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Control-relevant input excitation for system identification of ill-conditioned $n \times n$ systems with $n > 2$

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Abstract: The objective of this work is to generalize input excitation designs suggested for system identification of ill-conditioned 2×2 systems to cases with more than two inputs and outputs. The methods are evaluated using a four inputs-four outputs distillation column/stripper system as a case study. The performance of the various procedures is evaluated through the model fit and different plant-friendly indices. The obtained models, and thus the quality of the associated input excitations, are also evaluated through cross validations.

Keywords: control-relevant system identification, ill-conditioned system, gain directionality, input excitation

1. INTRODUCTION

Typically, 75% of the cost associated with an advanced control project goes into model development (Gevers, 2005). Hence, efficient modelling and system identification techniques suited for industrial use and tailored for control design applications are crucial (Ljung, 1999; Hjalmarsson, 2005).

Unlike single-input single-output (SISO) systems, multi-input multi-output (MIMO) systems contain multiple interactions. In some cases the interactions are so strong that the system becomes difficult to handle. Such systems are ill-conditioned and they are characterized by a strong directionality. As a result, there may be difficulties in exciting the various "directions" adequately in system identification. It is desirable to make the identification (equally) informative for all relevant directions through explicit input excitations. It has been found that it is easier to obtain good information about the high-gain direction than the low-gain direction (Koung and MacGregor, 1993). Explicit excitation of the low-gain direction is required to obtain a model including the low-gain properties; otherwise, the model may be inadequate for control design (Koung and MacGregor, 1993; Häggblom and Böling, 1998).

The excitation signals can be grouped into three major categories: general-purpose signals, optimized test signals and advanced dedicated signals. It has been shown that the advanced dedicated signals are system specific and difficult to construct (Pintelon and Schoukens, 2012). Typically, three types of perturbations are used: step, PRBS (pseudo-random binary sequence) and sinusoidal inputs. In this contribution, combinations of PRBS and step signals are considered.

A considerable amount of literature exists on identification of ill-conditioned systems (e.g., Koung and MacGregor, 1993; Häggblom and Böling, 1998, 2013, Lee at al, 2003; Zhu and Stec, 2006; Rivera et al., 2009). However, most studies are limited to 2×2 systems. In this study, the suggested methods are generalized to systems with more inputs and outputs. The methods are tested on a 4×4 ill-conditioned system (Alatiqi and Luyben. 1986). The methods are compared in the following ways: fit of the identified models to data, condition number and singular values of the obtained models, plant-friendliness of the excitation signals, and an extensive cross validation. As plant friendliness factors we consider the crest factor (CF) and the performance index for perturbation signals (PIPS).

2. INPUT EXCITATION

For system identification, the system has to be perturbed beyond its normal operation. Yet, this input excitation should be plant friendly. This is essential in order to keep the variations of the inputs and the outputs within specified limits. However, this also limits the information available for system identification. Collecting data from industrial systems is complex and costly (Katayama et al., 2006). Thus, there is a trade-off between how much one is prepared to "pay" for the information and the information needed for system identification (Gevers, 2005). From this it follows that the input design is crucial (Häggblom and Böling, 1998).

2.1 Directional Inputs

Common practice is to use uncorrelated PRBS signals for input excitation, one at a time or simultaneously. However, this does not excite the system in all directions adequately, especially in the low-gain direction (Häggblom and Böling, 1998).

Consider a singular value decomposition of the steady state gain matrix, i.e., $G(0) = U\Sigma V^{\mathrm{T}} = \sum_{i=1}^{n} U_i \sigma_i V_i^{\mathrm{T}}$. If only the steady state of the system is considered, the input $u = u^i = V_i \sigma_i^{-1}$ will produce the output $y = y^i = U_i$ with the norm $\|y^i\| = 1$. To properly excite all directions i, $i = 1,\ldots,n$, it is necessary to apply inputs u^i that vary (symmetrically) between $u_-^i = -\sigma_i^{-1} V_i$ and $u_+^i = +\sigma_i^{-1} V_i$ (Häggblom and Böling, 2013). The excitations can be introduced one direction at a time, or all directions simultaneously if they are designed to be uncorrelated. This can achieved by any kind of input signal, e.g., sequence of steps, PRBS or multi-sinusoidal.

The duration of an experiment affects the overall cost of an experiment. The experiment length can be shortened by applying the input excitations simultaneously. Thus, it is of interest to study whether simultaneously applied directional inputs are sufficiently informative as compared to directional excitations one at a time or more standard input designs.

2.2 Plant-Friendly Inputs

Plant friendliness has been expressed in many ways in the control literature. One way is to express the plant friendliness of an input signal as $\Phi_i = 100\left(1 - n_t/(N-1)\right)\%$, where n_t is the total number of switches and N is the total input signal length (Parker et al., 2001). This means that a constant signal is 100 % "plant friendly", whereas a signal that changes at every instant is 0 % "plant friendly." For example, a single step change is a very plant friendly signal, whereas a Gaussian random signal is a least plant friendly signal.

However, from the system identification point of view, a single step change is least useful because it does not excite the system sufficiently. In this respect, a random signal is much better. Therefore, there is a trade-off between a good input excitation for identification and its plant friendliness.

2.3 Details on Identification Experiments

Eight different types of input excitations are studied and they are summarized in Table 1. The first six experiments are designed in a similar fashion as in Häggblom and Böling (2013) and the last two are similar to the design in Zhu and Stec (2006). The input excitations are step and PRBS signals.

The dominating time constant of the example process is on the order of 50 min. In the case of step excitations, this motivated a step length of 330 min for each step change.

The PRBS signals were designed using the bandwidth expression (Lee, 2006)

$$\omega_L = \frac{1}{\beta \tau_{high}} \le \omega \le \frac{\alpha}{\tau_{low}} = \omega_H \tag{1}$$

where τ_{high} and τ_{low} are high and low dominating time constants, respectively, τ_{low}/α is the closed-loop time constant of interest, and $\beta\tau_{high}$ is the settling time. The dominating time constants of the example process are estimated to be $\tau_{low} = 15$ min and $\tau_{high} = 50$ min, whereas α and β are chosen as 2 and 3, respectively. A minimum switching time $T_{sw} = 16$ min is used for the PRBS signals. Two PRBS lengths, 63 and 255, of the form $2^n - 1$, where n is an integer, are considered. When a PRBS of length 63 is used, it is applied separately for the 4 inputs in the example process, resulting in the total experiment length $T_{eL} \approx 4080$ min in both cases.

Table 1. Experiment designs

Experiment #	Input excitation	Signal description	Applied gain directionality	Average CF(u)	Average PIPS(u) (%)
1	stepSeq	Sequential step changes of inputs one at a time	No	2.487	40.22
2	stepDir	Step changes in gain directions	Yes	2.452	40.24
3	prbsSeq	Sequential PRBS perturbation of all inputs	No	2.012	49.70
4	prbsUnc	Simultaneous uncorrelated PRBS inputs	No	1.000	100.00
5	prbsSeqDir	PRBS according to gain directions, one at a time	Yes	1.984	49.73
6	prbsSimDir	Simultaneous PRBS excitation of all gain directions	Yes	1.153	97.31
7	prbsDirCorSeq	Sequential high-amplitude correlated PRBS and low-amplitude uncorrelated PRBS	Yes*	1.084	95.90
8	prbsDirCorSim	Simultaneous high-amplitude correlated PRBS and low-amplitude uncorrelated PRBS	Yes*	1.404	70.75

Two experiments are designed using step changes. In the first experiment, step signals are applied sequentially, one at a time in each input. In the second experiment, the step changes are applied simultaneously to all inputs in such a way that the various gain directions are excited sequentially. In the rest of the experiments, PRBS signals are used. In the third experiment, the PRBS signals are applied one at a time to each input, and in fourth experiment, uncorrelated PRBS signals are applied simultaneously to all four inputs. The uncorrelated PRBS signals are constructed according to the guidelines in Lee (2006). The same PRBS is used for all inputs, but they are suitably shifted to make them uncorrelated. In the fifth experiment, PRBS signals are applied simultaneously to all inputs, but correlated in such a way that the various gain directions are successively excited. The sixth experiment is similar to the fifth experiment, but the gain directions are excited simultaneously in such a way that they are uncorrelated with each other.

The last two experiments, i.e., the seventh and the eighth experiment, are designed according to the guidelines for 2×2 systems by Zhu and Stec (2006). According to them, it is sufficient to excite the low-gain direction explicitly; the high-gain direction is excited by a low-amplitude uncorrelated signal, which is added to the high-amplitude low-gain signal. The low-gain direction is estimated based on prior knowledge. In this study, the true low-gain direction is used. The signals are scaled so that comparable signal-to-noise ratios are obtained in all cases. In a practical case, the information to do this may not be available, but here the purpose is to compare ideal case performance in a fair way.

All experiments are summarized in Table 1, and a representative part of one input for each experiment (each subfigure) is shown in Figure 1. Figure 2 shows all four inputs and four outputs for one of the experiments.

3. EVALUATION OF INPUT SIGNALS

3.1 Fit percentage

The normalized root mean square error (NRMSE) is a measure of the model fit to data. The NRMSE fitness value is calculated for each fitted output by (Ljung, 2006)

$$NRMSE = \left(1 - \frac{\sqrt{\sum_i (\hat{y}_i - y_i)^2}}{\sqrt{\sum_i (y_i - \bar{y})^2}} \right) 100 \ \ [\%] \qquad (2)$$

where y_i and $\hat{y}_i$ are the measured and estimated output at the ith instant, respectively, and $\bar{y}$ is the mean value.

3.2 Crest Factor

The crest factor (CF) is defined as the ratio between the ℓ_∞-norm and the ℓ_2-norm,

$$CF_x = \frac{\ell_\infty(x)}{\ell_2(x)} \qquad (3)$$

where the ℓ_p-norm of a sequence $x(t)$ in the discrete time interval $[0, N]$ is defined as

$$\ell_p(x) = \left[\frac{1}{N} \sum_0^N |x(t)|^p \right]^{1/p} \qquad (4)$$

The ℓ_∞-norm represents $\max|x(t)|$, or the absolute peak value of the signal, and the ℓ_2-norm is the root mean square (rms) value of the signal.

3.3 Performance Index for Perturbation Signals

The performance index for perturbation signals (PIPS) for linear system is defined as

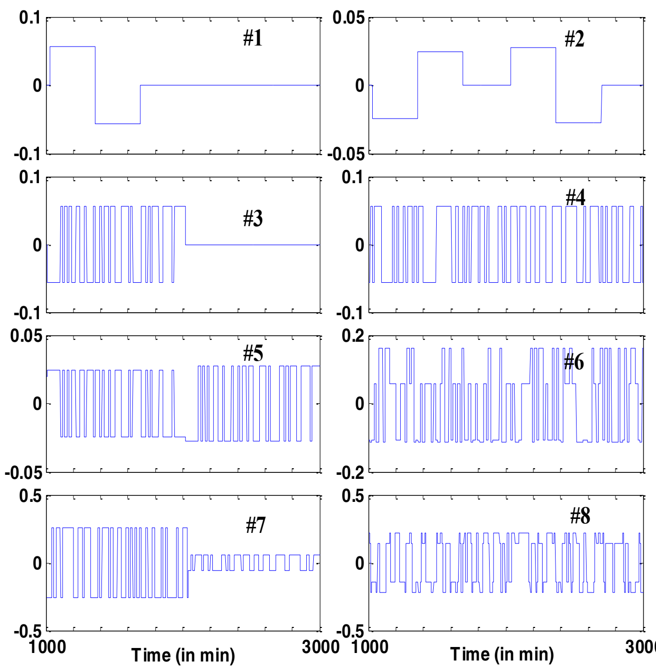

Fig. 1. Input excitation signals of one input for all experiments.

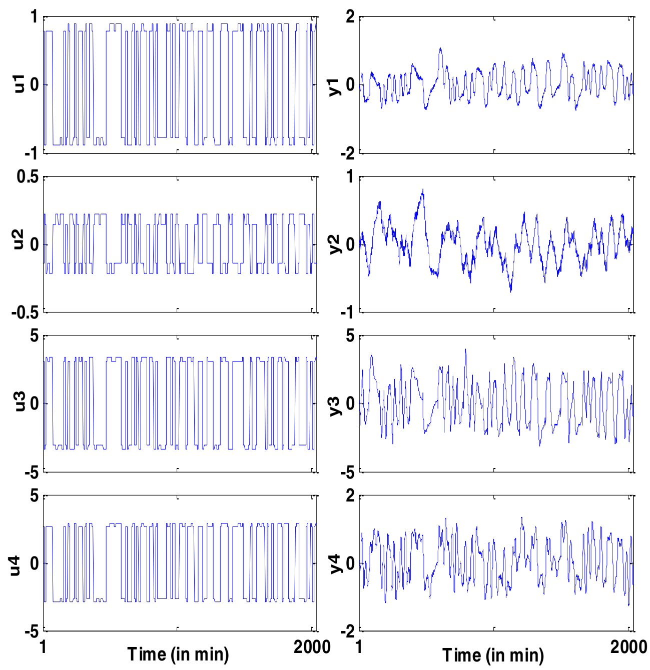

Fig. 2. Inputs and simulated outputs for one experiment.

$$PIPS_x = 200\frac{\sqrt{\left(x_{rms}^2 - x_{mean}^2\right)}}{\left(x_{\max} - x_{\min}\right)} \quad [\%] \tag{5}$$

where x_{rms}, x_{mean}, $x_{\max}$ and $x_{\min}$ are the rms, mean, maximum and minimum values of the signal, respectively (Godfrey 1999).

A measure related to PIPS is the peak factor (PF), defined as $PF = \frac{x_{\max} - x_{\min}}{2\sqrt{2}x_{rms}}$. However, in this work only CF and PIPS are considered as plant-friendly signal measures.

4. CASE STUDY DISTILLATION COLUMN/STRIPPER

A distillation column/stripper system presented by Alatiqi and Luyben (1986) is used as a case study. The transfer function matrix of the system is given in Eq. (7). The time constants of the system are in the range 5…48 min and the time delays are in the range 0.02…18 min. The system is ill-conditioned with a condition number 125.2. The steady-state relative gain array (RGA) is

$$\Lambda(0) = \begin{bmatrix} 1.077 & 0.296 & -0.118 & -0.256 \\ 4.505 & 1.772 & 0.207 & -5.484 \\ 0.020 & 0.019 & 1.086 & -0.124 \\ -4.603 & -1.086 & -0.175 & 6.864 \end{bmatrix} \tag{6}$$

The input designs in Table 1 were applied to this system. White noise with the variance 0.03 was added to all outputs. Matlab's System Identification Toolbox (Ljung, 2007) was used for model fitting.

In a distillation column, the dynamics in lower gain directions are typically much faster than the dynamics in higher gain directions. By means of step tests it was found that the system has a dominating time constant of 15 min in the low-gain direction and a dominating time constant of 50 min in the high-gain direction. To capture both dynamics, at least a second order model for each output is needed. Therefore, each transfer function was identified as a second-order transfer function with a time delay.

5. RESULTS AND DISCUSSION

To compare the performance of the different input excitations listed in Table 1, models were identified from the input-output data of each experiment and compared via cross validations. In the cross validation, each model is used to simulate the outputs of all other experiments. The agreement between simulations ("fits") and data is quantified by Eq. (2). Table 2 shows the average fit and the worst-case fit for every model. As can be seen, the models obtained from experiments where all directions were explicitly excited (stepDir and prbsSeqDir) are most robust. A notable exception is experiment prbsSimDir, where all directions were excited simultaneously

$$G(s) = \begin{bmatrix} \frac{4.09e^{-1.3s}}{(33s+1)(8.3s+1)} & \frac{6.36e^{-1.2s}}{(31.6s+1)(20s+1)} & \frac{-0.25e^{-1.4s}}{21s+1} & \frac{-0.49e^{-6s}}{22s+1} \\ \frac{-4.17e^{-5s}}{45s+1} & \frac{6.93e^{-1.02s}}{(44.6s+1)} & \frac{-0.05e^{-6s}}{(34.5s+1)^2} & \frac{1.53e^{-3.8s}}{(48s+1)} \\ \frac{1.73e^{-18s}}{(13s+1)^2} & \frac{5.11e^{-12s}}{(13.3s+1)^2} & \frac{4.61e^{-1.01s}}{(18.5s+1)} & \frac{-5.49e^{-1.5s}}{(15s+1)} \\ \frac{-11.2e^{-2.6s}}{(43s+1)(6.5s+1)} & \frac{14(10s+1)e^{-0.02s}}{(45s+1)(17.4s^2+3s+1)} & \frac{-0.1e^{-0.05s}}{(31.6s+1)(5s+1)} & \frac{4.49e^{-0.6s}}{(48s+1)(6.3s+1)} \end{bmatrix} \tag{7}$$

Table 2. NRMSE of cross validations

Model	Input excitation	Average fit (%)	Worst case fit (%)
1	stepSeq	83.97	48.41
2	stepDir	85.31	70.52
3	prbsSeq	84.56	58.85
4	prbsUnc	84.57	59.94
5	prbsSeqDir	85.43	70.97
6	prbsSimDir	82.56	55.92
7	prbsDirCorSeq	81.62	55.88
8	prbsDirCorSim	81.04	52.23

Table 3. Condition number and singular values

Model from	Condition number	Singular values			
Exp. 1	118.66	20.32	10.17	5.04	0.17
Exp. 2	124.60	20.23	10.11	5.05	0.16
Exp. 3	166.75	19.31	10.35	5.07	0.12
Exp. 4	108.13	18.99	10.37	5.06	0.18
Exp. 5	116.44	18.33	9.76	5.07	0.16
Exp. 6	543.22	47.56	8.21	4.94	0.09
Exp. 7	114.19	17.45	10.24	3.87	0.15
Exp. 8	129.18	19.66	10.47	5.17	0.15
True Value	125.20	20.23	10.11	5.05	0.16

in an uncorrelated way. It seems that the model fits did not adequately capture the directional properties. This also seems to be the problem with the models obtained from experiments, where only the low-gain direction was explicitly excited (prbsDirCorSeq and prbsDirCorSim).

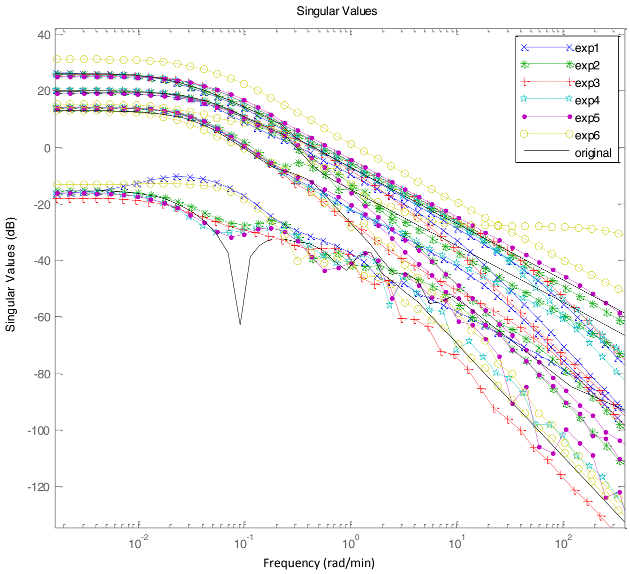

Fig. 3. Singular values of estimated models with measurement noise ($s^2 = 0.03$).

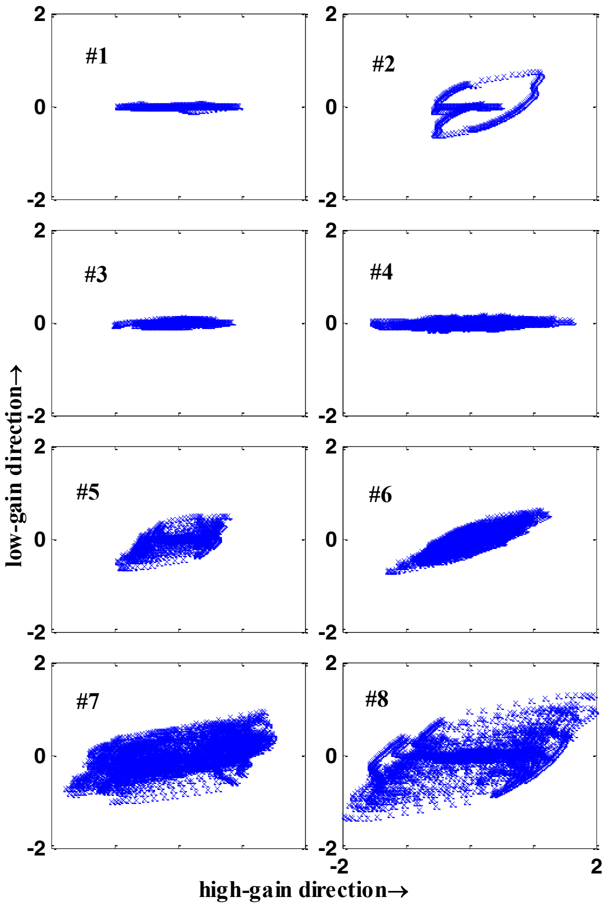

Fig. 4. Output space comparison (high-gain vs. low-gain direction).

The condition number and the singular values of the steady-state gain matrix of the obtained models as well as the true model are shown in Table 3. The models performing well in the cross validation, have a condition number and singular values very close to those of the true model. Also here, Exp. 6 (prbsSimDir) is very far off.

The singular values of the identified models and the true system model are shown in Figure 3 as a function of frequency. The figure shows that most models fit the system quite well for lower frequencies, but less so for higher frequencies. A likely explanation is that lower-order models than the true system order were fitted. Another reason might be that the input signals were mainly designed based on steady-state information.

It is of interest to study projections of the outputs, defined $y^*(t) = W^{\mathrm{T}} y(t)$, where the orthogonal matrix W is obtained from the singular value decomposition $G = W \Sigma V^{\mathrm{T}}$. This means that $y^*(t) = \Sigma V^{\mathrm{T}} u(t)$, where $y_i^*(t)$ is the ith output direction. Figure 4 shows the low-gain direction output $y_4^*(t)$ vs. the high-gain direction output $y_1^*(t)$ for all eight experiments. As can be seen, the experiments where the low-gain direction was explicitly excited, resulted in a better variability in the low-gain direction. Note, however, that $y^*(t)$ is scaling dependent in this formulation — larger input values result in larger $y^*(t)$ values. This might explain why experiments 7 and 8 seem to have the best variability, although an effort was made to use the same overall input norm in all experiments.

Table 4 shows the standard deviations of $y_i^*(t)$, $i = 1, \ldots, 4$, for all experiments. The standard deviations can be interpreted as follows: The more equal the standard deviations in the four gain directions are for an experiment, the better balanced the input excitations are in the various gain directions. The ratio between the maximum and the minimum standard deviation, which is also included in the table, can be used as a scaling-

Table 4. Standard deviation of projections along gain directions

Exp.	High gain	Middle gain 1	Middle gain 2	Low gain	Max/ min
1	0.502	0.463	0.291	0.015	33.467
2	0.252	0.268	0.204	0.179	1.497
3	0.316	0.336	0.227	0.029	11.586
4	0.573	0.672	0.443	0.058	11.586
5	0.275	0.457	0.236	0.123	3.715
6	0.447	0.967	0.479	0.231	4.186
7	0.788	1.307	0.671	0.337	3.878
8	0.767	1.259	0.649	0.359	3.507

independent measure of this balance. According to this measure, Exp. 2 (stepDir) is best balanced, but all experiments employing directional inputs are better balanced than the other experiments.

6. CONCLUSION

Constructing tailor-made input excitations in a smart way is crucial to a successful identification experiment. For an ill-conditioned system, input design is especially critical due to widely different singular values of the gain matrix. The usefulness of a number of input design methods for ill-conditioned MIMO systems was investigated in this paper. The main conclusion is that all gain directions have to be properly excited. Simplified methods that work for 2×2 systems may not be adequate for more inputs and outputs.

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