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Reliable Measurement-Based System Design: A New Paradigm ⋆

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Abstract: In this paper we introduce a new approach to the design of fault-tolerant systems. Unlike traditional methods which rely on a mathematical model of the system, the proposed method is based on measurements only and a model is not required. The key elements of this approach are the determination of the design element values to preserve acceptable system performance under predetermined failures of fault prone elements. It is shown that such designs can be carried out without the necessity of a model and based only on strategic processing of measurements. An illustrative example of an electrical circuit that continues to maintain acceptable operation despite component failures is included.

1. INTRODUCTION

In this paper, we propose a new technique to design fault tolerant systems by using a recently introduced measurement based approach. Our goal is to obtain appropriate design parameters, to maintain system variables that represent the performance of the system, within prespecified limits with or without the presence of faults. We assume that the mathematical model of the system is not known. The novel measurement based approach in Bhattacharyya [2013] is used in our solution. First, the relation between system variables, fault prone components and design parameters is obtained using measurements made on the system. Once this relation is determined, appropriate design parameters can be characterized for fault tolerant operation.

It is noted that all existing design techniques in Fault Tolerant System Design are based on the availability of mathematical models of the system. Such requirement of availability of a mathematical model is becoming more and more difficult as system complexity increases. Therefore, measurement based designs serve as an attractive alternative to model based design. These techniques are especially advantageous in real world engineering, where accurate models are difficult to obtain or often unavailable.

Recently, a new approach that completely relies on measurements only was introduced in the monograph Bhattacharyya [2013]. It determines the solution function of a system described by linear equations when the model is not known. This approach provides the functional dependency of the system variables on the design parameters by using a few measurements made on the system and processing them strategically without constructing the model to determine the parameters of the function. This approach introduces a new paradigm for describing the behavior of a system.

Some background literature on fault tolerant design follows. In Ikeda [1992], one plant and two controller configuration was considered to obtain a reliable design. In Gundes [1992], a stable factorization approach was used to design controllers with fault tolerance. Subsequently, this work led to design a software package implemented in Mathematica, Bhaya [1994]. In Han [1996] a state feedback control law which retains stability against arbitrary actuator failures and parameter perturbations is derived using a positive definite symmetric solution of a new Riccati-type matrix equation. In Ikeda [2004], a method to design a decentralized H_∞ output feedback controller to maintain stability of the system and a certain performance under failure of any one of the local controllers. All of the above methods are model-based.

The rest of this paper is organized as follows. Section 2 gives an overview of the measurement based approach to unknown linear systems. Following the problem formulation and our proposed approach in Sections 3 and 4, an illustrative example is given in Section 5.

2. MEASUREMENT BASED APPROACH TO LINEAR EQUATIONS

Consider the system of parametrized linear equations

$$\mathbf{A}(\mathbf{p})\mathbf{x} = \mathbf{b}(\mathbf{p}) \tag{1}$$

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where $\mathbf{A}(\mathbf{p})$ is an $n \times n$ matrix, and $\mathbf{x}$ and $\mathbf{b}(\mathbf{p})$ are $n \times 1$ vectors all with real or complex entries. Assuming that $|\mathbf{A}(\mathbf{p})| \neq 0$, there exists a unique solution $\mathbf{x}$ and the i^{th} component x_i of $\mathbf{x}$ is given by

$$x_i(\mathbf{p}) = \frac{|\mathbf{B}_i(\mathbf{p})|}{|\mathbf{A}(\mathbf{p})|} \tag{2}$$

where $\mathbf{B}_i(\mathbf{p})$ is the matrix obtained by replacing the i^{th} column of $\mathbf{A}(\mathbf{p})$ by $\mathbf{b}(\mathbf{p})$. This type of linear equation can represent a large class of systems in engineering, biology, and the social sciences. Here $\mathbf{x}$ represents the system variables such as currents, voltages, displacements, flow rates, and $\mathbf{p}$ represents a vector of design parameters which appears affinely in $\mathbf{A}(\mathbf{p})$ and $\mathbf{b}(\mathbf{p})$. Thus, we can write

$$\mathbf{A}(\mathbf{p}) := \mathbf{A}_0 + p_1\mathbf{A}_1 + p_2\mathbf{A}_2 + \cdots + p_l\mathbf{A}_l. \tag{3}$$

To proceed, consider the special case of a scalar parameter $\mathbf{p} = p_1$. Then we have the following.

Lemma 1. Let

$$\mathbf{A}(\mathbf{p}) = \mathbf{A}_0 + p_1\mathbf{A}_1. \tag{4}$$

Then $|\mathbf{A}(\mathbf{p})|$ is a polynomial of degree at most r_1 in p_1 where

$$r_1 = \text{rank}\,[\mathbf{A}_1]\,. \tag{5}$$

The proof follows easily from the properties of determinants.

Lemma 2. With $\mathbf{A}(\mathbf{p})$ as in (3), let

$$r_i = \text{rank}\,[\mathbf{A}_i]\,, \quad i = 1, 2, \cdots, l. \tag{6}$$

Then $|\mathbf{A}(\mathbf{p})|$ is a multivariate polynomial in $\mathbf{p}$ of degree r_i or less in p_i, $i = 1, 2, \cdots, l$ and

$$|\mathbf{A}(\mathbf{p})| = \sum_{i_l=0}^{r_l} \cdots \sum_{l_2=0}^{r_2} \sum_{i_1=0}^{r_1} \alpha_{i_1 i_2 \cdots i_l} p_1^{i_1} p_2^{i_2} \cdots p_l^{i_l} =: \alpha(\mathbf{p}) \tag{7}$$

$$|\mathbf{B}_i(\mathbf{p})| = \sum_{i_l=0}^{t_l} \cdots \sum_{l_2=0}^{t_2} \sum_{i_1=0}^{t_1} \beta_{i_1 i_2 \cdots i_l} p_1^{i_1} p_2^{i_2} \cdots p_l^{i_l} =: \beta(\mathbf{p}) \tag{8}$$

where $\mathbf{B}_i(\mathbf{p})$ is the matrix obtained by replacing the i^{th} column of $\mathbf{A}(\mathbf{p})$ by the vector $\mathbf{b}(\mathbf{p})$ and

$$t_i = \text{rank}\,[\mathbf{B}_i(\mathbf{p})] \quad i = 1, 2, \cdots, l. \tag{9}$$

This follows immediately from Lemma 1.

Based on the above, we have the following characterization of parametrized solutions.

Theorem 3. Let

$$\mathbf{A}(\mathbf{p})\mathbf{x} = \mathbf{b}(\mathbf{p}) \tag{10}$$

where

$$\mathbf{A}(\mathbf{p}) = \mathbf{A}_0 + p_1\mathbf{A}_1 + p_2\mathbf{A}_2 + \cdots + p_l\mathbf{A}_l. \tag{11}$$

Then

$$x_i(\mathbf{p}) = \frac{\beta_i(\mathbf{p})}{\alpha(\mathbf{p})}, \quad i = 1, 2, \cdots, n \tag{12}$$

where $\beta_i(\mathbf{p})$, $i = 1, 2, \cdots, n$ and $\alpha(\mathbf{p})$ are multivariate polynomials in $\mathbf{p}$.

The proof follows from (2) and Lemma 2.

For an unknown model, ($\mathbf{A}(\mathbf{p})$ and $\mathbf{b}(\mathbf{p})$ are not known. However we assume that the ranks r_i and t_i are known. The coefficient in (7) and (8) denoted by the vectors α and β are unknown, and the number of unknown coefficients is

$$\mu := \Pi_{i=1}^{l}\,(r_i + 1) + \Pi_{i=1}^{l}\,(t_i + 1) - 1. \tag{13}$$

However, these coefficients can be determined by setting the parameter vector $\mathbf{p}$ to μ different sets of values and solving a set of μ linear equations in the μ unknowns.

Theorem 4. (A Measurement Theorem). The function $x(\mathbf{p})$ can be determined from μ measurements and solution of a system of μ linear equations called the measurement equations in the unknown coefficient vectors α and β.

To proceed, let us first consider a system with one design parameter and one performance measurement shown in Fig. 1. In most physical systems the design parameter $\mathbf{p} = p_1$ appears in the the unknown system matrix $\mathbf{A}(\mathbf{p})$ with rank one dependency ($r_1 = 1$). Henceforth we assume this rank one dependency.

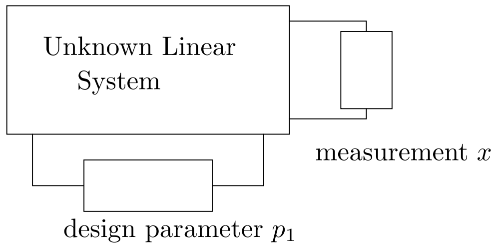

Fig. 1. An unknown linear system with one design parameter

In this case, the functional dependency of the measurement x on the design parameter p_1 can be expressed as

$$x(p_1) = \frac{\alpha_0 + \alpha_1 p_1}{\beta_0 + p_1} \tag{14}$$

where α_0, α_1, β_0 are constants to be determined. These coefficients can easily be determined by conducting 3 experiments by setting 3 different values to the design parameter p_1 and making the corresponding measurements. Then the following set of measurement equations can be formed:

$$\begin{bmatrix} 1 & p_{11} & -x_1 \\ 1 & p_{12} & -x_2 \\ 1 & p_{13} & -x_3 \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \beta_0 \end{bmatrix} = \begin{bmatrix} x_1 p_{11} \\ x_2 p_{12} \\ x_3 p_{13} \end{bmatrix} \tag{15}$$

Now let us consider a system with two design parameters (p_1, p_2) and one performance measurement x. Under the same assumption that the design parameter p_i, $i = 1, 2$ appears in the matrix $\mathbf{A}(\mathbf{p})$ with rank 1 ($r_1 = 1$, $r_2 = 1$), the functional dependency of the measurement x on the design parameter (p_1, p_2) can be expressed as

$$x(p_1, p_2) = \frac{\alpha_0 + \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_1 p_2}{\beta_0 + \beta_1 p_1 + \beta_2 p_2 + p_1 p_2} \tag{16}$$

where $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \beta_0, \beta_1, \beta_2$ are constants that can be determined by conducting 7 experiments by assigning 7 different sets of values to the design parameters (p_1, p_2) and the taking the corresponding measurements. Thus, we have the following.

$$\begin{bmatrix} 1 & p_{11} & p_{21} & p_{11}p_{21} & -x_1 & -x_1p_{11} & -x_1p_{21} \\ 1 & p_{12} & p_{22} & p_{12}p_{22} & -x_2 & -x_2p_{12} & -x_2p_{22} \\ 1 & p_{13} & p_{23} & p_{13}p_{23} & -x_3 & -x_3p_{13} & -x_3p_{23} \\ 1 & p_{14} & p_{24} & p_{14}p_{24} & -x_4 & -x_4p_{14} & -x_4p_{24} \\ 1 & p_{15} & p_{25} & p_{15}p_{25} & -x_5 & -x_5p_{15} & -x_5p_{25} \\ 1 & p_{16} & p_{26} & p_{16}p_{26} & -x_6 & -x_6p_{16} & -x_6p_{26} \\ 1 & p_{17} & p_{27} & p_{17}p_{27} & -x_7 & -x_7p_{17} & -x_7p_{27} \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} x_1p_{11}p_{21} \\ x_2p_{12}p_{22} \\ x_3p_{13}p_{23} \\ x_4p_{14}p_{24} \\ x_5p_{15}p_{25} \\ x_6p_{16}p_{26} \\ x_7p_{17}p_{27} \end{bmatrix} \quad (17)$$

This 2 design parameter problem motivates the following fault-tolerant system design problem:

3. FAULT TOLERANT SYSTEM DESIGN: SINGLE FAILURE

Consider the following system shown in Fig. 2. The parameter p represents the states of a component which is failure prone and x is the measurement of the performance of the system. The fault tolerant system design is accomplished by determining the design variable q so that the performance measure x remains within the prescribed range of acceptable values as the fault prone parameter p undergoes normal and failure states. For example, $p = p_0$ (some fixed value) for normal state and $p = 0$ (or ∞) if a failure occurs.

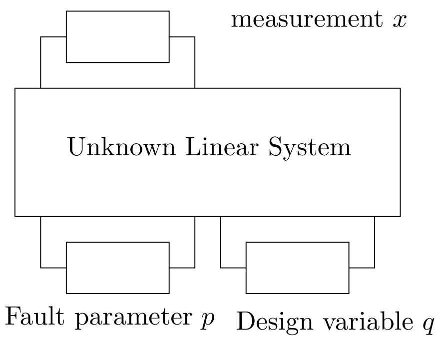

Fig. 2. Fault Tolerant System Design (Single Failure)

In this case, the functional dependency of the performance measurement x on the fault prone parameter p and the design variable q can be written as

$$x(p, q) = \frac{\alpha_0 + \alpha_1 p + \alpha_2 q + \alpha_3 pq}{\beta_0 + \beta_1 p + \beta_2 q + pq}. \quad (18)$$

As shown in the previous section, the coefficients α_0, α_1, α_2, α_3, β_0, β_1, β_2 can be determined by conducting 7 experiments with 7 different sets of values of the parameters p and q, and taking the corresponding measurements x.

Let the acceptable range of system performance values for x be limited by

$$x \in [x_{\min}, x_{\max}]\,. \quad (19)$$

Suppose that the fault prone parameter undergoes p^1, p^2, p^3 states indicating normal or failure conditions of the component. Then the task is to determine the design variable q or a range of such values such that

$$x_{\min} \leq \min_{p \in [p^1, p^2, p^3]} x(p, q) \quad (20)$$

and

$$x_{\max} \geq \max_{p \in [p^1, p^2, p^3]} x(p, q). \quad (21)$$

4. FAULT TOLERANT SYSTEM DESIGN: TWO FAILURES

We now consider a system with two vulnerable components, one design parameter, and one performance variable (see Fig. 3).

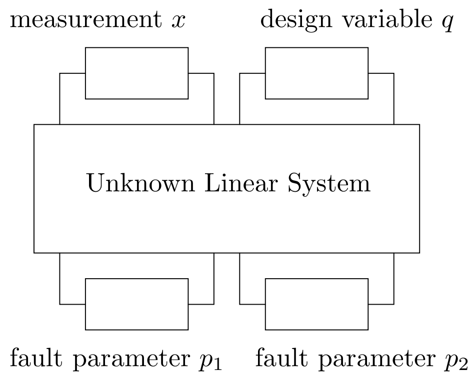

Fig. 3. Fault tolerant system design (two failure prone elements)

The functional dependency of the performance variable x on the two fault prone parameter $\mathbf{p} = [p_1, p_2]$ and the design parameter q can be written as

$$x(\mathbf{p}, q) = \frac{\beta(\mathbf{p}, q)}{\alpha(\mathbf{p}, q)} \quad (22)$$

where

$$\begin{aligned} \beta(\mathbf{p}, q) =& \alpha_0 + \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 q + \alpha_4 p_1 p_2 + \alpha_5 p_1 q \\ &+ \alpha_6 p_2 q + \alpha_7 p_1 p_2 q \\ \alpha(\mathbf{p}, q) =& \beta_0 + \beta_1 p_1 + \beta_2 p_2 + \beta_3 q + \beta_4 p_1 p_2 + \beta_5 p_1 q \\ &+ \beta_6 p_2 q + p_1 p_2 q. \end{aligned}$$

Clearly, 15 experiments with 15 different sets of values of $(\mathbf{p}, q)$ and the corresponding measurements of x suffice to determine all the coefficients representing the above functional dependency. Suppose that the fault prone parameter $\mathbf{p}$ undergoes states $\bar{\mathbf{p}} := \{\mathbf{p}^k, k = 1, 2, \cdots\}$ representing all possible failure states. Then the fault tolerant design is obtained by selecting the design variable q or a range of q values such that for the given acceptable tolerance $x \in [x_{\min}, x_{\max}]$, we achieve

$$x_{\min} \leq \min_{\mathbf{p} \in \bar{\mathbf{p}}} x(\mathbf{p}, q) \quad (23)$$

$$x_{\max} \geq \max_{\mathbf{p} \in \bar{\mathbf{p}}} x(\mathbf{p}, q). \quad (24)$$

Remark 1. The technique can easily be extended to systems with an arbitrary number of fault prone parameters $\mathbf{p}$, an arbitrary number of design variables $\mathbf{q}$, and an arbitrary number of performance measurements $\mathbf{x}$.

5. EXAMPLES

Example 1. (A single failure). Consider an *unknown* resistive circuit. A Simulink model (see Fig. 4) of the circuit was created to conduct experiments. No knowledge about the circuit is assumed.

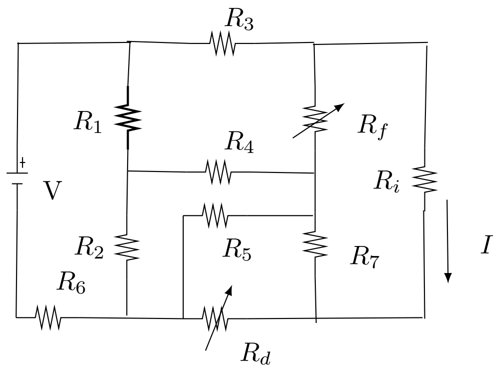

Fig. 4. Example resistive circuit

Suppose that resistor R_f is most vulnerable to faults. The goal is to design R_d so that the current through R_i denoted as I should be in the range $[3.2A, 6.8A]$. So we can write,

$$[I_{\min}, I_{\max}] = [3.2A, 6.8A].$$

We find the functional dependency of current I on the resistors R_d and R_f is given by,

$$I(R_d, R_f) = \frac{\alpha_0 + \alpha_1 R_d + \alpha_2 R_f + \alpha_3 R_d R_f}{\beta_0 + \beta_1 R_d + \beta_2 R_f + R_d R_f}. \quad (25)$$

To determine α_0, α_1, α_2, α_3, β_0, β_1 and β_2, seven experiments should be conducted by setting 7 different values for the resistors R_d, and R_f, and measuring the corresponding current values I. The experiments are conducted on the Simulink model of the circuit. Note that in the design process, it is assumed that the model is not known. By setting 7 different values for the resistors R_d, and R_f the corresponding currents I are measured. The measurements made are given in Table 1.

Table 1. Numerical values of $I(R_d, R_f)$ measurements for experiments with R_d, R_f.

Exp.No	$R_d(\Omega)$	$R_f(\Omega)$	$I(R_d, R_f)(A)$
1	1	1	4.52
2	7	2	3.07
3	12	7	4.09
4	20	13	4.41
5	35	24	4.65
6	67	43	4.79
7	90	58	4.85

Then solve

$$\begin{bmatrix} 1 & R_{d1} & R_{f1} & R_{d1}R_{f1} & -I_1 & -I_1R_{d1} & -I_1R_{f1} \\ 1 & R_{d2} & R_{f2} & R_{d2}R_{f2} & -I_2 & -I_2R_{d2} & -I_2R_{f2} \\ 1 & R_{d3} & R_{f3} & R_{d3}R_{f3} & -I_3 & -I_3R_{d3} & -I_3R_{f3} \\ 1 & R_{d4} & R_{f4} & R_{d4}R_{f4} & -I_4 & -I_4R_{d4} & -I_4R_{f4} \\ 1 & R_{d5} & R_{f5} & R_{d5}R_{f5} & -I_5 & -I_5R_{d5} & -I_5R_{f5} \\ 1 & R_{d6} & R_{f6} & R_{d6}R_{f6} & -I_6 & -I_6R_{d6} & -I_6R_{f6} \\ 1 & R_{d7} & R_{f7} & R_{d7}R_{f7} & -I_7 & -I_7R_{d7} & -I_7R_{f7} \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix} = \begin{bmatrix} I_1R_{d1}R_{f1} \\ I_2R_{d2}R_{f2} \\ I_3R_{d3}R_{f3} \\ I_4R_{d4}R_{f4} \\ I_5R_{d5}R_{f5} \\ I_6R_{d6}R_{f6} \\ I_7R_{d7}R_{f7} \end{bmatrix}. \quad (26)$$

Using the measurements from the experiments given in Table 1, and (26) the functional dependency of current I on the resistors R_d, and R_f is found to be

$$I(R_d, R_f) = \frac{42.6594 - 0.1964R_d + 30.5111R_f + 5.0467R_dR_f}{8.3847 + 3.6402R_d + 4.2363R_f + R_dR_f}. \quad (27)$$

Using (27), $I(R_d, R_f)|_{R_f=0}$ and $I(R_d, R_f)|_{R_f=\infty}$ for the range of resistance $R_d \in [0, 25]$ are calculated and are shown in Fig. 5.

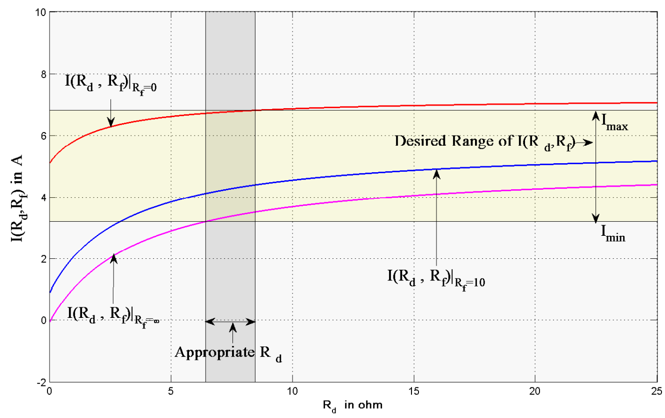

Fig. 5. I vs. R_d, for fixed R_f.

Then the appropriate range of R_d for fault tolerance has to be selected so that the conditions in (28) and (29) are satisfied,

$$I(R_{d,\min}, R_f)|_{R_f\in[0,\infty]} \in [3.2A, 6.8A], \quad (28)$$
$$I(R_{d,\max}, R_f)|_{R_f\in[0,\infty]} \in [3.2A, 6.8A]. \quad (29)$$

From Fig. 5. it is found that $R_{d,\min} = 6.43\Omega$ and $R_{d,\max} = 8.45\Omega$ is the appropriate range of R_d which satisfies (28) and (29). When R_d is set to any value with in the designed range the current I is maintained within the desired range for any fault in R_f.

Example 2. (Two Failures). Consider an unknown resistive circuit. A Simulink model is created to conduct experiments (Fig. 6). Resistors R_{f1} and R_{f2} are most vulnerable to faults. We want to design R_d so that the current through R_i denoted as I should be in the range $[0.5A, 4A]$.

$$[I_{\min}, I_{\max}] = [0.5A, 4A].$$

First, we find the functional dependency of current I on the resistors R_d, R_{f1}, and R_{f2}. The relation is given by,

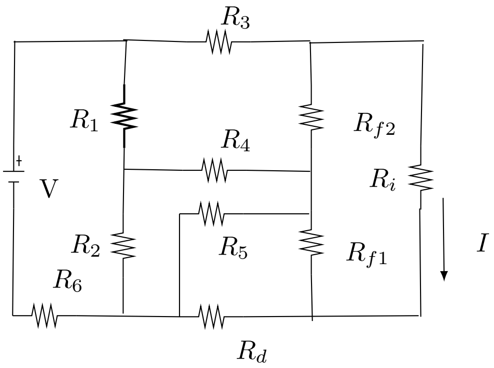

Fig. 6. Example resistive circuit

$$I(R_d, R_{f1}, R_{f2}) = \frac{\alpha(R_d, R_{f1}, R_{f2})}{\beta(R_d, R_{f1}, R_{f2})} \quad (30)$$

where

$$\alpha(R_d, R_{f1}, R_{f2}) = \alpha_0 + \alpha_1 R_d + \alpha_2 R_{f1} + \alpha_3 R_{f2} + \alpha_4 R_d R_{f1} + \alpha_5 R_{f1} R_{f2} + \alpha_6 R_{f2} R_d + \alpha_7 R_d R_{f1} R_{f2}$$

$$\beta(R_d, R_{f1}, R_{f2}) = \beta_0 + \beta_1 R_d + \beta_2 R_{f1} + \beta_3 R_{f2} + \beta_4 R_d R_{f1} + \beta_5 R_{f1} R_{f2} + \beta_6 R_{f2} R_d + R_d R_{f1} R_{f2}.$$

To determine α_0, α_1, α_2, α_3, α_4, α_5, α_6, α_7, β_0, β_1,β_2, β_3, β_4, β_5, and β_6, fifteen experiments should be conducted by setting 15 different sets of values for the resistors R_d,R_{f1}, and R_{f2}, and measuring the corresponding current I. The experiments are conducted on the Simulink model of the circuit. As before, in the design process, it is assumed that the model is not known. By setting 15 different values for the resistors R_d, R_{f1}, and R_{f2} corresponding current I is measured. The measurements made are given in Table 2.

Table 2. Numerical values of $I(R_d, R_{f1}, R_{f2})$ measurements for experiments with R_d, R_{f1}, R_{f2}.

Exp.No	$R_d(\Omega)$	$R_{f1}(\Omega)$	$R_{f2}(\Omega)$	$I(R_d, R_{f1}, R_{f2})(A)$
1	1	1	1	3.53
2	2	3	4	3.62
3	7	7	7	4.35
4	10	9	12	3.99
5	13	12	17	3.68
6	18	16	20	3.54
7	21	20	25	3.24
8	24	28	31	2.85
9	29	35	35	2.65
10	34	42	43	2.38
11	39	51	56	2.06
12	43	67	67	1.77
13	51	75	70	1.71
14	58	83	79	1.59
15	75	90	85	1.53

Using the measurements from the experiments given in Table 2, and (30) the functional dependency of current I on the resistors R_d,R_{f1}, and R_{f2} is found to be,

$$\begin{aligned}\alpha(R_d, R_{f1}, R_{f2}) =& 7.5 - 2.1R_d - 0.9R_{f1} + 1.28R_{f2} \\ &+ 0.03R_d R_{f1} + 0.08R_{f1} R_{f2} \\ &+ 0.11R_{f2} R_d - 0.0002R_d R_{f1} R_{f2} \\ \beta(R_d, R_{f1}, R_{f2}) =& 2 - 0.5R_d - 0.4R_{f1} + 0.5R_{f2} \\ &+ 0.01R_d R_{f1} + 0.006R_{f1} R_{f2} \\ &+ 0.02R_{f2} R_d + 0.001R_d R_{f1} R_{f2} \quad (31)\end{aligned}$$

For different failure conditions listed in Table 3 and for $R_d \in [0, 25]$, current $I(R_d, R_{f1}, R_{f2})$ is calculated using the expression (31)). The results are shown in Fig. 7.

Table 3. Single or Two resistor Failure Conditions Considered in Design

Fault Condition	R_{f1}	R_{f2}
1	Short	Normal
2	Open	Normal
3	Normal	Short
4	Normal	Open
5	Short	Short
6	Short	Open
7	Open	Short
8	Open	Open
9	Normal	Normal

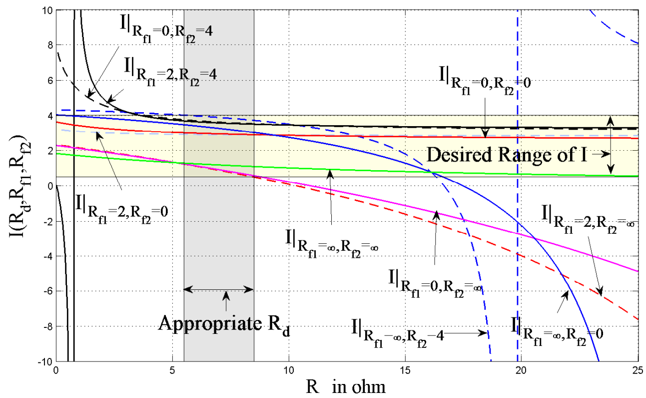

Fig. 7. I vs. R_d, for fixed R_{f1}, and R_{f2}.

Then the appropriate range of R_d for fault tolerance has to be selected so that the conditions in (32) and (33) are satisfied for all the fault conditions considered in Table 3.

$$I(R_{d,\min}, R_{f1}, R_{f2}) \in [0.5A, 4A], \quad (32)$$

$$I(R_{d,\max}, R_{f1}, R_{f2}) \in [0.5A, 4A] \quad (33)$$

From Fig. 7, it is found that $R_{d,\min} = 5.5\Omega$ and $R_{d,\max} = 8.5\Omega$ is the appropriate range of R_d. When R_d is set to any value within the above range, the current I is maintained within the desired range for any faults at R_{f1} and R_{f2}.

6. CONCLUSIONS

Here we have introduced a new approach to achieve fault tolerant system design without knowledge of its mathematical models. A measurement based approach to linear equations is used to obtain the relation between system variables and the design parameters. Then using this relation appropriate design parameter values are extracted, so that the system performance measure to be controlled lies within acceptable ranges even when faults occur. This is illustrated by examples of design of fault tolerant electrical circuits. Research is ongoing for extending this theory to fault tolerant system and controller design for general linear time invariant systems.

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