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Pinning Synchronization of Complex Networks via Cooperative Heterogeneous Information ⋆

Henghui ZHU * Jinhu LÜ * Yao CHEN **

* *Key Laboratory of Systems and Control, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China.*
** *School of Electronic and Information Engineering, Beijing Jiaotong University, Beijing 100044, China.*
(e-mails: jhlu@iss.ac.cn (Lü), chenyao07@gmail.com (Chen))

Abstract: Synchronization is a ubiquitous phenomenon in nature. Over the last two decades, the synchronization of complex networks receives an increasing attention. This paper aims at developing a new pinning synchronization approach for complex networks by using the heterogeneous information of different nodes cooperatively. Heterogeneous information means the different observation of the reference signal. In detail, different from the traditional pinning control idea, this paper utilizes different feedback gains to cope with the corresponding personalized observations of a specific static reference signal for each node to realize network synchronization. Based on our proposed method, this paper obtains three fundamental pinning synchronization theorems with necessary and sufficient conditions for undirected connected networks, strongly connected networks, and networks with a spanning tree, respectively. Following the above theorems, this paper also designs the feedback gains and coupling strength for networks to achieve synchronization. Finally, the numerical simulations are given to validate the proposed approach.

Keywords: Complex networks, synchronization, pinning control, heterogeneous information, multi-agent systems.

1. INTRODUCTION

Nowadays, complex networks are ubiquitous in nature and our daily life. A complex network is composed of a large set of autonomous nodes which are governed by some simple local rules. Over the last two decades, complex networks have developed very fast due to its wide applications in various disciplines, including physics (Czirok and Vicsek, 2000), biology (Reynolds, 1987; Toner and Tu, 1998), distributed optimization (Nedic and Ozdaglar, 2009), and unmanned air vehicles (Beard et al., 2002; Fax and Murray, 2004). Synchronization, as a typical collective behavior, is a fundamental phenomenon in nature (Lu and Chen, 2005). Recently, the synchronization of complex networks receives an increasing attention in both theoretical research and real-world applications. A fundamental question is to how the network synchronization or consensus can be generated by merging the local interactions of all nodes. In detail, the essential problem lies in how to design the suitable protocols (Jadbabaie et al., 2003; Ren and Beard, 2005; Olfati-Saber et al., 2007).

Over the last few years, numerous efforts has been focused on analyzing and understanding the inherent mechanism of network synchronization. Follow this line, Chen *et al.* developed some new convex analysis tools to deal with the consensus of discrete-time multi-agent systems (Chen et al., 2013a,b). Arcak proposed the design approach of controller based on passivity for the group coordination (Arcak, 2007). Li *et al.* introduced a output feedback control method based on state observers for network synchronization (Li et al., 2010). Most of the above approaches realize network synchronization by driving all nodes to a specific reference signal. In particular, pinning control is a useful technique for network synchronization (Li et al., 2004; Zhou et al., 2008; Yu et al., 2009).

It is well known that the basis idea of traditional pinning control is to design some suitable feedback controllers to guide the node dynamics to a specific reference signal. In detail, the pinning controllers utilizes some so-called "supernodes", each of which acquire all the necessary information of reference signal. However, it is often difficult or even impossible to obtain all the necessary information of reference signal for a single node because of the limit of observability, or the high dimensionality of reference signal. To overcome the above limits, this paper aims at developing an effective pinning synchronization method for complex networks by using the heterogeneous information of each node cooperatively. Heterogeneous information means the different observation of the reference signal. Compared with the traditional pinning control idea,

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this paper utilizes different feedback gains to integrate the corresponding personalized observations of a specific static reference signal for each node to reach network synchronization. Based on the above approach, this paper introduces three pinning synchronization theorems with necessary and sufficient conditions. And some numerical simulations are also given to verify the effectiveness of the proposed theorems.

The rest of this paper is organized as follows. In Section 2, some preliminaries and the main problem are briefly outlined. The three fundamental pinning synchronization theorems are deduced in Section 3 for the undirected connected networks, strongly connected networks, and networks with a spanning tree, respectively. In Section 4, the numerical simulations are given to validate the effectiveness of the proposed method, main theorems, and the design of feedback gains and coupling strength. Finally, conclusions are drawn in Section 5.

2. PRELIMINARIES AND PROBLEM STATEMENT

Let $\mathbb{R}^{N\times N}$ be the set of all real matrices. Denote $\mathbf{1}_N$ be the vector in $\mathbb{R}^N$ with all its entries 1. The notation $M > 0$ indicates matrix $M \in \mathbb{R}^{N\times N}$ is positive definite while $M < 0$ indicates M is negative definite. Let $\mathrm{diag}(G_1, \cdots, G_N)$ be the block-diagonal matrix with matrices G_i on its ith diagonal, $i = 1, \cdots, N$. The Kronecker product of matrices $G = [G_{ij}] \in \mathbb{R}^{N\times N}$ and $H \in \mathbb{R}^{n\times n}$ is defined as

$$G \otimes H = \begin{bmatrix} G_{11}H & \cdots & G_{1N}H \\ \vdots & \ddots & \vdots \\ G_{N1}H & \cdots & G_{NN}H \end{bmatrix}.$$

A (directed) graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of two sets, the set of *nodes* $\mathcal{V} = \{1, 2, \cdots, N\}$ and the set of *edges* $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. A graph with the property that $(i, j) \in \mathcal{E}$ if and only if $(j, i) \in \mathcal{E}$ is called *undirected* graph. There exists a *path* from node i to node j if there exist k different nodes $\{i_s\}$, $1 \le s \le k$, for some integer $k \ge 1$, with $i_1 = i$, $i_k = j$, and $(i_p, i_{p+1}) \in E$ for any $1 \le p \le s-1$. In this paper, all the graphs discussed are assumed with no self-loop, that is $(i, i) \notin \mathcal{E}$ for any i.

A graph $\mathcal{G}$ *contains a spanning tree* if there exists a node i from which there is a path to any other node on the graph, and the node i is called the *root* of the spanning tree. Furthermore, if there exists a path connecting any pair of nodes $i, j \in V$ with $i \neq j$, then the graph G is *strongly connected*. Clearly, a strongly connected graph contains a spanning tree. A strongly connected undirected graph is usually called *connected*.

The *adjacency matrix* $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ of a graph $\mathcal{G}$ is defined as follows: $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. The graph $\mathcal{G}$ *corresponding to a matrix* $G \in \mathbb{R}^{N\times N}$ is defined as follows: $\mathcal{G}$ has N nodes and (i, j) is an edge iff $G_{ji} \neq 0$. The *Laplacian* matrix $\mathcal{L} = [\mathcal{L}_{ij}] \in \mathbb{R}^{N\times N}$ is a matrix, whose diagonal entries $\mathcal{L}_{ii} = \sum_{j\neq i} a_{ij}$ for any i and off-diagonal entries $\mathcal{L}_{ij} = -a_{ij}$. If a graph $\mathcal{G}$ is undirected, then its adjacency matrix A and Laplacian $\mathcal{L}$ are symmetric.

This paper aims to design a cooperative method for pinning control under heterogeneous observation. The network is described as a graph $\mathcal{G}$ with N nodes. Let $A = [a_{ij}]$ be the adjacency matrix of the network $\mathcal{G}$. The static reference signal $x(t)$, which means $x(t) \equiv x$ for a constant $x \in \mathbb{R}^n$, is used as the ultimate synchronization state for all nodes. However, due to observation or configuration of each node, node i can only obtain the partial information static reference signal: $y_i = C_i x$, where $C_i \in \mathbb{R}^{m_i\times n}$ and $y_i \in \mathbb{R}^{m_i}$ for some m_i. This is called the heterogeneous information.

The dynamics of the network is as follows:

$$\dot{\hat{x}}_i = \nu \sum_{j=1}^{N} a_{ij}(\hat{x}_j - \hat{x}_i) - H_i(C_i\hat{x}_i - y_i), \qquad (1)$$

where $\hat{x}_i \in \mathbb{R}^n$, $H_i \in \mathbb{R}^{n\times m_i}$ for $i = 1, \cdots, N$. $\nu > 0$ is called coupling strength. The feedback gains H_i and the coupling strength ν are the parameters to design for the fulfillment of cooperative pinning.

The goal of pinning control is to make all the state of nodes in the network converge to reference signal x asymptotically, i.e.

$$\lim_{t\to\infty} \|\hat{x}_i(t) - x\| = 0, \qquad i = 1, \cdots, N \qquad (2)$$

for any initial value $\hat{x}_i(0)$, $i = 1, \cdots, N$. It is said that the *cooperative pinning synchronization is fulfilled* in a graph with observation matrices C_i if (2) holds for some suitable $\nu > 0$ and $H_i \in \mathbb{R}^{n\times m_i}$.

3. ON COOPERATIVE PINNING SYNCHRONIZATION OF NETWORKS WITH STATIC REFERENCE SIGNAL

This section shows the results of cooperative pinning synchronization of networks, which suggest that the cooperative pinning synchronization can be fulfilled under the hypothesis of connected topology, strongly connected topology, and topology with a spanning tree, respectively.

3.1 Cooperative Pinning Synchronization in Connected Networks

This subsection focuses on the synchronization of connected networks $\mathcal{G}$. At first, some properties on the Laplacian of undirected graph are given as follows.

Lemma 1. ((Godsil and Royle, 2001)). For a matrix G which satisfy $\sum_{j=1}^{N} G_{ij} = 0$ and $G_{ij} \le 0$ for all $i \neq j$, all the eigenvalue of G have no nonnegative real parts. Zero is an eigenvalue of G, with $\mathbf{1}_N$ as the corresponding right eigenvector.

It is known the Laplacian of a graph satisfies the condition of Lemma 1. So all of its eigenvalues have no nonnegative real parts. The following lemma indicates that, for a graph with a spanning tree, there is only one simple zero eigenvalue of its Laplacian.

Lemma 2. ((Ren and Beard, 2005)). For a matrix G satisfying $\sum_{j=1}^{N} G_{ij} = 0$ and $G_{ij} \le 0$ for all $i \neq j$, zero is a simple eigenvalue if the graph $\mathcal{G}$ corresponding to matrix G contains a spanning tree.

To deal with the problem of synchronization of a connected network under cooperative pinning control, one needs the following hypothesis.

Hypothesis 3. Matrix $\mathcal{C} = [C_1^T, \cdots, C_N^T]^T$ has full row-rank.

This hypothesis indicates that the all information of x can be acquired by combining the partial information of different nodes. If such hypothesis is violated, some information of x can not be achieved, resulting the non-fulfillment of the cooperative pinning synchronization.

Theorem 4. For a connected network $\mathcal{G}$, the cooperative pinning synchronization is fulfilled *if and only if* the Hypothesis 3 holds.

Proof. (Necessity) With loss of generality, choose $\nu = 1$ and $H_i = C_i^T$, And each node has the following form:

$$\dot{\hat{x}}_i = \sum_{j=1}^{N} a_{ij}(\hat{x}_j - \hat{x}_i) - Q_i(\hat{x}_i - x),$$

where $Q_i = C_i^T C_i$. Denote $z_i = \hat{x}_i - x$ and concatenated vectors $Z = [z_1^T, \cdots, z_N^T]^T$, then the above equation can be rewritten as follows

$$\dot{Z} = -(\mathcal{L} \otimes I_n)Z - \text{diag}(Q_1, \cdots, Q_N)Z. \quad (3)$$

If the asymptotic stability of (3) can be guaranteed, then clearly the cooperative pinning synchronization is fulfilled.

Consider the Lyapunov function candidate $V = \dfrac{1}{2}Z^T Z$. Since $\mathcal{L}$ is symmetric, the derivative of V is

$$\dot{V}(Z) = Z^T(-\mathcal{L} \otimes I_n - \text{diag}(Q_1, \cdots, Q_N))Z. \quad (4)$$

Let $\mathcal{S}_0$ be the eigenspace corresponding to zero eigenvector of $\mathcal{L} \otimes I_n$. From Lemma 1 and Lemma 2, $\mathcal{S}_0 = \{Z = [z^T, \cdots, z^T]^T | z \in \mathbb{R}^n\}$. Notice that the block diagonal matrix $\text{diag}(Q_1, \cdots, Q_N)$ is positive semi-definite because all of its blocks are positive semi-definite.

For any vector $Z \notin \mathcal{S}_0$, $\dot{V}$ can be estimated as

$$\begin{aligned}\dot{V}(Z) &= -Z^T(\mathcal{L} \otimes I_n)Z - Z^T\text{diag}(Q_1, \cdots, Q_N))Z \\ &\leq -Z^T(\mathcal{L} \otimes I_n)Z < 0. \end{aligned} \quad (5)$$

For any $Z \in \mathcal{S}_0$, one can find a $z \in \mathbb{R}^n$ such that $Z = [z^T, \cdots, z^T]^T$. Then

$$\begin{aligned}\dot{V}(Z) &= -Z^T(\mathcal{L} \otimes I_n)Z - Z^T\text{diag}(Q_1, \cdots, Q_N))Z \\ &= -Z^T\text{diag}(Q_1, \cdots, Q_N))Z \\ &= -\sum_{i=1}^{N} z^T Q_i z = -\sum_{i=1}^{N}(C_i z)^T(C_i z) \leq 0. \end{aligned} \quad (6)$$

If $-\sum_{i=1}^{N}(C_i z)^T(C_i z) = 0$, then $C_i z = 0$ for all $i = 1, \cdots, N$, implying $\mathcal{C}z = 0$. From Hypothesis 3, it can be concluded that $z = 0$. Together with (5), $\dot{V}(Z) < 0$ when $Z \neq 0$ and $\dot{V}(Z) = 0$ when $Z = 0$, that implies the negative definition of the Lyapunov function V. So (3) is asymptotically stable.

(Sufficiency) Suppose Hypothesis 3 is violated, then $\mathcal{C}$ does not have full row-rank. Choose a $w \in \mathbb{R}^n$, which satisfies $w \neq 0$ and w is orthogonal to row space of $\mathcal{C}$. By the definition of $\mathcal{C}$, $C_i w = 0$ holds for $i = 1, \cdots, N$. For any static reference signal x, choose $\hat{x}_i(0) = x + w$, $i = 1, \cdots, N$ as initial value of each node. From (1), it is clear that $\hat{x}_i(t) \equiv x + w$ for all $t > 0$ and $i = 1, \cdots, N$. Thus (2) can not hold and cooperative pinning synchronization can not be fulfilled.

Based on the proof of above theorem, the following corollary summarizes the design of feedback gains and coupling strength under which the network can achieve cooperative pinning synchronization.

Corollary 5. Suppose the network is connected. If Hypothesis 3 holds and H_i are chosen such that $\sum_{i=1}^{N} H_i C_i > 0$, then the network synchronizes to the reference signal x when $\nu > 0$.

Remark 6. The theorem and corollary above can be easily extended to general undirected networks. Define a *connected component* of an undirected graph to be a subgraph which is connected while adding any nodes else will violate the connectedness of this subgraph. Let $\mathcal{I}_i$ be the index of nodes in ith connected component of the graph, $i = 1, \cdots, k$. A necessity and sufficiency condition for fulfillment of cooperative pinning synchronization is all the matrices $\mathcal{C}_i = [C_{j_1}^T, \cdots, C_{j_{m_i}}^T]^T$, $\mathcal{I}_i = \{j_1, \cdots, j_{m_i}\}$ have full row-rank. One can arrive at this result by applying the above analysis in each connected component of $\mathcal{G}$. What is more, if one choose H_i such that $\sum_{j \in \mathcal{I}_i} H_j C_j > 0$ for $i = 1, \cdots k$, state of all nodes in the network synchronizes to the reference signal.

3.2 Cooperative Pinning !Synchronization in Strongly Connected Networks

This subsection discusses the cooperative pinning synchronization of strongly connected networks $\mathcal{G}$. It will be shown that Theorem 4 can be extended to such network. The main difference between undirected and directed graph is the Laplacian of directed graph is not symmetric. The analysis of Theorem 4 can not be directly applied to the directed graph. In order to derive the same result in the case of directed graph, we need the following lemmas.

Definition 7. ((Horn and Johnson, 1990)). A matrix $G \in \mathbb{R}^{N \times N}$ is *reducible* if there is a permutation matrix $P \in \mathbb{R}^{N \times N}$ and an integer $1 \leq m \leq N-1$, such that

$$P^T G P = \begin{bmatrix} \widetilde{G}_{11} & 0 \\ \widetilde{G}_{21} & \widetilde{G}_{22} \end{bmatrix},$$

where $\widetilde{G}_{11} \in \mathbb{R}^{m \times m}$, $\widetilde{G}_{21} \in \mathbb{R}^{(N-m) \times m}$ and $\widetilde{G}_{22} \in \mathbb{R}^{(N-m) \times (N-m)}$. If G is not reducible, then G is called irreducible.

An intuitive view of irreducible matrix is that the graph corresponding to the matrix is strongly connected. Otherwise, the nodes of subgraph corresponding to $\widetilde{G}_{11}$ have no path leads to the subgraph corresponding to $\widetilde{G}_{22}$.

Lemma 8. ((Horn and Johnson, 1990)). The matrix G is irreducible *if and only if* graph $\mathcal{G}$ corresponding to a matrix G is strongly connected.

Lemma 9. ((Yu et al., 2013)). Suppose that Laplicain $\mathcal{L}$ a directed graph is irreducible. Then there exists a positive definite diagonal matrix $\Xi = \text{diag}(\xi_1, \cdots, \xi_N)$ such that $\hat{\mathcal{L}} = \dfrac{1}{2}(\Xi\mathcal{L} + \mathcal{L}^T\Xi)$ is symmetric and $\sum_{j=1}^{N} \hat{\mathcal{L}}_{ij} = \sum_{j=1}^{N} \hat{\mathcal{L}}_{ji} = 0$ for all $i = 1, \cdots, N$.

It should be noticed that $\hat{\mathcal{L}}$ satisfies the condition of Lemma 1 and Lemma 2.

Theorem 10. For a directed and strong connected network $\mathcal{G}$, the cooperative pinning synchronization is fulfilled *if and only if* the Hypothesis 3 holds.

Proof. (Necessity) Choose the same H_i and ν in Theorem 4 and apply the same reasoning. One can have equation (3) and cast the cooperative pinning synchronization problem into the asymptotically stable analysis of (3).

For Laplacian $\mathcal{L}$, one can find a Ξ in Lemma 9. Use $V = \frac{1}{2}Z^T(\Xi \otimes I_n)Z$ as a Lyapunov function candidate, and then:

$$\dot{V}(Z) = Z^T(-\hat{\mathcal{L}} \otimes I_n - \mathrm{diag}(\xi_1 Q_1, \cdots, \xi_N Q_N))Z \quad (7)$$

It should be noticed that $\hat{\mathcal{L}}$ satisfy the requirements in Lemma 1 and Lemma 2 and $\xi_i > 0$ for $i = 1, \cdots, N$. The rest proof is similar to Theorem 4.

(Sufficiency) It is totally the same as Theorem 4, thus we omit is here.

Corollary 11. Suppose the network is strongly connected. If Hypothesis 3 holds and H_i are chosen such that $\sum_{i=1}^{N} H_i C_i > 0$, then the network synchronizes to the reference signal x when $\nu > 0$.

3.3 Cooperative Pinning Synchronization in Networks with Spanning Tree

This subsection copes with the cooperative pinning synchronization where the graph $\mathcal{G}$ contains a spanning tree. Since Lemma 9 does not hold for such graph, the proof of Theorem 4 or Theorem 10 can not be extended to such case directly. To overcome this difficulty, some lemmas must be established first.

Lemma 12. ((Wu, 2005)). If a graph $\mathcal{G}$ contains a spanning tree, then with proper permutation, $\mathcal{L}$ can be reduced to the Frobenius normal form

$$\mathcal{L} = \begin{bmatrix} L_{11} & L_{12} & \cdots & L_{1k} \\ 0 & L_{22} & \cdots & L_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & L_{kk} \end{bmatrix}, \quad (8)$$

where L_{ii}, $i = 1, \cdots, k-1$, are irreducible, each L_{ii} has at least one row with positive row sum and L_{kk} are irreducible or is a zero matrix of dimension one.

A graph theory view of the above lemma is that a graph which contains a spanning tree can be decomposed into some strongly connected components. If one contracts each strongly connected component to a single supernode, the resulting graph is a directed tree with root corresponding to the subgraph of L_{kk}. By (8), no node other that ones corresponding to L_{kk} links to nodes in L_{kk}. So a nature idea to fulfill the cooperative pinning synchronization is letting the nodes corresponding to L_{kk} track the reference signal x cooperatively and letting other nodes synchronize to nodes in L_{kk}. The former are shown in Theorem 10 since the subgraph corresponding to L_{kk} are strongly connected. Then, the following lemma are presented to cope with the synchronization of nodes other than L_{kk}.

Lemma 13. For the matrix $\mathcal{L}$ in Lemma 12, all the eigenvalues of matrix

$$\bar{\mathcal{L}} = \begin{bmatrix} L_{11} & L_{12} & \cdots & L_{1\,k-1} \\ 0 & L_{22} & \cdots & L_{2\,k-1} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & L_{k-1\,k-1} \end{bmatrix}$$

have positive real part.

The above lemma is a slight modification of the Lemma 5 in (Li et al., 2010). The proof of this lemma is trivial and omitted here. Without loss of generality, we assume that the index of nodes are already permuted in a way such that its Laplacian $\mathcal{L}$ is in form (8). Let M be the dimension of $\bar{\mathcal{L}}$ in Lemma 13. So the nodes corresponding to L_{kk} are indexed $M+1, \cdots, N$.

To guarantee the fulfillment of the cooperative pinning synchronization, we introduce the following assumption.

Hypothesis 14. The matrix $\widetilde{\mathcal{C}} = [C_{M+1}^T, \cdots, C_N^T]^T$ has full row-rank.

Theorem 15. For a network $\mathcal{G}$ contains a directed spanning tree, the cooperative pinning synchronization is fulfilled *if and only if* the Hypothesis 14 holds.

Proof. (Necessity) Choose $H_i = 0$ for $i = 1, \cdots M$ and $H_i = C_i^T$ for $i = M+1, \cdots, N$. Let $\nu = 1$. Denote $z_i = \hat{x}_i - x$, $Z_1 = [z_1^T, \cdots, z_M^T]^T$, $Z_2 = [z_{M+1}^T, \cdots, z_N^T]^T$, and $Z = [z_1^T, \cdots, z_N^T]^T$. Since Laplacian $\mathcal{L}$ is in form (8), the dynamics of Z_2 is

$$\dot{Z}_2 = -(\mathcal{L}_{kk} \otimes I_n)Z_2 + \mathrm{diag}(Q_{M+1}, \cdots, Q_N)Z_2, \quad (9)$$

where $Q_i = C_i^T C_i$, $i = M+1, \cdots, N$. Clearly, the subgraph corresponding to $\mathcal{L}_{kk}$ is strongly connected. From Theorem 10, one has

$$\lim_{t\to\infty} Z_2 = 0 \quad (10)$$

for all initial value of Z_2. Now we consider the dynamics of Z_1:

$$\dot{Z}_1 = -(\bar{\mathcal{L}} \otimes I_n)Z_1 + BZ_2, \quad (11)$$

where $B = [L_{1k}^T, L_{2k}^T, \cdots, L_{k\,k-1}^T]^T$. From Lemma 13, $-(\bar{\mathcal{L}} \otimes I_n)$ is Hurwitz. Thus Z_1 approaches zero asymptotically and cooperative pinning synchronization is fulfilled.

(Sufficiency) If Hypothesis 14 was violated, the dynamics of nodes corresponding to $\mathcal{L}_{kk}$ is

$$\dot{\hat{x}}_i = \nu \sum_{j=M+1}^{N} a_{ij}(\hat{x}_j - \hat{x}_i) - H_i(C_i\hat{x}_i - y_i),\ i = M+1, \cdots, N. \quad (12)$$

Apply the same reasoning in Theorem 4 for above equation. It is trivial to see that the cooperative pinning synchronization problem can not be fulfilled.

Corollary 16. Suppose the network contains a spanning tree. If Hypothesis 3 holds, let $H_i = 0$ for $i = 1, \cdots, M$ and choose H_i, where $i = M+1, \cdots, N$, such that $\sum_{i=M+1}^{N} H_i C_i > 0$, then the network synchronizes to the reference signal x when $\nu > 0$.

Remark 17. For a general directed graph, one can apply the strongly connected component decomposition to the graph and contract each component to a single supernode. For all the subgraph $\mathcal{G}_i$ corresponding to the supernode with no in-degree, let $\mathcal{I}_i$ be the set of index of nodes in

$\mathcal{G}_i$, $i = 1, \cdots, k$. A necessity and sufficiency condition under which the cooperative pinning synchronization can be fulfilled is all matrices $\mathcal{C}_i = [C_{j_1}^T, \cdots, C_{j_{m_i}}^T]^T$, $\mathcal{I}_i = \{j_1, \cdots, j_{m_i}\}$ have full row-rank. One can arrive this result by applying the above analysis in each connected component of graph $\mathcal{G}$. What is more, if one choose H_i such that $\sum_{j \in \mathcal{I}_i} H_j C_j > 0$ for $i = 1, \cdots k$ and $H_j = 0$ if $j \notin \cup_i \mathcal{I}_i$, then state of all nodes in the network synchronizes to reference signal.

4. SIMULATION EXAMPLES

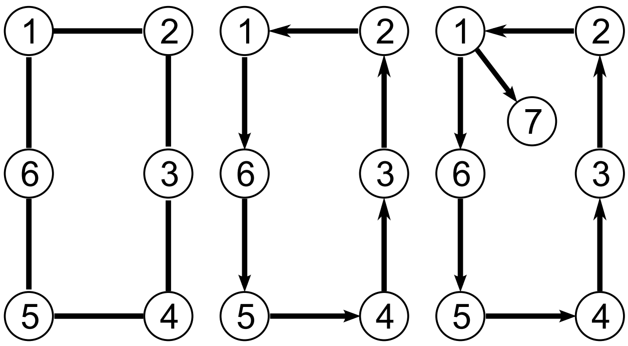
In this section, some simulation examples are given to verify the theorem above.

4.1 Cooperative Pinning Control in a Connected Network

We choose a reference signal $x = [1\,1\,1\,1\,1\,1]^T$ and a ring-shape networks $\mathcal{G}_1$ described in Fig. 1(a). The observation matrices C_i are:

$$C_1 = [1\,1\,1\,1\,1\,1], \quad C_2 = [0\,1\,1\,1\,1\,1], \quad C_3 = [0\,0\,1\,1\,1\,1],$$
$$C_4 = [0\,0\,0\,1\,1\,1], \quad C_5 = [0\,0\,0\,0\,1\,1], \quad C_6 = [0\,0\,0\,0\,0\,1].$$

For better perform in convergence, let $\nu = 5$ and $H_i = 10\,C_i^T$, $i = 1, \cdots, n$. Here, we define $\bar{x}_i(t) = \|\hat{x}_i(t) - x\|$ as synchronization error. Fig. 2 shows the synchronization error with random generated initial values. It is clear that the cooperative pinning synchronization is fulfilled.



(a) The network $\mathcal{G}_1$ (b) The network $\mathcal{G}_2$ (c) The network $\mathcal{G}_3$

Fig. 1. Topology of networks $\mathcal{G}_1$, $\mathcal{G}_2$ and $\mathcal{G}_3$, which are used in simulation examples.

4.2 Cooperative Pinning Control in a Strongly Connected Network

Choose the same reference signal and observation matrices in subsection 4.1 except the network topology. In this simulation example, a directed ring-shape network $\mathcal{G}_2$ is used, which is shown in Fig. 1(b). Again Fig. 3 indicates the fulfillment of the cooperative pinning synchronization.

4.3 Cooperative Pinning Control in a Network with a Spanning Tree

Again we choose the same reference signal x and observation matrices. But in this case the graph $\mathcal{G}_3$ is a modification of $\mathcal{G}_2$ and shown in Fig. 1(c). The observation matrix for node 7 is $C_7 = [0\,0\,0\,0\,0\,0]$.

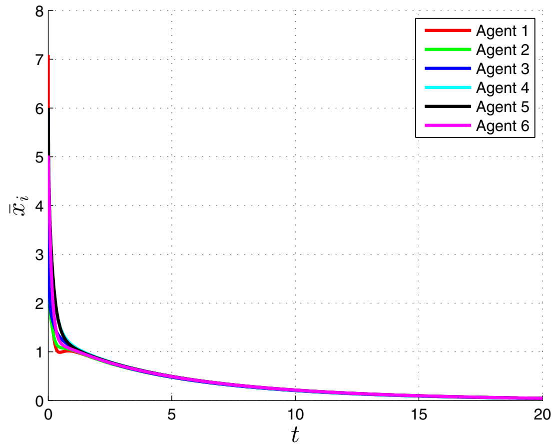


Fig. 2. Synchronization error of the ring-shape network $\mathcal{G}_1$ under cooperative pinning control.

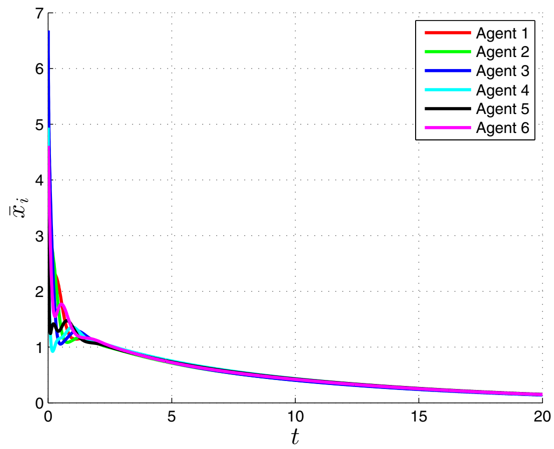


Fig. 3. Synchronization error of the directed ring-shape network $\mathcal{G}_2$ under cooperative pinning control.

Use the same ν and H_i, $i = 1, \cdots, 6$ in subsection 4.1 and let $H_7 = [0\,0\,0\,0\,0\,0]^T$. The synchronization error in Fig. 4 suggests the network synchronizes.

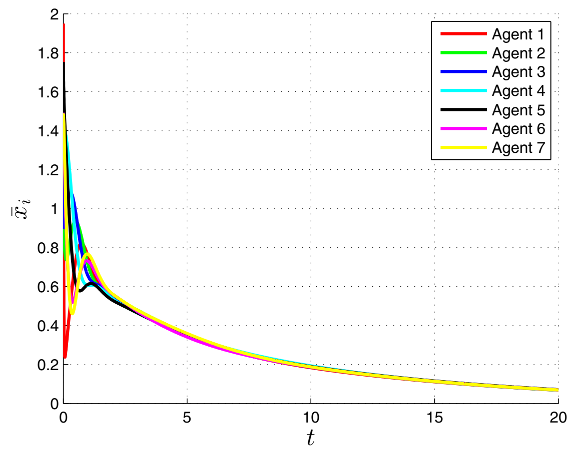


Fig. 4. Synchronization error of the network $\mathcal{G}_3$ under cooperative pinning control.

5. CONCLUSION

This paper has developed a new pinning synchronization method for complex networks via heterogeneous information. In particular, when the reference signal is static, three necessary and sufficient conditions are deduced on existing parameters for the cooperative pinning synchronization of undirected connected networks, strongly connected networks, and networks with a spanning tree, respectively. By merging information each node received together, this proposed approach overcomes the limitation that none of individual node can access all the necessary information of reference signal. Based on the above theoretical results, this paper also develops the corresponding design of feedback gains for the cooperative pinning controllers and coupling strength of networks for cooperative pinning synchronization. Numerical simulation results illustrated the theoretical results at last.

However, for the general reference signals, it is also a challenging problem to design the optimal cooperative pinning controllers for network synchronization. Nevertheless, these theoretical results shed some light on the future real-world engineering applications, such as smart grid and networked location-based services.

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