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Cascaded Control of Superheat Temperature of an HVAC System Via Super Twisting Sliding Mode Control

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Abstract: In this paper two separate cascaded control approaches to control superheat temperature, and length of two-phase flow in an evaporator of Heating Ventilation Air Conditioning Systems(HVAC) are presented. In the first approach, superheat temperature is controlled by inlet mass flow rate of the evaporator, while in the second approach, length of two-phase flow is controlled by outlet mass flow rate. Two inner loop and outer loop of the cascaded controller are designed using sliding mode along with feedback linearization. Superheat temperature, T_{sh} (first approach), and the length of two-phase flow, l (second approach) are controlled in the outer loop, and evaporating temperature of refrigerant, T_e, is controlled in the inner-loop for both approaches. To improve performance, a second order super twisting method has been utilized. The controller is equipped with a generalized super-twisting sliding mode observer to estimate the length of two phase flow of the refrigerant inside the evaporator. The robustness of the suggested control strategy against disturbance and parameter uncertainties is illustrated through simulation in MATLAB/Simulink environment.

Keywords: Superheat temperature, Sliding mode control, Super twisting, HVAC, Evaporator.

1. INTRODUCTION

HVAC and Refrigeration systems as a major source of energy consumption are being subject of many researches. Several works are allocated to improve the efficiency of those systems. Superheat temperature has a key role in energy optimization, and safety of the HVAC and Refrigeration systems.

Superheat is defined as the difference between the temperature of refrigerant at the outlet of evaporator, T_2 (Fig.1), and the evaporating temperature, T_e of the refrigerant. The refrigerant should be in gaseous state (superheated) to ensure that no liquid enters the compressor to avoid serious damage to the compressor. At the same time, superheat temperature should not be very high, because as much as it increases, the heat transfer in the evaporator decreases causing low efficiency. Usually the normal superheat temperature is between 5-10 (°C).

Thermodynamic cycle of an HVAC-Refrigeration system is illustrated in the pressure, P, enthalpy, h, diagram, Fig.1. When the refrigerant enters the evaporator, it is in a two-phase state at the point 1, then it absorbs the heat from the ambient and changes to superheated gas at point 2, at this point it enters the compressor and pressurized to the point 3. The pressurized refrigerant enters the condenser at this point. During stage 3 to 4, in the condenser, it releases its heat to the outer ambient. It is assumed that the refrigerant is in saturated liquid form when it goes out from the condenser. During stage 4 to 1, the pressure of refrigerant decreases while passing through the expansion valve.

As it is mentioned due to the major effect of HVAC and Refrigeration systems on energy consumption, several research activities have been carried out on modelling and control of those systems, and it is still a subject of many researches. In the evaporator two state variables which are usually controlled are superheat temperature and evaporating temperature. He and Asada [2003] proposed a feedback linearization controller to control the length of two phase flow inside the evaporator indicating the superheat and evaporating temperature. In their method, evaporating temperature is controlled by the mass flow rate of the refrigerant at the outlet of the evaporator, $\dot{m}_{out}$ which is a function of compressor speed. However, in their work, the length of two-phase flow is determined just for the steady state case by a constant inlet mass flow rate $\dot{m}_{in}$. Rasmussen and Larsen [2009] presented a backstepping method to control an

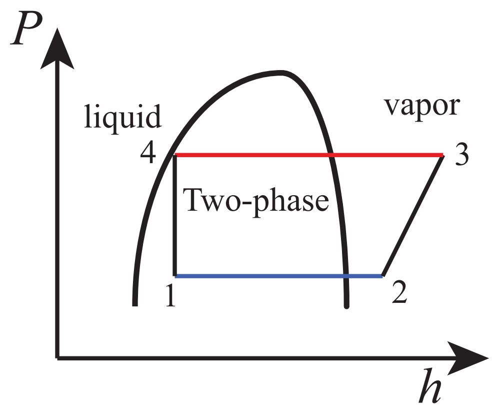

Fig. 1. Pressure(P)-Enthalpy(h) diagram of HVAC-Refrigeration Cycle

HVAC system. They developed a nonlinear model based on the assumption that the inlet and outlet mass flow rate of the refrigerant is equal. However, this assumption is valid for the steady state case. Li et al. [2009], Fallahsohi et al. [2010], Qi et al. [2010] utilized linear models of the superheat (i.e. transfer function between the opening degree of the expansion valve and the superheat temperature), and proposed different types of linear controller. Elliott et al. [2010] proposed a cascade control for a linear model of superheat. They used two feedback loops in a cascaded way to improve the transient response of the superheat control.

Cascade control (Matraji et al. [2011]) is an appropriate control method to improve the response of system against disturbances and uncertainties. In this work two separate cascaded control approaches are proposed. In the first approach, the control variable is the superheat temperature controlled by the refrigerant mass flow rate at the inlet of the evaporator (by using expansion valve). However, in the second approach, the length of two-phase flow is the control variable controlled by the mass flow rate of the refrigerant at the outlet of the evaporator (by compressor speed). The cascaded controller is designed based on a nonlinear model of HVAC system. The controller includes two feedback loops designed using super twisting sliding mode and feedback linearization. While outer loop of the cascade controller, regulates the superheat temperature or length of two-phase flow, and generates the reference value of the evaporating temperature of the refrigerant for the inner loop. The inner loop in both approaches controls the evaporating temperature. The robustness of sliding mode for nonlinear systems makes it as a good candidate for control design. A super-twisting sliding mode approach is utilized to over come some of the limitations of classical sliding mode such as chattering. Super twisting method is a common type of second order sliding mode method (i.e. control input is designed such that it enforces both sliding variable, s, and its time derivative, $\dot{s}$, to converge to zero) (Levant [2001], Bartolini et al. [1999]). Kianfar et al. [2013] proposed a multi-input multi-output super twisting sliding mode based on the nonlinear model of the evaporator, to control the evaporating temperature and superheat by using both inlet and outlet mass flow rate of the refrigerant. The common practice to reliably calculate the superheat value is to use two sensor measurements: 1 - a temperature sensor attached at evaporator outlet providing the temperature of the refrigerant in gaseous state and 2 - a pressure sensor that provides the evaporating pressure of the refrigerant (denoted P_e) (and hence the evaporating temperature T_e as there is a one-to-one relation between these two). It should be noted that it is possible to utilize a temperature sensor at the inlet of the evaporator to provide the evaporating temperature. However, due to practical difficulties encountered during operation the two temperature sensor configuration is not commonly utilized.

The proposed method in our work distinguishes itself from existing methods by using two temperature measurements: measurement of the evaporating temperature T_e (or by measuring the pressure P_e and computing the corr. T_e) and using the air temperature, T_a. As the air temperature is available in all HVAC systems, we are effectively utilizing one temperature sensor less than the commonly used control strategies presented in the literature. In this work, the estimation of the length of two phase region is used to determine the reference set-point for the evaporating temperature based on a desired superheat temperature for the outer loop feedback. To estimate the length of two phase flow, and correspondingly the superheat temperature, a super twisting sliding mode observer (Salgado et al. [2011], Tao Cheng et al. [2004]) is utilized. The sliding observer has the advantage of insensitivity to the unknown inputs, finite time convergence over the other types of observers Slotine et al. [1986], Moreno and Osorio [2008].

In this paper, in section 2, a lumped nonlinear dynamic model of the evaporator is presented. In section 3, two separate cascaded control design approaches are presented. Section 4 presents, the super twisting sliding observer design. And, in section 5, simulation results are shown to illustrate the control performance of the system. Last section is allocated to conclusions.

2. DYNAMIC MODEL

The presented dynamic model of the evaporator is a nonlinear model with two states (He and Asada [2003]), length of two phase flow, and the evaporating temperature, $x = [l, T_e]^T$. By combining mass, and energy balance in the two phase flow section of the evaporator (Fig.2), the length of two phase flow variations along the time can be determined by:

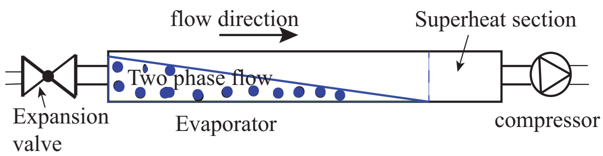

Fig. 2. Two phase flow and superheat section in the evaporator

$$\dot{l} = -\frac{\pi D_i \alpha_i}{\rho_l (1-\gamma) A h_{lg}} (T_a - T_e) l + \frac{1-x_0}{\rho_l (1-\gamma) A} \dot{m}_{in} \tag{1}$$

where D_i is the inside diameter of the evaporator, x_0 is the vapour quality of the refrigerant, γ is the mean void fraction, α_i is the inside heat transfer coefficient between evaporator refrigerant and ambient in the two phase section, A is the cross section of evaporator pipe, h_{lg} is the difference between enthalpy of refrigerant in liquid and gas forms, T_a is the ambient temperature, ρ_l is the refrigerant density in saturated liquid form, and $\dot{m}_{in}$ is the inlet mass flow rate of the refrigerant controlled by the expansion valve. Variation of evaporating temperature of the refrigerant is given as follows,

$$\dot{T}_e = \frac{\pi D_i \alpha_i}{k h_{lg}} (T_a - T_e) l + \frac{x_0}{k} \dot{m}_{in} - \frac{1}{k} \dot{m}_{out} \tag{2}$$

where, $k = V \frac{d\rho_g}{dT_e}$, and $\frac{d\rho_g}{dT_e} = k_1 T_e + k_2$, where k_1, and k_2 are considered constants and depend on the refrigerant fluid characteristics, V is the inside volume of low pressure side, $\dot{m}_{out}$ is the mass flow rate of the refrigerant at the outlet of the evaporator. By neglecting the axial conduction, superheat temperature can be calculated by the following formula

$$T_{sh} = (T_a - T_e)(1 - e^{\frac{-c(L-l)}{\dot{m}_{out}}}) \tag{3}$$

where, L is the length of evaporator, and $c = \frac{\pi D_i \alpha_i}{C_p}$, C_p is the specific heat capacity of the refrigerant.

3. SECOND ORDER SLIDING MODE CONTROL

As it is said, a second order sliding mode (SOSM) is a type of higher order sliding mode (HOSM), introduced to over come the limitation of standard sliding mode such as chattering problem Levant [2003], Pisano and Usai [2011]. Two types of the SOSM are twisting sliding mode and super twisting sliding mode. Here, super twisting strategy is used to design the controller. The nonlinear model of the system as a single input system is considered as follows:

$$\dot{x} = f(x) + g(x)u \tag{4}$$

where $x \in \mathbb{X} \subset \mathbb{R}^n$ are the state variables, and $u \in \mathbb{U} \subset \mathbb{R}$ is the control input, f and g are smooth nonlinear functions. The sets $\mathbb{X}$ and $\mathbb{U}$ are defined as $\mathbb{X} = \{x \in \mathbb{R} || x_i| \leq x_{i_{max}}, i = 1, ..., n\}$, and $\mathbb{U} = \{u \in \mathbb{R} || u| \leq u_{max}\}$. By defining the sliding variable, $s(x) = y - y_{ref}$, the second order sliding mode steers the sliding variable $(s(x))$ and its time derivative $(\dot{s}(x))$ to zero. If the sliding variable has a relative degree 1, the sliding surface derivatives are

$$\dot{s} = \frac{\partial}{\partial t}s + \frac{\partial}{\partial x}s(f(x) + g(x)u) \tag{5}$$

$$\begin{aligned}\ddot{s} &= \frac{\partial}{\partial t}\dot{s} + \frac{\partial}{\partial x}\dot{s}(f(x) + g(x)u) + \frac{\partial}{\partial u}\dot{s}\dot{u} \\ &= \phi(x) + \gamma(x)v\end{aligned} \tag{6}$$

where $v = \dot{u}$. Considering there exist positive constant values Φ, Γ_m, and Γ_M such that for $x \in \mathbb{X}$, and $u \in \mathbb{U}$

$$|\frac{\partial}{\partial t}\dot{s} + \frac{\partial}{\partial x}\dot{s}(f(x) + g(x)u)| \leq \Phi, \tag{7}$$

$$0 < \Gamma_m < \frac{\partial}{\partial u}\dot{s} < \Gamma_M \tag{8}$$

The super twisting sliding mode is defined by

$$\begin{aligned} u =& \quad u_1 + u_2, \\ u_1 =& \quad -\alpha\sqrt{|s|}sign(s) \\ \dot{u}_2 =& \quad -\beta sign(s) \\ \alpha > 0& \;, \quad \beta > 0 \end{aligned} \tag{9}$$

the sufficient condition for the finite time convergence to the sliding surface is

$$\beta > \frac{\Phi}{\Gamma_m}, \alpha^2 \geq \frac{4\Phi\Gamma_M(\beta + \Phi)}{{\Gamma_m}^3(\beta - \Phi)} \tag{10}$$

4. CONTROL DESIGN

Considering $[x_1, x_2]^T = [l, T_e]^T$, and $u = \dot{m}_{in}$, dynamic equations of the system, (1), and (2), can be written in the form of $\dot{x} = f(x) + g(x)u$, as follows:

$$\begin{aligned}\dot{x}_1 &= a_1(T_a - x_2)x_1 + b_1 u, \\ \dot{x}_2 &= a_2\frac{(T_a - x_2)x_1}{k_1 x_2 + k_2} + \frac{b_2}{k_1 x_2 + k_2}u - \\ &\quad \frac{1}{k_1 x_2 + k_2}\dot{m}_{out}\end{aligned} \tag{11}$$

where $a_1 = -\frac{\pi D_i \alpha_i}{\rho_l(1-\gamma)A h_{lg}}$, $b_1 = \frac{1-x_0}{\rho_l(1-\gamma)A}$, $a_2 = \frac{\pi D_i \alpha_i}{h_{lg}}$.

$$f = \begin{pmatrix} a_1(T_a - x_2)x_1 \\ \frac{a_2}{k}(T_a - x_2)x_1 \end{pmatrix}, g = \begin{pmatrix} b_1 \\ \frac{b_2}{k} \end{pmatrix} \tag{12}$$

4.1 First Approach: Superheat Control by Inlet Mass Flow

Here the output of evaporator is

$$y = h(x) = [T_{sh}, T_e]^T \tag{13}$$

however, in the next sections it is assumed that the measured output of the system is the evaporating temperature, T_e, and the superheat temperature is estimated by implementing an observer presented in the next sections.

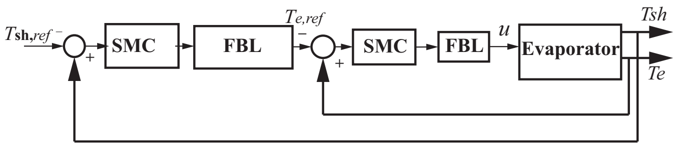

Fig. 3. Block diagram of the cascade control scheme

I. Outer loop The sliding surface for the outer loop of the proposed cascaded control system (Fig. 3) is defined as follows:

$$\begin{aligned} s_1 &= T_{sh} - T_{sh_{ref}} \\ s_1 &= (T_a - x_{2ref})(1 - e^{\frac{-c(L-x_1)}{\dot{m}_{out}}}) - T_{sh_{ref}} \end{aligned} \tag{14}$$

where ref denotes the reference signal, considering $u^* = -x_2$ as the control output, leads to an input dependent, i.e. $s_1 = s_1(x_1, u^*)$ (s_1 does not have any relative degree), so by taking the first derivative of s_1 with respect to time, $\dot{u}^*$ appears in that:

$$\dot{s}_1 = -\dot{x}_2(1 - e^{\frac{-c(L-x_1)}{\dot{m}_{out}}}) - \frac{c}{\dot{m}_{out}}\dot{x}_1(T_a - x_2)e^{\frac{-c(L-x_1)}{\dot{m}_{out}}} - \dot{T}_{sh_{ref}} \tag{15}$$

considering $u_1 = -\dot{x}_2$ as the control output, it can be written as follows:

$$\dot{s}_1 = \phi + \gamma u_1 \tag{16}$$

where $\gamma = 1 - e^{\frac{-c(L-x_1)}{\dot{m}_{out}}}$. Considering the uncertainties, γ and ϕ can be written in the following form:

$$\begin{aligned} \phi_1 &= \phi_{01} + \delta\phi_1 \\ \gamma_1 &= \gamma_{01} + \delta\gamma_1 \end{aligned} \tag{17}$$

where ϕ_{01}, and γ_{01} are nominal values of γ_1 and ϕ_1. Using feedback linearization method (FBL) for the nominal values, that is equating $\dot{s}_1$ with v_1, the evaporating temperature of the refrigerant as the reference for the inner loop, x_{2ref}, i.e. $T_{e_{ref}}$ can be determined. Therefore, x_{2ref} is:

$$\begin{aligned} u_1 &= {\gamma_{01}}^{-1}(v_1 - \phi_{01}) \\ T_{e_{ref}} &= -\int(u_1)dt \end{aligned} \tag{18}$$

where $v_1 = v_{11} + v_{12}$, can be determined using a super twisting control method,

$$v_{11} = -\alpha_1\sqrt{|s_1|}sign(s_1), \dot{v}_{12} = -\beta_1 sign(s_1) \tag{19}$$

Bounds of the convergence conditions can be found by taking the second derivatives of s_1 with respect to time: which is in the form of $\ddot{s}_1 = \phi_1^* + \gamma_1^* v$, where the coefficient of $v = -\ddot{x}_{2_{ref}}$, is $\gamma_1^* = (1 - e^{\frac{-c(L-x_1)}{\dot{m}_{out}}})$. Because of bounded states and inputs, ϕ_1^*, and γ_1^* are bounded as well.The lower and upper bounds of γ_1^* can be determined based on the physical limitations of the system, (e.g. $0 < L_0 < x_1 < L$). Therefore, the lower and upper bounds are $\Gamma_m = \delta > 0$ (where δ has a small positive value), and $\Gamma_M = 1 - e^{-\frac{cL}{\dot{m}_{out}}}$, respectively.

II. Inner loop Now in order to design the inner loop controller, the following sliding surface is defined:

$$s_2 = T_e - T_{e_{ref}}, \tag{20}$$

which can be written as

$$s_2 = x_2 - x_{2_{ref}}, \tag{21}$$

considering the inlet mass flow rate $\dot{m}_{in}$ as the control input (u), the relative degree of s_2 is one. Then by differentiating the (21)

$$\dot{s}_2 = \dot{x}_2 - \dot{x}_{2_{ref}} = \mathscr{L}_{f_2} s_2 + \mathscr{L}_{g_2} s_2 u, \tag{22}$$

where $\mathscr{L}$ denotes Lie derivative. By substituting (2) into (22), and using feedback linearization method (i.e. $\dot{s}_2 = \nu_2$), the inlet mass flow rate $\dot{m}_{in}$ is determined as:

$$\dot{m}_{in} = \frac{k}{x_0}(\nu_2 - \frac{\pi D_i \alpha_i}{k h_{lg}}(T_a - T_e)l + \dot{T}_{e_{ref}} + \frac{1}{k}\dot{m}_{out}) \tag{23}$$

where, ν_2 is the super twisting sliding mode control output,

$$\nu_2 = \nu_{21} + \nu_{22} \tag{24}$$

$$\nu_{21} = -\alpha_2\sqrt{|s_2|}sign(s_2), \dot{\nu}_{22} = -\beta_2 sign(s_2) \tag{25}$$

4.2 Second Approach: Control of Length of two-phase Flow by Outlet Mass Flow

In this part of paper, second approach which is control of the two-phase flow using the mass flow rate at the outlet of evaporator is proposed. As it was earlier mentioned, $\dot{m}_{out}$ is controlled by the compressor speed. So here the control input is $\dot{m}_{out}$ instead of $\dot{m}_{in}$ as it was the control input in the previous section. The cascaded control here has two loops, where the outputs of system are $y = [l, T_e]^T$. For the outer-loop the length of two phase flow, l, and for the the inner loop the evaporating temperature, T_e are used. The control design is presented in the following subsections.

I. Outer-loop Control Design Here, by defining the sliding variable:

$$s_1 = l - l_{ref} \tag{26}$$

where l_{ref} is the reference value for the length of two-phase flow. It is assumed it is a constant value. By taking the first derivative with respect to time, and substituting with $\dot{l}$

$$\dot{s}_1 = \dot{l} - \dot{l}_{ref} = a_1(T_a - x_2)x_1 + b_1\dot{m}_{in} \tag{27}$$

considering x_2 as the control input u as the reference signal for the inner loop, x_{2ref} can be generated from and using FBL (i.e. equating $\dot{s}_1$ with ν),

$$x_{2ref} = T_a - \frac{1}{a_1 x_1}(\nu_1 - b_1\dot{m}_{in}) \tag{28}$$

where ν_1 is the control input determined by utilizing STW method. Second derivative of s_1 is:

$$\ddot{s}_1 = a_1\dot{x}_1(T_a - x_2) + a_1 x_1 \dot{x}_2 + b_1\ddot{m}_{in} \tag{29}$$

So $\ddot{s}_1$ can be written as $\ddot{s}_1 = \phi_1 + \gamma_1\dot{x}_2$. Since the states and inputs are bounded, then γ_1 and ϕ_1 are bounded, which means that $\Gamma_{m1} < \gamma_1 = a_1x_1 < \Gamma_{M1}$, and $|\phi_1| = |a_1\dot{x}_1(T_a - x_2) + b_1\ddot{m}_{in}| < \Phi_1$. Then we can find gains of controller to guarantee the convergence conditions.

II. Inner-loop Control Design -

The sliding surface is defined as:

$$s_2 = T_e - T_{e_{ref}} \tag{30}$$

Considering the mass flow rate at the outlet $\dot{m}_{out}$, as the control input, the relative degree of the sliding variable is 1 with respect to that. By taking the first time derivative of the sliding variable and substituting (2):

$$\dot{s}_2 = \dot{T}_e - \dot{T}_{e_{ref}} = a_2(T_a - x_2)x_1 + \frac{x_0}{k_2}\dot{m}_{in} - \frac{1}{k_2}u - \dot{T}_{e_{ref}} \tag{31}$$

by using FBL technique, and equating $\dot{s}_2$ with ν_2, outlet mass flow rate i.e. control input can be determined:

$$u = \dot{m}_{out} = -k_2(\nu_2 - a_2(T_a - x_2)x_1 - \frac{x_0}{k_2}\dot{m}_{in} - \dot{T}_{e_{ref}}) \tag{32}$$

where ν_2 is determined by super twisting sliding mode method. For the convergence of super twisting sliding mode, it is assumed that the bounds on coefficients of $\ddot{s}_2 = \phi + \gamma\dot{u}$ (i.e. $\gamma = \frac{1}{k_2}$, and $\phi = a_2(T_a - x_2)\dot{x}_1 - a_2x_1\dot{x}_2 + \frac{x_0}{k_2}\ddot{m}_{in} + \ddot{T}_{e_{ref}}$), are known ($\Gamma_{m2} < \frac{1}{k_2} < \Gamma_{M2}$), and $|\frac{d\phi}{dt}| < \Phi_2$, then gains of controller can be chosen such that the sufficient condition (as it was mentioned in the previous section) for the convergence is satisfied. To determine the control input we need to have the time derivative of $\dot{T}_{e_{ref}}$. To do so, in the next section a super-twisting sliding mode differentiator is proposed.

5. DIFFERENTIATOR

In order to generate derivative of reference evaporating temperature of refrigerant, $\dot{T}_{e_{ref}}$, in real time, a second order super twisting differentiator is used which is more accurate and robust than the regular differentiators Levant [2003]. By defining the sliding surface

$$s_3 = \hat{T}_{e_{ref}} - T_{e_{ref}} \tag{33}$$

The output of differentiator is determined by

$$\dot{\hat{T}}_{e_{ref}} = \nu_{31} + \nu_{32} \tag{34}$$

where

$$\nu_{31} = -\alpha_3\sqrt{|s_3|}sign(s_3), \dot{\nu}_{32} = -\beta_3 sign(s_3) \tag{35}$$

6. SUPER TWISTING SLIDING OBSERVER DESIGN

The only measured output of system y is T_e. The other output, T_{sh}, can be determined by estimating of the value of the length of two phase flow inside the evaporator l. By using the Lie derivative, (36), and considering that, k is a constant value, observability matrix is determined by (37)

$$y = h(x) = x_2$$
$$\mathscr{L}_f h = \frac{a_2}{k}(T_a - x_2)x_1 \tag{36}$$

$$\mathscr{O} = \begin{pmatrix} 0 & 1 \\ \frac{a_2}{k}(T_a - x_2) & -\frac{a_2}{k}x_1 \end{pmatrix} \tag{37}$$

The rank of (37) is 2, so the system is observable.

Here an observer using super twisting sliding mode method is designed to estimate l.

$$\dot{\hat{x}}_1 = f_1(x) + g_1(x)u + L_1\tilde{x}_2 + L_2|\tilde{x}_2|^{1/2}sign(\tilde{x}_2) \tag{38}$$

$$\dot{\hat{x}}_2 = f_2(x) + g_2(x)u + L_3\tilde{x}_2 + L_4 sign(\tilde{x}_2) \tag{39}$$

where L_i, $i = 1, \ldots, 4$ are the gains for the observer (Salgado et al. [2011]). The value of those gains are: L1=1, L2=1, L3=10, and L4=0.1.

In other words (38), and (39), for the evaporator model can be written as follows

$$\dot{\hat{l}} = a_1(T_a - \hat{T}_e)\hat{l} + b_1\dot{m}_{in} + L_1\tilde{T}_e + L_2|\tilde{T}_e|^{1/2}sign(\tilde{T}_e) \quad (40)$$

$$\dot{\hat{T}}_e = \frac{a_2}{k}(T_a - \hat{T}_e)\hat{l} + \frac{b_2}{k}\dot{m}_{in} - \frac{b_3}{k}\dot{m}_{out} + L_3\tilde{T}_e + L_4 sign(\tilde{T}_e) \quad (41)$$

$$\tilde{T}_e = T_e - \hat{T}_e \quad (42)$$

where $\tilde{T}_e$ is the observer state error used to estimate the states.

7. SIMULATION

To validate the designed control system we simulated the system by using MATLAB/Simulink. The values of parameters used in simulations are shown in table I. The parameters of the evaporator in table I, for simulation are according to Eldredge et al. [2008]. The gains of the super twisting controllers for both outer and inner loops are $\alpha_1 = 0.4$ and $\beta_1 = 0.8$, $\alpha_2 = 0.5$ and $\beta_2 = 1$, respectively.

Table 1. Evaporator parameters

Parameters	Units	Value
Slip ratio		4.6
mean void fraction		0.93
ρ_l	kg/m^3	1300
Superheat heat transfer c.	kW/m^2 K	0.04
Two Phase heat transfer c.	kW/m^2 K	2
Inlet vapor quality		0.2
length of evaporator	m	11.45
Flow cross sectional area	m^2	0.00005156
Internal volume	m^3	0.0028665
Refrigerant R134a		
k_1		0.0368
k_2		0.4005
C_p		1

Specific heat capacity of the refrigerant, C_p, in the superheat section of the evaporator is a function of pressure and enthalpy. Here, in the simulation an average value of 1 is chosen from the refrigerant R-134a tables. The parameters k_1, and k_2 are the coefficient of $\frac{d\rho_g}{dT_e}$, where $\rho_g(T_e)$ is approximated by curve fitting of a quadratic polynomial from the table of properties of R-134a. Fig.4 shows the superheat temperature (°C) (top), and the inlet mass flow rate kg/s (bottom). As it is illustrated the controller objective is to maintain the superheat temperature on 6(°C), and it reaches to the reference value in less than 10 sec..

As it is shown in Fig.5, both the evaporating temperature (top), and the length of two phase flow (bottom) reach to their steady state values in less than 10 sec.. In this part the results are for the case when there is a disturbance on the system due to the variations of outlet mass flow rate of the refrigerant (Fig.6), which is a function of the compressor speed, and there is parameters uncertainty in the model of system. The parameters uncertainties of the dynamic model of the system are in the additive form,i.e. $a_1 + \delta a_1$, and those values for the simulations are, $\delta a_1 = +10\%$, $\delta b_1 = +10\%$, $\delta a_2 = -5\%$.

Fig.7 shows the good convergence of the estimated states, $\hat{T}_{sh}$, and $\hat{l}$, to their real values by the proposed observer.

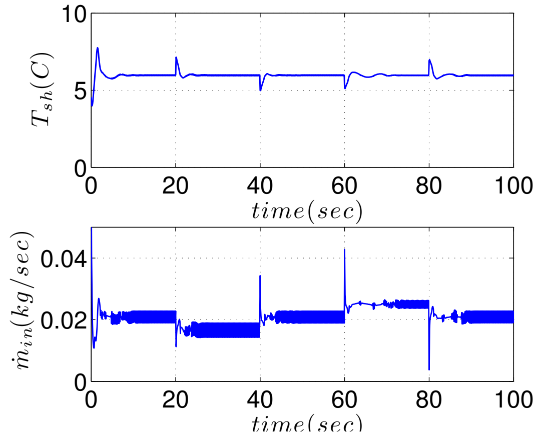

Fig. 4. Superheat temperature(top) and inlet mass flow rate(bottom)

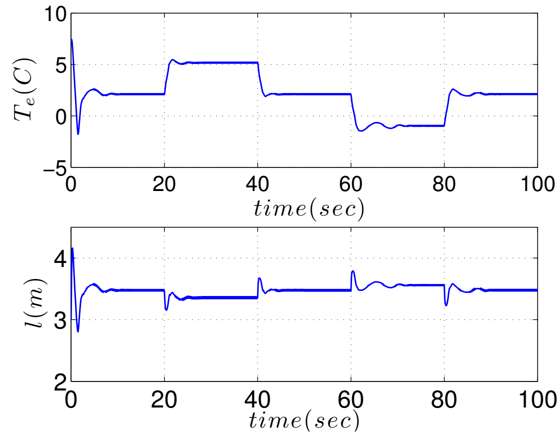

Fig. 5. Evaporating temperature of refrigerant(top) and length of two phase flow inside the evaporator(bottom)

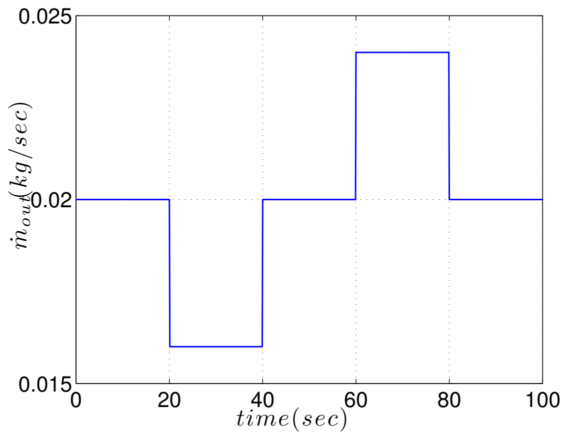

Fig. 6. outlet mass flow rate variations as the disturbance on system

In this part the simulation results of the second approach is presented. Here the goal is to control the length of two-phase flow, and the control input is mass flow rate at the outlet of the evaporator, $\dot{m}_{out}$, controlled by compressor speed. In the following simulations the reference value for the length of two phase flow is $4m$. In this part, to have better evaluation of simu-

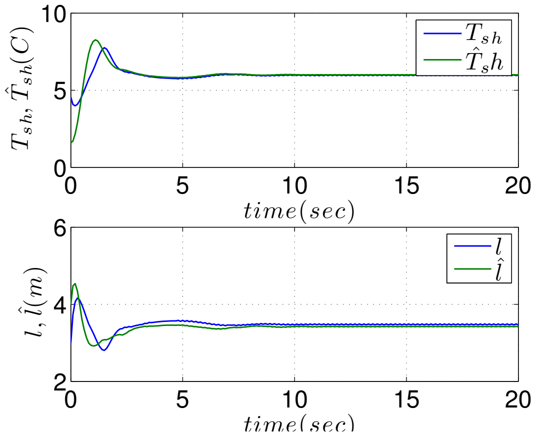

Fig. 7. Estimation of superheat(top) and length of two phase flow(bottom) with presence of white noise using super-twisting sliding mode observer

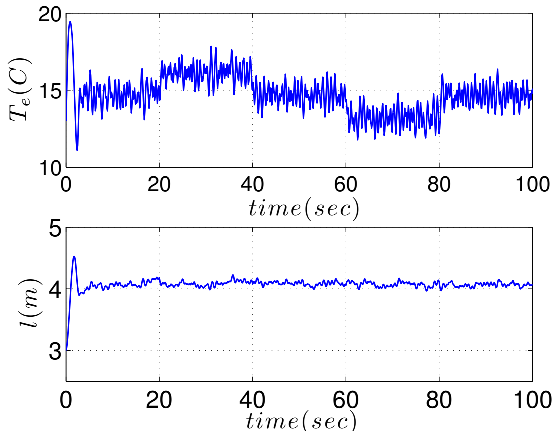

Fig. 8. Evaporating temperature (top), length of two-phase flow (bottom)

lations, a measurement white noise is added to the evaporating temperature. The disturbance is the variation of inlet mass flow rate $\dot{m}_{in}$. The gains of the super twisting controllers for both outer and inner loops are $\alpha_1 = 3$ and $\beta_1 = 1$, $\alpha_2 = 2$ and $\beta_2 = 1$, respectively.

Fig.8 (bottom) shows the length of two-phase flow regulated for the reference value $4m$. Fig.8 (top) shows the variations of the evaporating temperature, which is regulated in the inner-loop of the controller. The corresponding superheat temperature and the control input, $\dot{m}_{out}$ is shown in Fig.9.

Fig.10 shows the variation of inlet mass flow rate as the disturbance acted on the system.

8. CONCLUSIONS

In this paper two cascade controller approaches proposed to control the HVAC systems superheat. In the first approach, superheat is controlled directly by controlling the inlet mass flow rate as the control input, however in the second approach, length of two phase flow controlled by the outlet mass flow rate as the control input. Simulation results showed that the proposed super twisting sliding mode control for both inner loop and outer loop, provided satisfactory results, and robustness

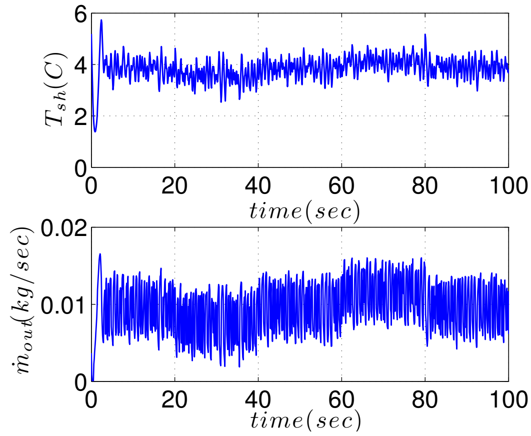

Fig. 9. Superheat temperature (top), outlet mass flow rate (control input) (bottom)

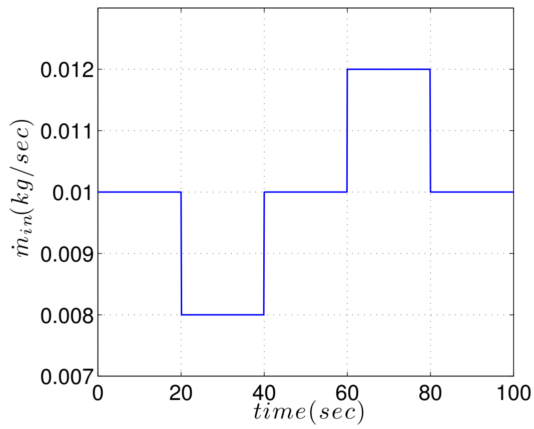

Fig. 10. Inlet mass flow rate, $\dot{m}_{in}$, as the disturbance

against uncertainties and disturbances. In this work a method of using one sensor instead of using two sensors (as it is common in practice), to determine superheat temperature is proposed. Superheat temperature was determined by measuring of evaporating temperature, and utilizing a robust sliding mode observer to determine the length of two phase flow. The generalized super twisting sliding mode observer appropriately estimates the length of two phase flow of the refrigerant in the evaporator. In future work, we are going to implement the proposed control system on a real HVAC system.

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