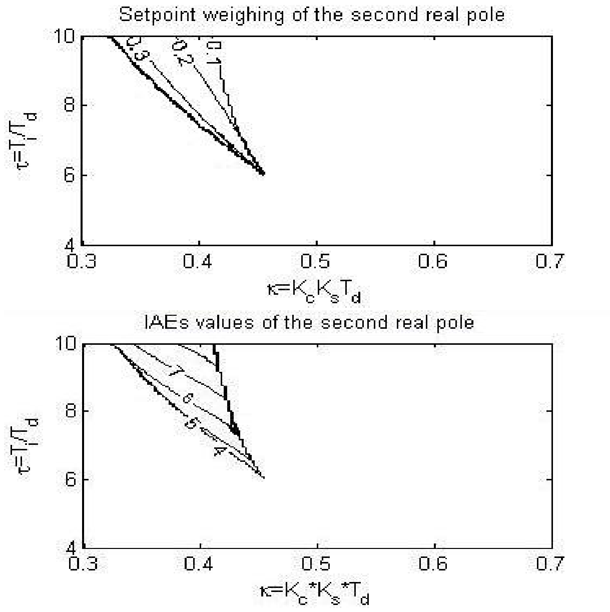

Fig. 4. Setpoint weighing b and IAE_s values calculated for the second root

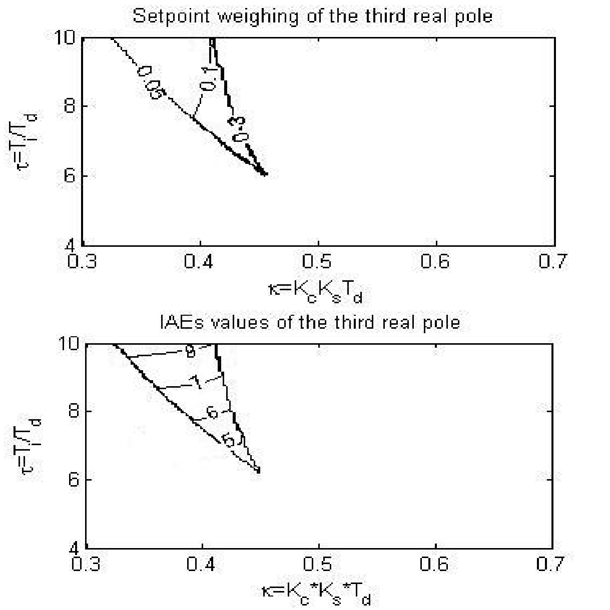

Fig. 5. Setpoint weighing b and IAE_s values calculated for the third root

These setpoint weighings, however, provide higher IAE values than the values gained from the first pole. The lowest achievable IAE value for the second pole is 4.0883 and for the third pole 4.7394, whereas, for the first pole, the corresponding IAE value is 3.9094. Every IAE value of the second and third pole is greater than the IAE values calculated for the first pole. They also show a growing tendency, whereas the values of the first pole decrease. This means that even if we consider several possible real poles of the characteristic equation, it is enough to find the slowest one.

4. PP: 3D APPROACH

If we consider the setpoint weighing b as an independent parameter that needs to be tuned, then the computed problem receives a new dimension. This avoids calculation of the characteristic polynomial roots, but, at the same time, considerably increases computational time and effort. We also need to consider an appropriate value range and resolution for the new dimension. While the range of the setpoint weighing is usually considered to be $< 0, 1 >$, the resolution is entirely up to us. Due to computational constraints, the 3D PP was calculated over a grid of 21x21x100 points, with a resolution in b axis set to 0.01. Since the number of points used for κ and τ are lower than in the 2D approach, there might be disparities in some values due to difference in sampling. Table 1 shows a comparison of the tuned values for the 2D and 3D method while table 2 shows the calculated controller parameters. It is to note that all these points respect the shape related constraint with $\epsilon = 0.001$.

Table 1. Comparison of achieved IAE values

PP	optimal IAE		tuned optimum		
$\epsilon = 0.001$	IAE_s	IAE_d	IAE_s	IAE_d	J
2D	2.7623	11.4241	3.3129	12.1824	1.3677
3D	2.783	7.931	3.491	7.931	1.1319

Table 2. Comparison of computed controller parameters

PP	controller parameters		
$\epsilon = 0.001$	K_c	T_i	b
2D	0.4723	5.7538	0.4239
3D	0.58	4.6	0.24

We can see that the found optimal setpoint step controller is nearly the same for both approaches, with slight numerical disparities caused by the different grid quantization. The optimal disturbance step controller, however, displays a rather significant difference for the two approaches. This is caused by the different shapes of the setpoint TV value sets. Because we require the input and output signals to respect our given shape restraint $\epsilon = 0.001$ for **both** the setpoint and disturbance step, our valid search area of the 2D PP is limited by the $TV_1(u_s) = \epsilon$ and $TV_0(y_s) = \epsilon$ contours to a triangular shape. This does not allow the disturbance tuning of the 2D approach to reach a smaller IAE value than the found $IAE_d = 11.4241$.
However, the shape of the $TV_1(u_s)$ and $TV_0(y_s)$ contours changes in the 3D approach according to the value of b. As can be seen in figures A.1 and A.2 the area given by the $TV_1(u_s) <= \epsilon$ and $TV_0(y_s) <= \epsilon$ contours is considerably larger for smaller values of b. As b increases, the area becomes smaller and moves towards the upper left corner of the PP—to lower κ and higher τ values. The value of b has no effect on the shape of the $TV_1(u_d)$ and $TV_1(y_d)$ contours, as the disturbance step simulation is defined for $w = 0; d_i = 1$. But the restraint ϵ on the $TV_1(u_s)$ and $TV_0(y_s)$ value sets reduces the admissible area to a subset of the areas defined by the ϵ contours over the $TV_1(u_d)$ and $TV_1(y_d)$ values. Therefore, the found optimal disturbance step tuning depends on the value of b, even though the disturbance input and output do not. This causes the main difference in the two approaches.

Thus, the setpoint weighting also infuences the time related measure of the disturbance responses. This property of the PP can be observed in figures A.1 and A.2. As b increases, the $IAE_s(y_s)$ values decrease. But since the recession of the $TV_1(u_s)$ and $TV_0(y_s)$ areas is faster than the IAE value decrease, the optimal setpoint controller can be found for $b = 0.66$. After this point all values of $IAE_s(y_s)$ situated within the restrained $TV_1(u_s)$ and $TV_0(y_s)$ areas are greater than the setpoint optimum.

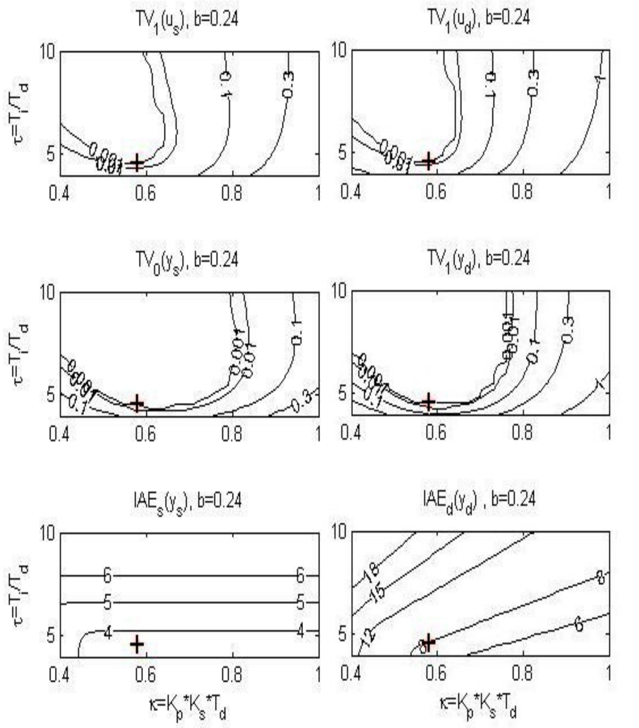

Fig. 6. Optimal nominal controller calculated by the 3D PP

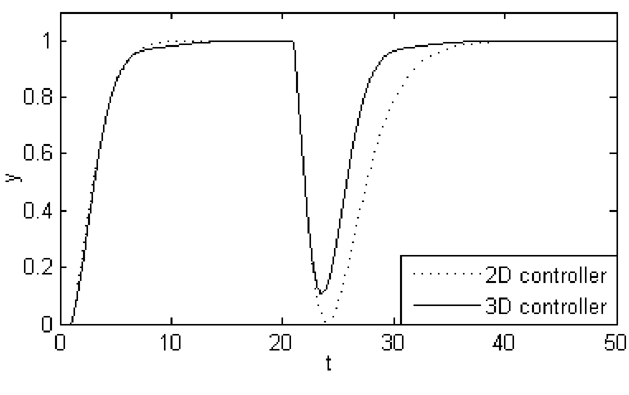

Fig. 7. Comparison of the 2D and 3D controller performance.

5. CONCLUSION

The performance portrait is an exceptional tool for both nominal and robust controller tuning. But it is still a relatively new approach and some its algorithms might not be entirely optimized. Therefore it is worth to confront, analyze and evaluate its application results. In a 2D approach we started with an assumption that when choosing the setpoint weighing b to cancel the "slowest" real pole of the characteristic equation, i.e. the slowest transient mode, one gets the fastest transients. Our results indicate that such a pole-zero cancellation, that in different modifications represents the core of many control design approaches, does not necessarily lead to the best achievable performance.

The larger search areas of the 3D method provide us with broader acceptable controller parameter combinations than the 2D approach does. It is important to notice this fact, since some of these combinations achieve better performance than the optimal 2D one. Therefore, even though the 3D approach is computationally more demanding, it is recommended to treat b as an independently tuned parameter for both the nominal and robust controller tuning. It not only provides a controller with better disturbance rejection, but the generated PP is simultaneously appropriate for a robust tuning. Similar 3D approach will also be required for a robust tuning of the disturbance observer based filtered PI control introduced in Huba [2013b].

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Appendix A. SAMPLE WINDOWS OF THE 3D PP

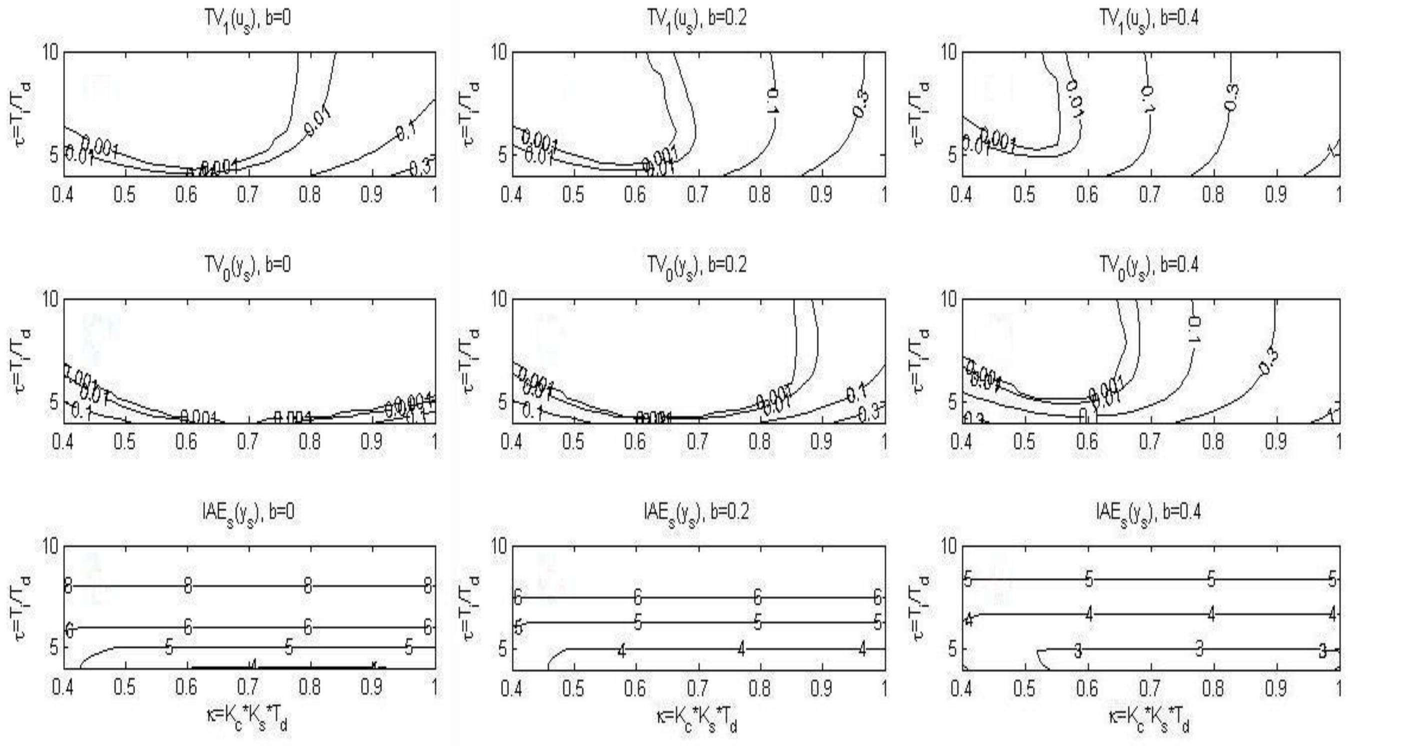

Fig. A.1. Some windows of the 3D PP (a)

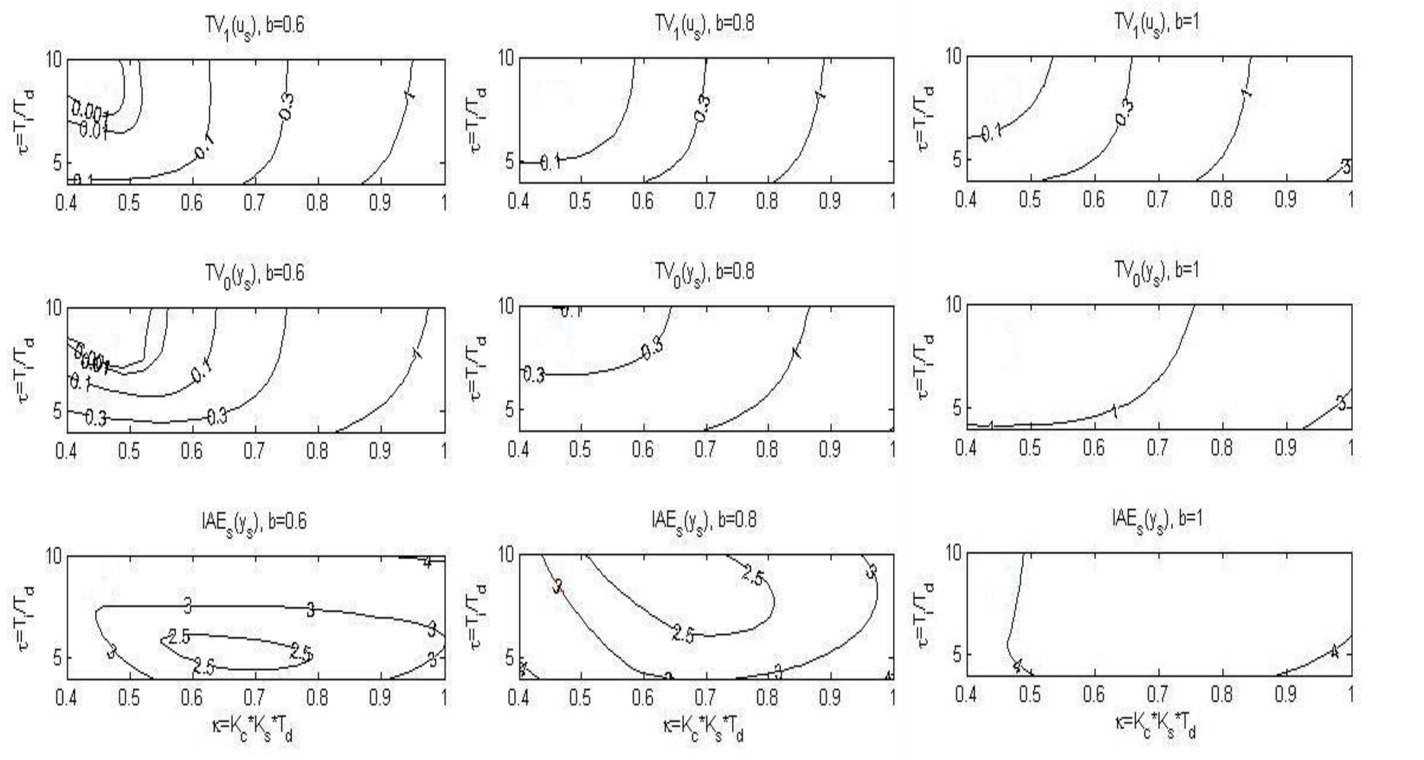

Fig. A.2. Some windows of the 3D PP (b)