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Control of Uncertain Teleoperators with Time-Delays using Artificial Neural Networks

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Abstract: This work proposes a Neural Network controller for uncertain bilateral teleoperators with variable time-delays. The controller employs a Radial Basis Function (RBF) network and dynamically finds the an estimation of the neural network interconnection weights. Furthermore, assuming bounded delays, it is analytically shown that, if the human operator and the environment do not inject forces in the local and the remote manipulators, respectively, position error and velocities asymptotically converge to zero. Simulations with a couple of nonlinear manipulators depict the performance of the proposed controller.

Keywords: Bilateral Teleoperation, Neural Networks, Time-Delays.

1. INTRODUCTION

The practical scenarios of bilateral teleoperators span multiple areas that range from medical to aeronautical applications. Controlling these systems has attracted the attention of the research community since the days of N. Tesla, with his U.S. Patent *Method of and Apparatus for Controlling Mechanism of Moving Vessels or Vehicles*, to our date. The complexity of the teleoperator nonlinear dynamics and the time-delays in the communications are among the most important control challenges. A major breakthrough to the treatment of these problems has been the use of scattering signals (wave variables) to transform the pure time–delays of the communications into a passive transmission line (Anderson and Spong, 1989; Niemeyer and Slotine, 1991). Despite the fact that scattering-based schemes have paved the way of modern teleoperation control, they are prone to position drift (Chopra et al., 2006). In order to improve their performance, several approaches have been reported: transmitting *wave* integrals Niemeyer and Slotine (2004), *wave* filtering Tanner and Niemeyer (2005), *wave* prediction Munir and Book (2002), power scaling Secchi et al. (2007), amongst others. Refer to (Hokayem and Spong, 2006), for a historical survey, and to (Nuño et al., 2011), for a control tutorial.

On one hand, the use of simple PD controllers, without employing the scattering transformation, has been proposed in (Lee and Spong, 2006; Nuño et al., 2008) for constant time-delays. Later, Nuño et al. (2009) have shown that these proportional plus damping controllers are capable of providing position tracking for bilateral teleoperators with variable time-delays. On the other hand, Chopra et al. (2008) have proposed the use of adaptive schemes to overcome the effects of position drift in uncertain teleoperators. Along the same line Nuño et al. (2010) report a different adaptive scheme that is capable of synchronizing the local and remote positions despite constant time-delays. The main, simple but essential, difference between the controller in (Nuño et al., 2010) and the one in (Chopra et al., 2008) is the use of a linear combination of the velocity and the position error —instead of the position— in the, so-called, synchronizing signal. In the recent work (Hashemzadeh et al., 2012) an adaptive controller together with a high-gain sliding term is proposed for teleoperators with variable delays. Furthermore, based on the small gain theorem and assuming that the physical parameters are known, (Polushin et al., 2013) proposes a controller for the asymptotical stabilization of a cooperative teleoperation system with variable time-delays.

The scattering-based schemes and the simpler PD controllers need to exactly compensate the gravity effects and the adaptive controllers rely on the property that the teleoperator dynamics are linearly parameterizable with regards to its physical parameters and they require an explicit *a priori* knowledge of the dynamical model structure. Due to the universal approximation property of the Neural Networks (NN), part of the nonlinear teleoperator dynamics can be approximated using a model free controller (Hua et al., 2013). The use of NN for single robot control dates back to the 90's (Lewis et al., 1996) and, since such date, several works have proposed different NN controllers to solve regulation and tracking problems for single robots (Patiño et al., 2002; Yildirim, 2004; Lee and Choi, 2004; Li et al., 2013).

This work proposes a Radial Basis Function (RBF) NN controller that ensures position tracking in uncertain bilateral teleoperators with variable time-delays in the communications without any explicit knowledge of the teleoperator dynamics. Compared to (Hua et al., 2013), our work deals with variable delays and the stability proofs are properly established. Simulations are provided to verify the theoretical results in this paper.

Notation. Bold capital letters are used for matrices and bold lower case letters for vectors. $\mathbb{R} := (-\infty, \infty)$, $\mathbb{R}_{>0} := (0, \infty)$, $\mathbb{R}_{\geq 0} := [0, \infty)$. $|\mathbf{x}|$ stands for the standard Euclidean norm of vector $\mathbf{x}$. For any function $\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, the $\mathcal{L}_\infty$-norm is defined as $\|\mathbf{f}\|_\infty = \sup_{t\in[0,\infty)} |\mathbf{f}(t)|$, and square of the $\mathcal{L}_2$-norm as $\|\mathbf{f}\|_2^2 = \int_0^\infty |\mathbf{f}(t)|^2 dt$. The $\mathcal{L}_\infty$ and $\mathcal{L}_2$ spaces are defined as the sets $\{\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|\mathbf{f}\|_\infty < \infty\}$ and $\{\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|\mathbf{f}\|_2 < \infty\}$, respectively. For any matrix $\mathbf{A} := [a_{ij}] \in \mathbb{R}^{n\times n}$, its trace is defined as $\mathrm{tr}(\mathbf{A}) := \sum_{i=1}^n a_{ii}$ and it satisfies $\mathrm{tr}(\mathbf{A}^\top \mathbf{A}) := \sum_{i,j=1}^n a_{ij}^2$. When clear from the context, the argument of signals and operators is removed. The subscript i takes the values l and r for local and remote manipulators, respectively.

The following lemma, that is instrumental in the stability proofs, has been borrowed from (Nuño et al., 2009).

Lemma 1. (Nuño et al., 2009) For any vector signals $\mathbf{y}, \mathbf{z} \in \mathbb{R}^n$, any variable time-delay $0 \leq T(t) \leq {}^*T < \infty$ and any constant $\delta > 0$, the following inequality holds

$$-\int_0^t \mathbf{y}^\top(\sigma) \int_{\sigma - T(\sigma)}^{\sigma} \mathbf{z}(\theta) d\theta d\sigma \leq \frac{\delta}{2}\|\mathbf{y}\|_2^2 + \frac{{}^*T^2}{2\delta}\|\mathbf{z}\|_2^2.$$

$\diamond$

2. BILATERAL TELEOPERATOR MODEL

The nonlinear dynamic behavior of a n-DOF robot manipulator can be derived from the Euler-Lagrange equations of motion $\frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{q}}} - \frac{\partial L}{\partial \mathbf{q}} = \boldsymbol{\tau}$, where $L(\mathbf{q}, \dot{\mathbf{q}})$ is the system Lagrangian which relates the kinetic and the potential energy as $L(\mathbf{q}, \dot{\mathbf{q}}) = \frac{1}{2}\dot{\mathbf{q}}^\top \mathbf{M}(\mathbf{q})\dot{\mathbf{q}} - U(\mathbf{q})$, where $U(\mathbf{q})$ is the potential energy, $\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{n\times n}$ is the inertia matrix and $\dot{\mathbf{q}}, \mathbf{q} \in \mathbb{R}^n$ are the joint velocities and positions, respectively. In compact form, these equations can be written as

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \mathbf{d}(t) = \boldsymbol{\tau} \quad (1)$$

where $\ddot{\mathbf{q}} \in \mathbb{R}^n$ is the joint acceleration vector, $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \in \mathbb{R}^{n\times n}$ is the Coriolis and centrifugal effects matrix defined as: $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) := \dot{\mathbf{M}}(\mathbf{q})\dot{\mathbf{q}} - \frac{1}{2}\frac{\partial}{\partial \mathbf{q}}\dot{\mathbf{q}}^\top \mathbf{M}(\mathbf{q})\dot{\mathbf{q}}$; $\mathbf{g}(\mathbf{q}) := \frac{\partial U(\mathbf{q})}{\partial \mathbf{q}} \in \mathbb{R}^n$ is the gravitational force vector, $\mathbf{d}(t)$ is an unknown external perturbation and $\boldsymbol{\tau} \in \mathbb{R}^n$ is a generalized controller force vector.

Throughout the paper, the following standard assumptions are adopted:

A1. $\mathbf{M}(\mathbf{q})$ is symmetric positive definite and bounded for all $\mathbf{q}$.

A2. The external perturbation $\mathbf{d}(t)$ is bounded. Hence, there exists $d \in \mathbb{R}_{>0}$ such that $|\mathbf{d}(t)| \leq d$.

The dynamical system (1) possesses some important and well–known properties (Kelly et al., 2005; Spong et al., 2005):

P1. $\forall \mathbf{x} \neq \mathbf{0} \in \mathbb{R}^n :\ \mathbf{x}^\top [\dot{\mathbf{M}}(\mathbf{q}) - 2\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})]\mathbf{x} = 0.$

P2. $\exists k_c \in \mathbb{R}_{>0} :\ |\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}| \leq k_c|\dot{\mathbf{q}}|^2.$

A bilateral teleoperator can be modeled as a pair of n-DOF manipulators with serial links of the form (1). Its nonlinear dynamics, together with the human operator and environment interactions, are given by

$$\mathbf{M}_l(\mathbf{q}_l)\ddot{\mathbf{q}}_l + \mathbf{C}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l)\dot{\mathbf{q}}_l + \mathbf{g}_l(\mathbf{q}_l) + \mathbf{d}_l(t) = \boldsymbol{\tau}_l - \boldsymbol{\tau}_h \quad (2)$$
$$\mathbf{M}_r(\mathbf{q}_r)\ddot{\mathbf{q}}_r + \mathbf{C}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r)\dot{\mathbf{q}}_r + \mathbf{g}_r(\mathbf{q}_r) + \mathbf{d}_r(t) = \boldsymbol{\tau}_r - \boldsymbol{\tau}_e,$$

where $\boldsymbol{\tau}_l, \boldsymbol{\tau}_r \in \mathbb{R}^n$ are the local and remote control signals and $\boldsymbol{\tau}_h, \boldsymbol{\tau}_e \in \mathbb{R}^n$ are the joint torques produced by the forces exerted by the human and the environment, respectively. It is assumed that the manipulators contain fully actuated revolute joints and that friction can be neglected.

3. PROPOSED CONTROLLER

The objective in this paper is to design a robust controller such that the local and remote position errors

$$\mathbf{e}_l := \mathbf{q}_l - \mathbf{q}_r(t - T_r(t)) \qquad \mathbf{e}_r := \mathbf{q}_r - \mathbf{q}_l(t - T_l(t)), \quad (3)$$

asymptotically converge to zero despite the variable time delays $T_i(t)$ and without any explicit knowledge of the teleoperator dynamical parameters.

It is assumed that the interconnecting delays $T_i(t)$ satisfy the following:

A3. The variable time-delay $T_i(t)$ has a known upper bound ${}^*T_i$, *i.e.*, $0 \leq T_i(t) \leq {}^*T_i < \infty$, and its first and second time-derivatives are bounded.

In order to simplify the presentation, let us rewrite the teleoperator dynamics (2) as

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \mathbf{g}_i(\mathbf{q}_i) + \mathbf{d}_i(t) = \boldsymbol{\tau}_i - \boldsymbol{\tau}_{h/e}, \quad (4)$$

where the subindex $i \in \{l, r\}$.

The universal approximation property ensures that, for a network composed of N artificial neurons and for any continuous function $\mathbf{a}_i \in \mathbb{R}^n$,

$$\mathbf{M}_i(\mathbf{q}_i)\dot{\mathbf{a}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\mathbf{a}_i + \mathbf{g}_i(\mathbf{q}_i) + \mathbf{d}_i(t) = \mathbf{W}_i^* \boldsymbol{\phi}_i(\mathbf{x}_i) + \boldsymbol{\varepsilon}_i(\mathbf{x}_i) \quad (5)$$

where $\mathbf{W}_i^* \in \mathbb{R}^{n\times N}$ is the ideal constant network weight matrix, $\boldsymbol{\phi}_i \in \mathbb{R}^N$ is the RBF vector, $\boldsymbol{\varepsilon}_i \in \mathbb{R}^n$ is a bounded approximation error which satisfies $|\boldsymbol{\varepsilon}_i| \leq \bar{\varepsilon}_i$, for $\bar{\varepsilon}_i > 0$, and $\mathbf{x}_i := \mathrm{col}(\mathbf{q}_i^\top, \dot{\mathbf{q}}_i^\top, \mathbf{a}_i^\top, \dot{\mathbf{a}}_i^\top) \in \mathbb{R}^{4n}$ (Patiño et al., 2002; Lee and Choi, 2004).

Each jth element of the RBF vector $\boldsymbol{\phi}_i$ is defined using the following Gaussian distribution

$$\phi_{ij}(\mathbf{x}_i) := e^{-\frac{|\mathbf{x}_i - \boldsymbol{\mu}_{ij}|^2}{2\sigma_{ij}}}$$

where $\boldsymbol{\mu}_{ij} \in \mathbb{R}^{4n}$ is the mean, $\sigma_{ij} \in \mathbb{R}$ is the variance and $j \in [1, \ldots, N]$. It should be noted that these RBF networks can be replaced by any linearly parameterized network, e.g., polynomial, splines and wavelets, without changing the main results in this work (Lewis et al., 1996).

Using (5), the dynamics of (1) can be expressed as

$$\mathbf{M}_i(\mathbf{q}_i)(\ddot{\mathbf{q}}_i + \dot{\mathbf{a}}_i) + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)(\dot{\mathbf{q}}_i + \mathbf{a}_i) = \mathbf{W}_i^* \boldsymbol{\phi}_i(\mathbf{x}_i) + \boldsymbol{\varepsilon}_i(\mathbf{x}_i) + \boldsymbol{\tau}_i - \boldsymbol{\tau}_{h/e},$$

which, defining $\boldsymbol{\epsilon}_i := \dot{\mathbf{q}}_i + \mathbf{a}_i$, can be further written as

$$\mathbf{M}_i(\mathbf{q}_i)\dot{\boldsymbol{\epsilon}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\boldsymbol{\epsilon}_i = \mathbf{W}_i^* \boldsymbol{\phi}_i(\mathbf{x}_i) + \boldsymbol{\varepsilon}_i(\mathbf{x}_i) + \boldsymbol{\tau}_i - \boldsymbol{\tau}_{h/e}.$$

The proposed robust controller is given by

$$\boldsymbol{\tau}_i = -\hat{\mathbf{W}}_i\boldsymbol{\phi}_i - \mathbf{K}_i\boldsymbol{\epsilon}_i - \frac{\alpha_i}{|\boldsymbol{\epsilon}_i| + b_i e^{-c_i t}}\boldsymbol{\epsilon}_i, \tag{6}$$

where $\hat{\mathbf{W}}_i \in \mathbb{R}^{n\times N}$ is the estimation of the network interconnection weights, $b_i, c_i > 0$ and $\mathbf{K}_i = \mathbf{K}_i^\top > 0 \in \mathbb{R}^{n\times n}$.

The closed-loop system (1) and (6) is

$$\mathbf{M}_i(\mathbf{q}_i)\dot{\boldsymbol{\epsilon}}_i + \mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\boldsymbol{\epsilon}_i + \mathbf{K}_i\boldsymbol{\epsilon}_i + \frac{\alpha_i}{|\boldsymbol{\epsilon}_i| + b_i e^{-c_i t}}\boldsymbol{\epsilon}_i = \tilde{\mathbf{W}}_i\boldsymbol{\phi}_i + \boldsymbol{\varepsilon}_i - \boldsymbol{\tau}_{h/e}. \tag{7}$$

where $\tilde{\mathbf{W}}_i = \mathbf{W}_i^* - \hat{\mathbf{W}}_i$ is the network weight estimation error, which satisfies $\dot{\tilde{\mathbf{W}}}_i = -\dot{\hat{\mathbf{W}}}_i$.

At this point we are ready to state our main result.

Proposition 1. Consider the bilateral teleoperator (2), satisfying Assumptions **A1**–**A3**, controlled by (6) with the following estimation laws

$$\begin{aligned}\dot{\hat{\mathbf{W}}}_i &= \boldsymbol{\Gamma}_i\boldsymbol{\epsilon}_i\boldsymbol{\phi}_i^\top,\\ \dot{\alpha}_i &= \frac{|\boldsymbol{\epsilon}_i|^2}{|\boldsymbol{\epsilon}_i| + b_i e^{-c_i t}},\end{aligned} \tag{8}$$

where $\boldsymbol{\Gamma}_i = \boldsymbol{\Gamma}_i^\top > 0 \in \mathbb{R}^{n\times n}$.

If the human operator and the remote environment forces are zero and defining $\mathbf{a}_i := \lambda\mathbf{e}_i$, with $\lambda \in \mathbb{R}_{>0}$ set such that

$$1 > \lambda({}^*T_l + {}^*T_r) \tag{9}$$

holds, then the local and remote position errors (3) asymptotically converge to zero. Furthermore, under these conditions, local and remote velocities also asymptotically converge to zero. ⋄

Proof. Consider the following function

$$\mathcal{V}_i = \frac{1}{2}\left[\boldsymbol{\epsilon}_i^\top\mathbf{M}_i(\mathbf{q}_i)\boldsymbol{\epsilon}_i + \mathrm{tr}(\tilde{\mathbf{W}}_i^\top\boldsymbol{\Gamma}_i^{-1}\tilde{\mathbf{W}}_i) + (\alpha_i - \bar{\varepsilon}_i)^2\right].$$

Evaluating $\mathcal{V}_i$ along the closed-loop system (1) and (6), using Property **P1**, yields

$$\begin{aligned}\dot{\mathcal{V}}_i = &-\boldsymbol{\epsilon}_i^\top\mathbf{K}_i\boldsymbol{\epsilon}_i - \frac{|\boldsymbol{\epsilon}_i|^2}{|\boldsymbol{\epsilon}_i| + b_i e^{-c_i t}}\alpha_i + \boldsymbol{\epsilon}_i^\top\tilde{\mathbf{W}}_i\boldsymbol{\phi}_i +\\ &+ \boldsymbol{\epsilon}_i^\top\boldsymbol{\varepsilon}_i - \mathrm{tr}(\tilde{\mathbf{W}}_i^\top\boldsymbol{\Gamma}_i^{-1}\dot{\hat{\mathbf{W}}}_i) + (\alpha_i - \bar{\varepsilon}_i)\dot{\alpha}_i.\end{aligned}$$

Using the fact that $\boldsymbol{\epsilon}_i^\top\tilde{\mathbf{W}}_i\boldsymbol{\phi}_i = \mathrm{tr}(\tilde{\mathbf{W}}_i^\top\boldsymbol{\epsilon}_i\boldsymbol{\phi}_i^\top)$ and the estimation laws (8), it holds that

$$\dot{\mathcal{V}}_i = -\boldsymbol{\epsilon}_i^\top\mathbf{K}_i\boldsymbol{\epsilon}_i + \boldsymbol{\epsilon}_i^\top\boldsymbol{\varepsilon}_i - \frac{|\boldsymbol{\epsilon}_i|^2}{|\boldsymbol{\epsilon}_i| + b_i e^{-c_i t}}\bar{\varepsilon}_i.$$

Now, since $\boldsymbol{\epsilon}_i^\top\boldsymbol{\varepsilon}_i \le |\boldsymbol{\epsilon}_i||\boldsymbol{\varepsilon}_i| \le |\boldsymbol{\epsilon}_i|\bar{\varepsilon}_i$ and

$$|\boldsymbol{\epsilon}_i| - \frac{|\boldsymbol{\epsilon}_i|^2}{|\boldsymbol{\epsilon}_i| + b_i e^{-c_i t}} = \frac{|\boldsymbol{\epsilon}_i|}{|\boldsymbol{\epsilon}_i| + b_i e^{-c_i t}}b_i e^{-c_i t} \le b_i e^{-c_i t},$$

it can be further shown that

$$\dot{\mathcal{V}}_i \le -k_i|\boldsymbol{\epsilon}_i|^2 + \bar{\varepsilon}_i b_i e^{-c_i t},$$

where k_i has been defined as the smallest eigenvalue of $\mathbf{K}_i$.

Since $\mathbf{a}_i := \lambda\mathbf{e}_i$, then

$$\boldsymbol{\epsilon}_i = \dot{\mathbf{q}}_i + \lambda\mathbf{e}_i. \tag{10}$$

Defining the function

$$\mathcal{V} = \frac{1}{k_l}\mathcal{V}_l + \frac{1}{k_r}\mathcal{V}_r + \lambda|\mathbf{q}_l - \mathbf{q}_r|^2,$$

yields

$$\dot{\mathcal{V}} \le \beta_l e^{-c_l t} + \beta_r e^{-c_r t} - |\boldsymbol{\epsilon}_l|^2 - |\boldsymbol{\epsilon}_r|^2 + 2\lambda(\mathbf{q}_l - \mathbf{q}_r)^\top(\dot{\mathbf{q}}_l - \dot{\mathbf{q}}_r),$$

where $\beta_i := \frac{\bar{\varepsilon}_i b_i}{k_i}$.

At this point, it is useful to note that

$$\mathbf{q}_i - \mathbf{q}_i(t - T_i(t)) = \int_{t-T_i(t)}^{t}\dot{\mathbf{q}}_i(\theta)d\theta. \tag{11}$$

Substituting (10) and using (11) on $\dot{\mathcal{V}}$ and doing some algebraic manipulations yields

$$\begin{aligned}\dot{\mathcal{V}} \le &\beta_l e^{-c_l t} + \beta_r e^{-c_r t} - \lambda^2(|\mathbf{e}_l|^2 + |\mathbf{e}_r|^2) - |\dot{\mathbf{q}}_l|^2 - |\dot{\mathbf{q}}_r|^2 -\\ &- 2\lambda\dot{\mathbf{q}}_l^\top\int_{t-T_r(t)}^{t}\dot{\mathbf{q}}_r(\theta)d\theta - 2\lambda\dot{\mathbf{q}}_r^\top\int_{t-T_l(t)}^{t}\dot{\mathbf{q}}_l(\theta)d\theta.\end{aligned}$$

Integrating $\dot{\mathcal{V}}$, from 0 to t and using the fact that $\beta_i\int_0^t e^{-c_i\eta}d\eta \le \frac{\beta_i}{c_i}$, yields

$$\begin{aligned}\mathcal{V}(t) - \mathcal{V}(0) \le &\frac{\beta_l}{c_l} + \frac{\beta_r}{c_r} - \lambda^2(\|\mathbf{e}_l\|_2^2 + \|\mathbf{e}_r\|_2^2) - \|\dot{\mathbf{q}}_l\|_2^2 -\\ &- \|\dot{\mathbf{q}}_r\|_2^2 - 2\lambda\int_0^t\dot{\mathbf{q}}_l^\top(\eta)\int_{\eta-T_r(\eta)}^{\eta}\dot{\mathbf{q}}_r(\theta)d\theta d\eta -\\ &- 2\lambda\int_0^t\dot{\mathbf{q}}_r^\top(\eta)\int_{\eta-T_l(\eta)}^{\eta}\dot{\mathbf{q}}_l(\theta)d\theta d\eta.\end{aligned}$$

Invoking Lemma 1 (see Appendix A) to the last double integral terms, with $\delta_l > 0$ and $\delta_r > 0$, respectively, yields

$$\mathcal{V}(t) - \mathcal{V}(0) \le \frac{\beta_l}{c_l} + \frac{\beta_r}{c_r} - \lambda^2(\|\mathbf{e}_l\|_2^2 + \|\mathbf{e}_r\|_2^2) - \psi_l\|\dot{\mathbf{q}}_l\|_2^2 - \psi_r\|\dot{\mathbf{q}}_r\|_2^2$$

where

$$\psi_l := 1 - \lambda\delta_l - \frac{\lambda^*T_l^2}{\delta_r}, \quad \psi_r := 1 - \lambda\delta_r - \frac{\lambda^*T_r^2}{\delta_l}.$$

Solving simultaneously for $\psi_i > 0$ and for $\delta_i > 0$, it is straightforward to show that there always exist a possible solution if λ is set fulfilling $1 > \lambda({}^*T_l + {}^*T_r)$.

Hence, setting λ such that (9) holds ensures that there exists $\psi_i > 0$ and thus

$$\mathcal{V}(t) + \lambda^2(\|\mathbf{e}_l\|_2^2 + \|\mathbf{e}_r\|_2^2) + \psi_l\|\dot{\mathbf{q}}_l\|_2^2 + \psi_r\|\dot{\mathbf{q}}_r\|_2^2 \le \mathcal{V}(0) + \frac{\beta_l}{c_l} + \frac{\beta_r}{c_r}.$$

Since $\mathcal{V}(t) \ge 0$ for all $t \ge 0$, $\mathbf{e}_i, \dot{\mathbf{q}}_i \in \mathcal{L}_2$ and $\mathcal{V} \in \mathcal{L}_\infty$. This last and the fact that $\mathcal{V}$ is positive definite and radially unbounded with respect to $\boldsymbol{\epsilon}_i$, $(\alpha_i - \bar{\varepsilon}_i)$ and $|\mathbf{q}_l - \mathbf{q}_r|$, shows that $\boldsymbol{\epsilon}_i, \alpha_i$ and $|\mathbf{q}_l - \mathbf{q}_r|$ are bounded. Additionally, $\tilde{\mathbf{W}}_i$ is a bounded operator and, from (10), $\boldsymbol{\epsilon}_i \in \mathcal{L}_2$.

Assumption **A3**, $|\mathbf{q}_l - \mathbf{q}_r| \in \mathcal{L}_\infty$ and $\dot{\mathbf{q}}_i \in \mathcal{L}_2$ imply that $\mathbf{e}_i \in \mathcal{L}_\infty$. This last, from (10), and $\boldsymbol{\epsilon}_i \in \mathcal{L}_\infty$ in turn imply that $\dot{\mathbf{q}}_i \in \mathcal{L}_\infty$. Furthermore, Assumption **A3** and $\dot{\mathbf{q}}_i \in \mathcal{L}_\infty$ support the fact that $\dot{\mathbf{e}}_i \in \mathcal{L}_\infty$. Invoking Barbălat's Lemma, with $\mathbf{e}_i \in \mathcal{L}_\infty \cap \mathcal{L}_2$ and $\dot{\mathbf{e}}_i \in \mathcal{L}_\infty$, it is proved that $\lim_{t\to\infty}|\mathbf{e}_i(t)| = 0$.

Now, since the RBF vector $\boldsymbol{\phi}_i$ is, by construction, bounded then all the previous bounded signals ensure from the closed-loop system (7) that $\dot{\boldsymbol{\epsilon}}_i \in \mathcal{L}_\infty$. This last and $\boldsymbol{\epsilon}_i \in \mathcal{L}_\infty \cap \mathcal{L}_2$ show, applying Barbălat's Lemma, that $\lim_{t\to\infty}|\boldsymbol{\epsilon}_i(t)| = 0$.

Finally, from (10) it holds that $\dot{\mathbf{q}}_i = -\lambda\mathbf{e}_i + \boldsymbol{\epsilon}_i$. Hence, $\lim_{t\to\infty}|\dot{\mathbf{q}}_i(t)| = 0$. This concludes the proof. □

4. SIMULATION RESULTS

To show the effectiveness of the proposed scheme, some simulations, in which the local and remote manipulators are modeled as a pair of 2 DOF serial links with revolute joints, are presented. Their corresponding nonlinear dynamics are modeled by (2). In what follows $\alpha_i := l_{2_i}^2 m_{2_i} + l_{1_i}^2(m_{1_i}+m_{2_i})$, $\beta_i := l_{1_i} l_{2_i} m_{2_i}$ and $\delta_i := l_{2_i}^2 m_{2_i}$. The inertia matrices $\mathbf{M}_i(\mathbf{q}_i)$ are given by

$$\mathbf{M}_i(\mathbf{q}_i) = \begin{bmatrix} \alpha_i + 2\beta_i \mathrm{c}_{2_i} & \delta_i + \beta_i \mathrm{c}_{2_i} \\ \delta_i + \beta_i \mathrm{c}_{2_i} & \delta_i \end{bmatrix}.$$

c_{2_i} is the short notation for $\cos(q_{2_i})$. q_{k_i} is the articular position of link k of manipulator i, with $k \in \{1, 2\}$. The Coriolis and centrifugal effects are modeled by

$$\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) = \begin{bmatrix} -2\beta_i \mathrm{s}_{2_i} \dot{q}_{2_i} & -\beta_i \mathrm{s}_{2_i} \dot{q}_{2_i} \\ \beta_i \mathrm{s}_{2_i} \dot{q}_{1_i} & 0 \end{bmatrix}.$$

s_{2_i} is the short notation for $\sin(q_{2_i})$. $\dot{q}_{1_i}$ and $\dot{q}_{2_i}$ are the respective revolute velocities of the two links. The gravity forces $\mathbf{g}_i(\mathbf{q}_i)$ for each manipulator are represented by

$$\mathbf{g}_i(\mathbf{q}_i) = \begin{bmatrix} \frac{1}{l_{2_i}} g\delta_i \mathrm{c}_{12_i} + \frac{1}{l_{1_i}}(\alpha_i - \delta_i)\mathrm{c}_{1_i} \\ \frac{1}{l_{2_i}} g\delta_i \mathrm{c}_{12_i} \end{bmatrix}.$$

c_{12_i} stands for $\cos(q_{1_i} + q_{2_i})$. l_{k_i} and m_{k_i} are the respective lengths and masses of each link. For simplicity, the external disturbance $\hat{\mathbf{d}}_i$ is set to zero.

The physical parameters for the manipulators are: the length of links l_{1_i} and l_{2_i}, for both manipulators, is 0.38m; the masses of the links are $m_{1_l} = 3.9473$kg, $m_{2_l} = 0.6232$kg, $m_{1_r} = 3.2409$kg and $m_{2_r} = 0.3185$kg. The initial conditions are $\dot{\mathbf{q}}_i(0) = \mathbf{0}$ and $\mathbf{q}_l^\top(0) = [-0.7\pi; 0]$, $\mathbf{q}_r^\top(0) = [-0.3\pi; -0.15\pi]$.

For simplicity, both time-delays are equal and are given by $T_l(t) = T_r(t) = 0.4 + 0.2\sin(\omega_1 t) + 0.1\sin(\omega_2 t)$, with $\omega_1 = 10$rad/s and $\omega_2 = 25$rad/s. Clearly, ${}^*T_i = 0.7$. The time-delay and its derivatives are shown in Fig. 1.

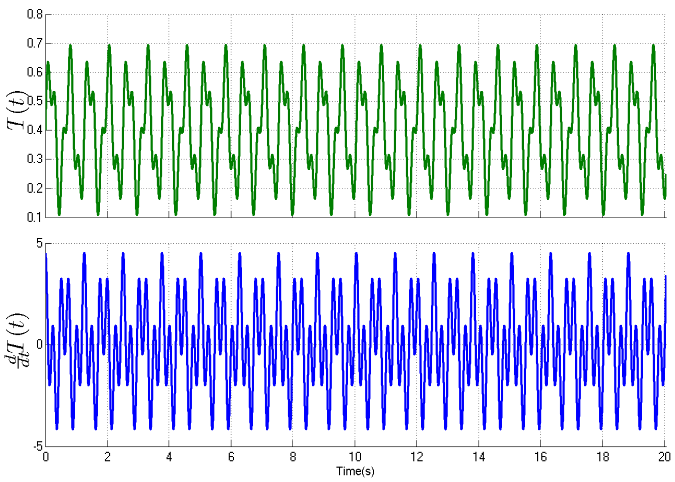

Fig. 1. Variable time-delay employed in the simulations.

The NN has two neurons and thus $\hat{\mathbf{W}}_i \in \mathbb{R}^{2\times 2}$ and $\boldsymbol{\varepsilon}_{ij} : \mathbb{R}^8 \mapsto \mathbb{R}^2$. The controller gains are set as: $\mathbf{K}_i = 30\mathbf{I}_2$; $\mathbf{\Gamma}_l = 0.1 \begin{bmatrix} 20 & 0 \\ 0 & 30 \end{bmatrix}$; $\mathbf{\Gamma}_r = 2 \begin{bmatrix} 20 & 0 \\ 0 & 30 \end{bmatrix}$; $b_i = 10$; $c_i = 0.05$, where $\mathbf{I}_2$ is the identity matrix of size two. $\mu_{ij} \in \mathbb{R}^8$ is given by $\mu_{i1} = [2\ 2\ 0\ 0\ 0\ 0\ 0\ 0]^\top$ and $\mu_{i2} = [1\ 1\ 0\ 0\ 0\ 0\ 0\ 0]^\top$, for $\sigma_{ij} \in \mathbb{R}$ the values are $\sigma_{i1} = 10$ and $\sigma_{i2} = 2$. Further, λ is set fulfilling (9) as $\lambda = 0.7$.

The human operator is modeled as the spring–damper system $\boldsymbol{\tau} = K_h(\mathbf{q}_h - \mathbf{q}_l) - K_d\dot{\mathbf{q}}_l$ where $K_h = 100$ and $K_d = 1$. Fig. 2 depicts the desired human position $\mathbf{q}_h$.

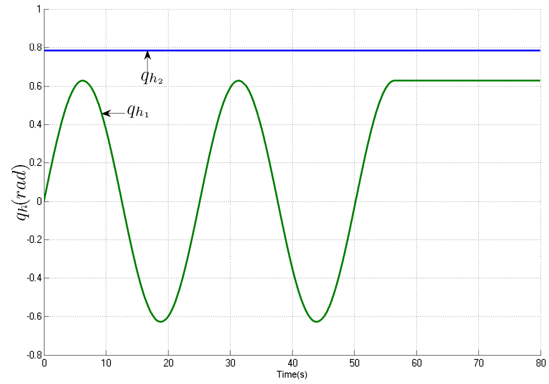

Fig. 2. Trajectory of the human operator.

Two different simulations have been performed, one in which the remote manipulator moves freely in its environment and another in which the remote manipulator interacts with a stiff wall.

4.1 Remote Manipulator in Free Space

Figs. 3–5 show the behavior of the angular local and remote joint positions and the evolution of the NN weights. From Fig. 3 it can be seen that, despite the difference in initial conditions and variable time-delays, position tracking is asymptotically achieved. Fig. 4, for the local controller, and Fig. 5, for the remote controller, depict the time evolution of the NN weights. It should be noted that, since the estimation gains for the two neurons are the same, the response of the two neurons is the same. This is why, instead of four and two signals in the weight matrix and the RBF vector, respectively, one can only see two and one signals.

4.2 Remote Manipulator Interacting with a Wall

In this set of simulations, a stiff wall is added in the remote environment. The wall is located in the $xz-$plane at $y = -0.1$m and it is modeled as a spring–damper Cartesian system with stiffness equal to 50000Nm and damping equal to 200Nm/s.

In this case, Fig. 6 and Fig. 7 show the position tracking capabilities of the proposed controller in the joint space and in the Cartesian y-axis, respectively. From these figures it is concluded that, despite variable time–delays and a stiff interaction with the environment, position error converges to zero and hence position tracking is established. Fig. 8 and Fig. 9 present the local and the remote time evolution of the estimated NN weights.

5. CONCLUSIONS

This paper reports a robust neural network controller, using radial basis function vectors, that is capable of providing position tracking in bilateral teleoperators with uncertain parameters and variable time-delays. These claims are analytically proven using the standard Barbălat's Lemma. Simulations using two 2-DOF manipulators support the theoretical results of this work.

Future research includes the extension of this controller to the control of multiple manipulators, following the idea reported in Nuño et al. (2013).

6. ACKNOWLEDGEMENTS

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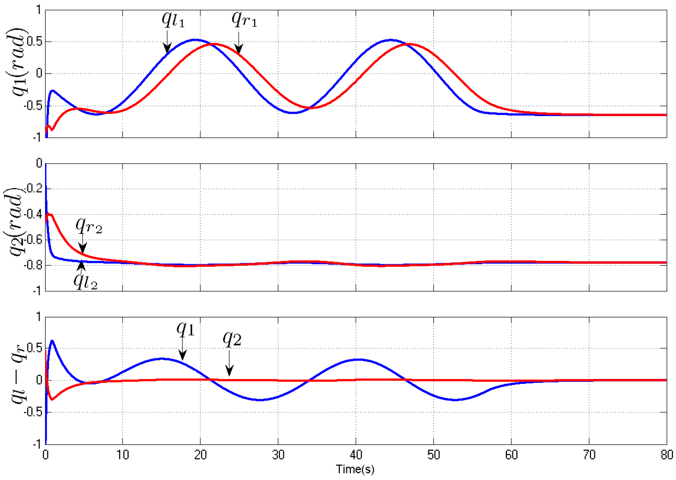

Fig. 3. Local and remote positions in free space.

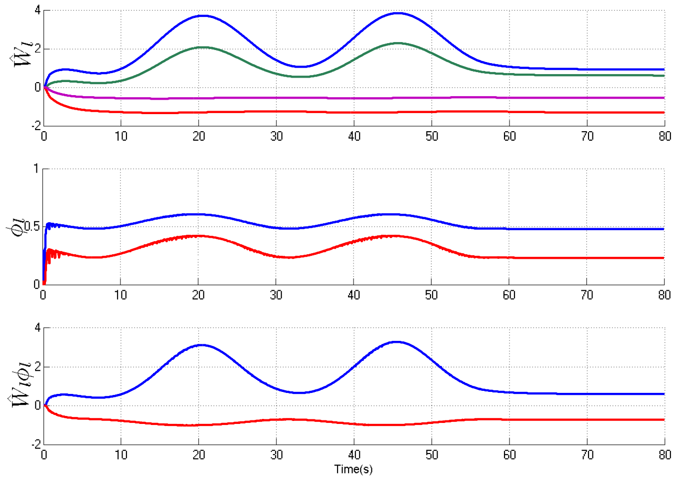

Fig. 4. Evolution of the local weight matrix and the local RBF vector in free space.

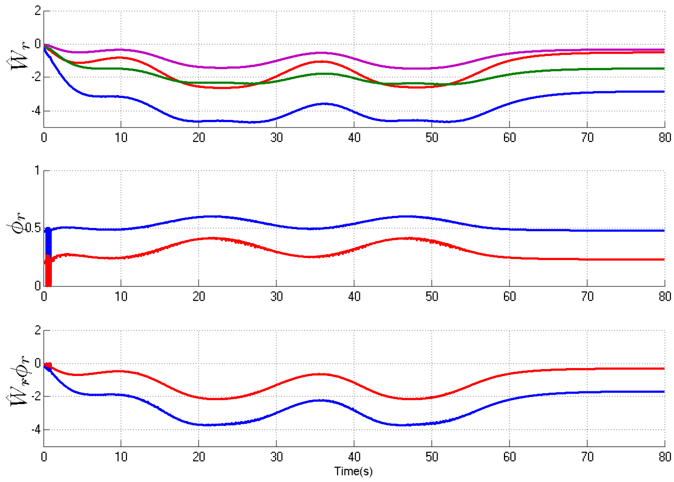

Fig. 5. Evolution of the remote weight matrix and the remote RBF vector in free space.

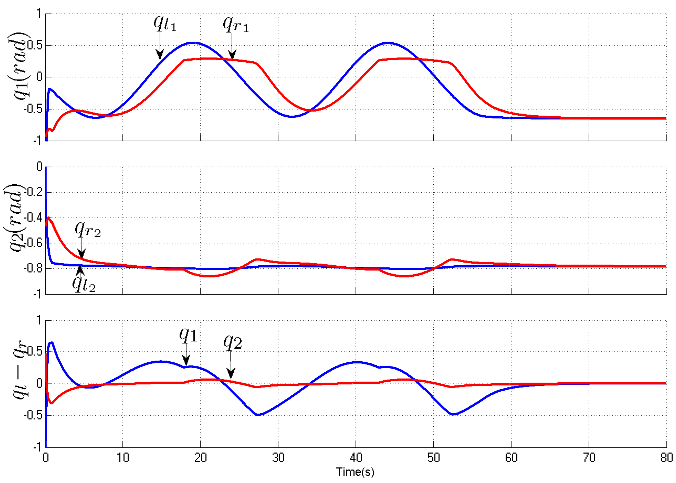

Fig. 6. Local and remote joint positions when the remote manipulator interacts with a wall.

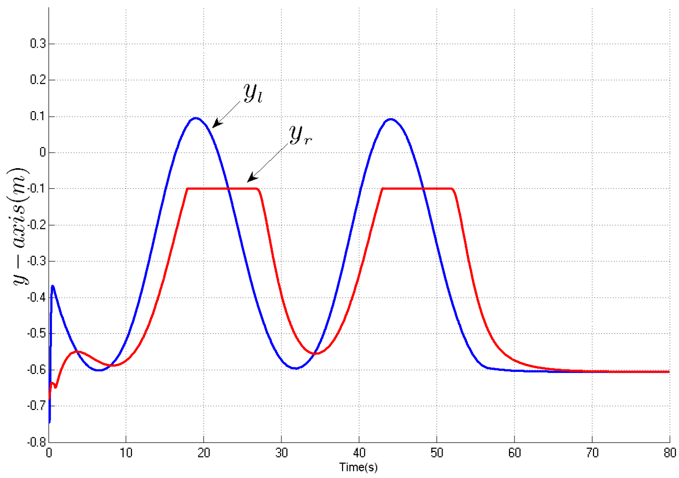

Fig. 7. y-axis Cartesian position of the local and the remote manipulators when interacting with a stiff wall.

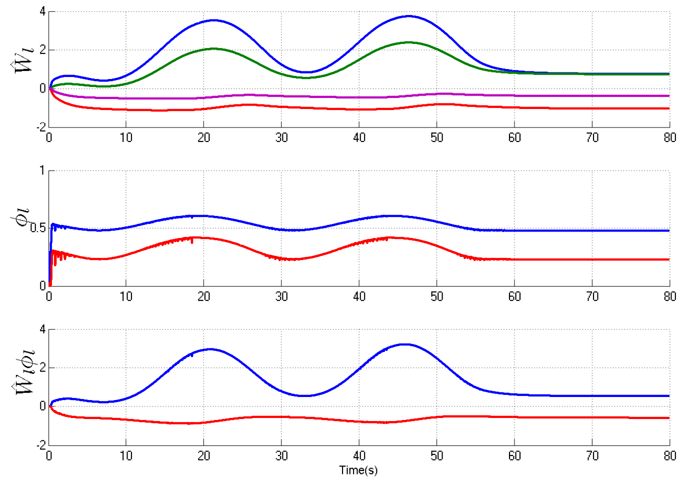

Fig. 8. Evolution of the local weight matrix and the local RBF vector when interacting with a stiff wall..

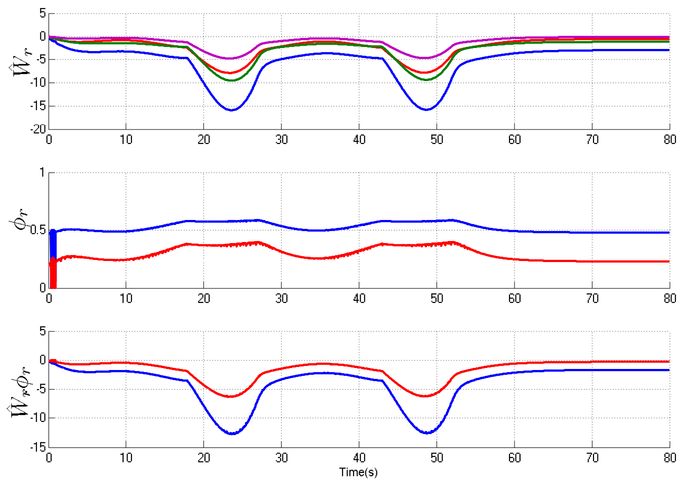

Fig. 9. Evolution of the remote weight matrix and the remote RBF vector when interacting with a stiff wall.