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Motion Coordination of Thrust-Propelled Underactuated Vehicles in the Presence of Communication Delays

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Abstract: This paper addresses the coordinated control problem of underactuated thrust-propelled vehicles in the presence of irregular communication delays under a directed communication graph topology. We propose a distributed control algorithm that drives the vehicles to a prescribed formation provided that the communication graph contains a spanning tree. Using the multi-dimensional small-gain approach, we show that the proposed control scheme is robust with respect to varying communication delays, which can be discontinuous with unknown upper bounds. Simulations are provided to illustrate the effectiveness of the proposed control algorithm.

Keywords: Formation control; thrust-propelled vehicles; Communication delays; Directed graph; Spanning tree.

1. INTRODUCTION

We consider the coordinated control problem of a class of thrust-propelled vehicles evolving in the special Euclidean group SE(3). This class includes autonomous underwater vehicles (AUVs) and vertical take-off and landing (VTOL) unmanned aerial vehicles (UAVs), such as classical helicopters as well as ducted fan and multi-rotor drones. By construction, these vehicles have fully-actuated attitude dynamics and underactuated translational dynamics; the translational motion is controlled by a thrust force along a single body-fixed axis. The control of thrust-propelled vehicles has been widely addressed in the literature from different perspectives [See, for instance, Frazzoli et al., 2000, Madani and Benallegue, 2006, Aguiar and Hespanha, 2007, Do and Pan, 2009, Hua et al., 2009, Abdessameud and Tayebi, 2010a, Roberts and Tayebi, 2011, Hua et al., 2013, Roberts and Tayebi, 2013, and references therein]. In contrast, only few works address the cooperative and coordinated control of teams of these vehicles. In fact, in addition to the under-actuation of the systems, coordinated control design faces increased difficulty due to several constraints related to the interconnection topology between vehicles and communication-delays that are generally unknown, time-varying and possibly discontinuous.

In Børhaug et al. [2011], the nonlinear cascaded systems theory is used to design a control law guaranteeing straight-line path following for surface vessels interconnected according to a directed graph with a globally reachable node. This scheme is applicable for planar motion with no communication delays. In Lee [2012], a backstepping design is proposed for the coordination of thrust-propelled vehicles over a directed and balanced graph. The work of Madani and Benallegue [2006] has been extended in Wang et al. [2013] to solve the consensus problem of a team of quadrotors under a directed communication topology with a spanning tree. In Lee [2012] and Wang et al. [2013], ideal communication between the systems is considered and the applied thrust for the vehicles is assumed to be always strictly positive, which might not be guaranteed, at least globally, by the proposed control schemes. This assumption is not required in Abdessameud and Tayebi [2009, 2010b], where formation control schemes for VTOL UAVs are proposed under an undirected graph with and without linear-velocity measurements. The latter results have been further extended in Abdessameud and Tayebi [2013] to the case of a directed graph containing a spanning tree and, in addition, in the presence of constant communication delays. The case of time-varying communication delays, with known upper bounds, has been addressed in Abdessameud and Tayebi [2011, 2013], where formation control schemes for a class of thrust-propelled vehicles have been proposed, yet under an undirected communication graph.

In this paper, we propose a distributed coordinated control scheme for thrust-propelled vehicles in the presence of irregular communication delays. The control objective is to achieve a prescribed formation for a team of vehicles interconnected according to a directed graph that contains a spanning tree. The small-gain framework is used to show that formation is achieved under some conditions on the communication delays. These conditions are easily realizable via an appropriate choice of the control gains without imposing additional constraints on the upper bounds of

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the possibly discontinuous communication delays. The effectiveness of the proposed approach is shown through a numerical example of a team of VTOL UAVs. It should be mentioned that a similar, although not identical, approach has been proposed in Abdessameud et al. [2014] for fully-actuated networked Euler-Lagrange systems whose extension to the case of under-actuated thrust-propelled vehicles requires additional considerations addressed in this paper.

Notations: Throughout the paper, we use $|\mathbf{x}|$ to denote the Euclidean norm of a vector $\mathbf{x} \in \mathbb{R}^m$. We also use $\mathcal{I}$ to denote the inertial frame rigidly attached to a position on the Earth (assumed flat) in North-East-Down coordinates. The orthonormal (right-handed) basis associated to $\mathcal{I}$ is denoted by $\{\hat{x}, \hat{y}, \hat{z}\}$. Also, $\mathcal{B}_i$ denotes the reference frame rigidly attached to the center of gravity of the i^{th}-vehicle. The orthonormal basis of $\mathcal{B}_i$ is denoted by $\{\hat{x}_{b_i}, \hat{y}_{b_i}, \hat{z}_{b_i}\}$, where $\hat{x}_{b_i}$ is directed towards the front of the vehicle, $\hat{y}_{b_i}$ is taken towards the right side, and $\hat{z}_{b_i}$ is directed downwards.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1 System Model

We consider a team of n thrust-propelled vehicles modeled as

$$\begin{aligned} \dot{\mathbf{p}}_i &= \mathbf{v}_i, \\ \dot{\mathbf{v}}_i &= g\hat{z} - \frac{\mathcal{T}_i}{m_i}\mathbf{R}(\mathbf{Q}_i)^\top \hat{z}, \\ \dot{\mathbf{Q}}_i &= \frac{1}{2}\mathbf{T}(\mathbf{Q}_i)\boldsymbol{\omega}_i, \\ \mathbf{J}_i\dot{\boldsymbol{\omega}}_i &= \boldsymbol{\Gamma}_i - \mathbf{S}(\boldsymbol{\omega}_i)\mathbf{J}_i\boldsymbol{\omega}_i, \end{aligned} \qquad (1)$$

where $\mathbf{p}_i$ and $\mathbf{v}_i$ denote, respectively, the position and linear-velocity of the i-th system expressed in $\mathcal{I}$, the system's mass and gravitational acceleration are denoted by m_i and g, respectively, the vector $\hat{z} := (0,0,1)^\top$, $\mathbf{J}_i \in \mathbb{R}^{3\times 3}$ is the symmetric positive definite constant inertia matrix of the i-th system with respect to $\mathcal{B}_i$. The scalar $\mathcal{T}_i$ and the vector $\boldsymbol{\Gamma}_i \in \mathbb{R}^3$ represent, respectively, the magnitude of the thrust applied to the i-th system in the direction of $\hat{z}_{b_i}$, and the external torque applied to the system expressed in $\mathcal{B}_i$. The vector $\boldsymbol{\omega}_i$ denotes the angular velocity of the $i-$th vehicle with respect to $\mathcal{I}$ expressed in $\mathcal{B}_i$. The orientation of each system is represented by the four-elements vector $\mathbf{Q}_i$, called unit-quaternion, which evolves in the unit three-sphere embedded in $\mathbb{R}^4$; $\mathbb{S}^3 = \{\mathbf{Q} \in \mathbb{R}^4 | \mathbf{Q}^T\mathbf{Q} = 1\}$. For any $\mathbf{Q}_i := (\mathbf{q}_i^\top, \eta_i)^\top$, with $\mathbf{q}_i \in \mathbb{R}^3$, the rotation matrix related to $\mathbf{Q}_i$, that brings $\mathcal{I}$ into $\mathcal{B}_i$, is given by $\mathbf{R}(\mathbf{Q}_i) \in SO(3) := \{R | R^\top R = \mathbf{I}_3, \det(R) = 1\}$ with $\mathbf{R}:\ \mathbb{S}^3 \to SO(3)$ and

$$\mathbf{R}(\mathbf{Q}_i) = (\eta_i^2 - \mathbf{q}_i^\top \mathbf{q}_i)\mathbf{I}_3 + 2\mathbf{q}_i\mathbf{q}_i^\top - 2\eta_i\mathbf{S}(\mathbf{q}_i),$$

where $\mathbf{I}_m$ denotes the identity matrix of dimension m and $\mathbf{S}(\mathbf{x})$ is the skew-symmetric matrix such that $\mathbf{S}(\mathbf{x}_1)\mathbf{x}_2 = \mathbf{x}_1 \times \mathbf{x}_2$ for any vectors $\mathbf{x}_1 \in \mathbb{R}^3$ and $\mathbf{x}_2 \in \mathbb{R}^3$, with '$\times$' denoting the vector cross product. Also, $\mathbf{T}(\mathbf{Q}_i)$ is defined such that $\mathbf{T} : \mathbb{S}^3 \to \mathbf{R}^{4\times 3}$ with

$$\mathbf{T}(\mathbf{Q}) = \begin{pmatrix} \eta\ \mathbf{I}_3 + \mathbf{S}(\mathbf{q}) \\ -\mathbf{q}^\top \end{pmatrix}, \qquad (2)$$

for any unit-quaternion $\mathbf{Q} := (\mathbf{q}^\top, \eta)^\top$.

2.2 Problem Formulation

We assume that the vehicles in the team transmit some of their state information according the interconnection topology described by the directed communication graph $\mathcal{G} = (\mathcal{N}, \mathcal{E}, \mathcal{A})$. The set $\mathcal{N}$ is the set of nodes or vertices, describing the set of thrust-propelled vehicles in the network, $\mathcal{E} \in \mathcal{N} \times \mathcal{N}$ is the set of ordered pairs of nodes, called edges, and $\mathcal{A} = \{a_{ij}\} \in \mathbb{R}^{n\times n}$ is the weighted adjacency matrix. An edge (i,j) indicates that vehicle j can receive information from vehicle i, but not necessarily *vice-versa*. The weighted adjacency matrix is defined such that $a_{ii} := 0$, $a_{ij} > 0$ if $(j,i) \in \mathcal{E}$, and $a_{ij} = 0$ if $(j,i) \notin \mathcal{E}$. A directed path is a sequence of edges in a directed graph of the form $(i_1, i_2), (i_2, i_3), \ldots$, where $i_l \in \mathcal{N}$. A directed graph is said to contain a directed spanning tree if there exists at least one node having a directed path to all the other nodes. The Laplacian matrix $\mathbf{L} := [l_{ij}] \in \mathbb{R}^{n\times n}$ of the directed graph $\mathcal{G}$ is defined such that: $l_{ii} = \sum_{j=1}^n a_{ij}$, and $l_{ij} = -a_{ij}$ for $i \neq j$.

In addition, the communication between the vehicles in the team is delayed by $\tau_{ij}(t)$, for each $(j,i) \in \mathcal{E}$, which satisfy the following assumption.

Assumption 1. For each $(j,i) \in \mathcal{E}$, the communication delay $\tau_{ij} \colon \mathbb{R}_+ \to \mathbb{R}_+$ can be decomposed into the sum of two terms,

$$\tau_{ij}(t) = \tau_{ij}^s(t) + \tau_{ij}^r(t), \qquad (3)$$

where the components $\tau_{ij}^s(\cdot)$ and $\tau_{ij}^r(\cdot)$ have the following properties:

i) There exists a function $\tau^* \colon \mathbb{R}_+ \to \mathbb{R}_+$ such that $\tau^*(t_2) - \tau^*(t_1) \leq t_2 - t_1$ for all t_1, $t_2 \in \mathbb{R}_+$, and $\left|\tau_{ij}^s(t)\right| \leq \tau^*(t)$ holds for all $t \geq 0$.
ii) The function $\tau_{ij}^s(t)$ satisfies: $t - \tau_{ij}^s(t) \to +\infty$ as $t \to +\infty$.
iii) There exists $\Upsilon_{ij} \geq 0$ such that the inequality: $\left|\tau_{ij}^s(t_2) - \tau_{ij}^s(t_1)\right| \leq \Upsilon_{ij} \cdot |t_2 - t_1|$ holds for almost all t_2, $t_1 \in \mathbb{R}_+$, with $t_2 \geq t_1$.
iv) There exists $\Delta_{ij}^\tau \geq 0$ such that: $\left|\tau_{ij}^r(t)\right| \leq \Delta_{ij}^\tau$ holds for almost all $t \geq 0$.

Assumption 1 implies that the communication delays contain smooth and irregular components, indicated by subscripts s and r, respectively. The smooth component, $\tau_{ij}^s(\cdot)$, is assumed to be upper bounded by a possibly time-varying unbounded function, given by $\tau^*(\cdot)$, that does not grow faster than the time itself. Also, the time-derivative of this component $d\tau_{ij}^s(t)/dt$ is well-defined for almost all $t \geq 0$ and is bounded by Υ_{ij} where defined. The irregular component, $\tau_{ij}^r(\cdot)$, is only assumed to be bounded. It is clear that Assumption 1 does not impose an upper bound on the communication delays $\tau_{ij}(t)$, and only Υ_{ij} and Δ_{ij}^τ are assumed to be known, which can be easily satisfied.

Our objective in this work is to design the thrust and torque inputs in (1) such that all vehicles converge to a prescribed stationary geometric formation in the presence of time-varying communication delays. Formally, we aim to guarantee that

$$\mathbf{v}_i(t) \to 0 \ \text{ and } \ (\mathbf{p}_i(t) - \mathbf{p}_j(t)) \to \mathbf{d}_{ij}, \qquad (4)$$

as $t \to +\infty$ for $i,j \in \mathcal{N}$, where $\mathbf{d}_{ij} \in \mathbb{R}^3$, satisfying $\mathbf{d}_{ij} = -\mathbf{d}_{ji}$, defines the desired formation pattern.

2.3 Preliminary results

Consider an affine nonlinear system of the form

$$\begin{aligned} \dot{x} &= f(x) + g_1(x)u_1 + \ldots + g_p(x)u_p, \\ y_1 &= h_1(x), \\ \vdots &\;\vdots\; \vdots \\ y_q &= h_q(x), \end{aligned} \tag{5}$$

where $x \in \mathbb{R}^N$, $u_i \in \mathbb{R}^{\tilde{m}_i}$ for $i \in \mathcal{N}_p := \{1,\ldots,p\}$, $y_j \in \mathbb{R}^{\bar{m}_j}$ for $j \in \mathcal{N}_q := \{1,\ldots,q\}$, and $f(\cdot)$, $g_i(\cdot)$, for $i \in \mathcal{N}_p$, and $h_j(\cdot)$, for $j \in \mathcal{N}_q$, are locally Lipschitz functions of the corresponding dimensions, $f(0) = 0$, $h(0) = 0$. Suppose that for any initial condition $x(t_0)$ and any inputs $u_1(t)$, ..., $u_p(t)$ that are uniformly essentially bounded on $[t_0, t_1)$, the corresponding solution $x(t)$ is well defined for all $t \in [t_0, t_1]$.

The convergence analysis in this work is based on the following small gain theorem.

Theorem 1. Consider a system of the form (5). Suppose the system is input-to-output stable[1] (IOS) with linear IOS gains $\gamma_{ij}^0 \geq 0$. Suppose also that each input $u_j(\cdot)$, $j \in \mathcal{N}_p$, is a Lebesgue measurable function satisfying

$$u_j(t) \equiv 0 \quad \text{for } t < 0, \tag{6}$$

and

$$|u_j(t)| \leq \sum_{i\in\mathcal{N}_q} \mu_{ji} \cdot \sup_{s\in[t-\vartheta_{ji}(t),t]} |y_i(s)| + |\delta_j(t)|, \tag{7}$$

for almost all $t \geq 0$, where $\mu_{ji} \geq 0$, all $\vartheta_{ji}(t)$ satisfy Assumption 1, and $\delta_j(t)$ is a uniformly essentially bounded signal that satisfy $|\delta_j(t)| \to 0$ at $t \to +\infty$. Let $\Gamma := \Gamma^0 \cdot \mathcal{M} \in \mathbb{R}^{q\times q}$, where $\Gamma^0 := \{\gamma_{ij}^0\}$, $\mathcal{M} := \{\mu_{ji}\}$, $i \in \mathcal{N}_q$, $j \in \mathcal{N}_p$. If $\rho(\Gamma) < 1$, where $\rho(\Gamma)$ is the spectral radius of the matrix Γ, then the trajectories of the system (5) with input-output constraints (6)-(7) are well defined for all $t \geq 0$ and such that all the outputs $y_i(t)$, $i \in \mathcal{N}_q$, and all the inputs $u_j(\cdot)$, $j \in \mathcal{N}_p$, are uniformly bounded and satisfy $|y_i(t)| \to 0$, $|u_j(t)| \to 0$ as $t \to +\infty$. □

Theorem 1 is a special case of a more general result given in Polushin et al. [2013], and the proof follows similar lines as the proof of Theorem 1 in Abdessameud et al. [2014], and is omitted due to space limitations.

3. CONTROL DESIGN

Consider the linear acceleration of each vehicle in (1), which can be rewritten as

$$\dot{\mathbf{v}}_i = \mathbf{F}_i - \frac{\mathcal{T}_i}{m_i}\left(\mathbf{R}(\mathbf{Q}_i)^\top - \mathbf{R}(\mathbf{Q}_{d_i})^\top\right)\hat{z}, \tag{8}$$

with

$$\mathbf{F}_i := g\hat{z} - \frac{\mathcal{T}_i}{m_i}\mathbf{R}(\mathbf{Q}_{d_i})^\top\hat{z}, \tag{9}$$

where the variable $\mathbf{F}_i \in \mathbb{R}^3$ is an intermediary control input to be designed later, and $\mathbf{Q}_{d_i} := (\mathbf{q}_{d_i}^\top, \eta_{d_i})^\top$ is the unit quaternion representing a desired orientation of the i^{th} vehicle. Note that, for a given $\mathbf{F}_i$, one possible solution for the necessary thrust and desired attitude can be obtained from (9) as

$$\mathcal{T}_i = m_i|g\hat{z} - \mathbf{F}_i|, \tag{10}$$

$$\eta_{d_i} = \sqrt{\frac{\mathcal{T}_i + m_i(g - \hat{z}^\top\mathbf{F}_i)}{2\mathcal{T}_i}}, \quad \mathbf{q}_{d_i} = \frac{m_i}{2\mathcal{T}_i\eta_{d_i}}\mathbf{S}(\mathbf{F}_i)\hat{z}, \tag{11}$$

from which one can verify that, under the sufficient condition

$$\hat{z}^\top\mathbf{F}_i < g, \tag{12}$$

the thrust input (10) is strictly positive and the desired attitude (11) is nonsingular (See Lemma 5.1 in Abdessameud and Tayebi [2013] for more details).

We consider the following intermediary control input

$$\mathbf{F}_i = -K_i^p\chi(\boldsymbol{\theta}_i) - K_i^d\chi(\dot{\boldsymbol{\theta}}_i), \quad \ddot{\boldsymbol{\theta}}_i = \mathbf{F}_i - \boldsymbol{\phi}_i, \tag{13}$$

where K_i^p and K_i^d are diagonal positive definite gain matrices and the function $\chi(\cdot)$ is defined as

$$\chi(\varrho) = (\sigma(x), \sigma(y), \sigma(z))^\top \in \mathbb{R}^3, \tag{14}$$

for any vector $\varrho = (x, y, z)^\top \in \mathbb{R}^3$, and $\sigma : \mathbb{R} \to \mathbb{R}$ is a strictly increasing continuously differentiable function satisfying the following properties: *i)* $\sigma(0) = 0$ and $x\sigma(x) > 0$ for $x \neq 0$, *ii)* $|\sigma(x)| \leq \sigma_b$, for $\sigma_b > 0$, *iii)* $\frac{\mathrm{d}\sigma(x)}{\mathrm{d}x}$ is bounded.
Also, the vector $\boldsymbol{\phi}_i \in \mathbb{R}^3$ is given as

$$\boldsymbol{\phi}_i = -k_i^r(\dot{\boldsymbol{\xi}}_i - \boldsymbol{\zeta}_i) + \dot{\boldsymbol{\zeta}}_i, \tag{15}$$

with

$$\boldsymbol{\xi}_i := \mathbf{p}_i - \boldsymbol{\theta}_i, \tag{16}$$

$$\dot{\boldsymbol{\zeta}}_i = -L_i\boldsymbol{\zeta}_i - \lambda_i(\kappa_i\boldsymbol{\xi}_i - \sum_{j=1}^n a_{ij}\mathbf{d}_{ij} - \boldsymbol{\psi}_i), \tag{17}$$

$$\dot{\boldsymbol{\psi}}_i = -\boldsymbol{\psi}_i + \sum_{j=1}^n a_{ij}\boldsymbol{\xi}_j(t - \tau_{ij}(t)), \tag{18}$$

where k_i^r, L_i and λ_i are strictly positive scalar gains, $a_{ij} \geq 0$ is the $(i,j)^{th}$ element of the adjacency matrix of the directed graph $\mathcal{G}$, and $\kappa_i := \sum_{j=1}^n a_{ij}$.

It is straightforward to verify from (13) and (14) that $|\hat{z}^\top\mathbf{F}_i| \leq \sigma_b(K_i^p + K_i^d)\hat{z}$, which indicates that condition (12) can be satisfied if

$$\sigma_b(K_i^p + K_i^d)\hat{z} < g. \tag{19}$$

This guarantees a nonsingular solution for the desired attitude for the i-th vehicle that can be used as a reference input for the rotational dynamics to achieve the desired motion of the vehicle. In addition, the input thrust (10) satisfies

$$0 < \mathcal{T}_i \leq m_i(g + \sigma_b\sqrt{3}(k_{i,\max}^p + k_{i,\max}^d)), \tag{20}$$

for $i \in \mathcal{N}$, where $k_{i,\max}^p$ and $k_{i,\max}^d$ are the maximum eigenvalues of K_i^p and K_i^d, respectively.

To design the input torque for the rotational dynamics, let $\tilde{\mathbf{Q}}_i := (\tilde{\mathbf{q}}_i^\top, \tilde{\eta}_i)^\top$ denote the attitude tracking error, describing the discrepancy between the vehicle's attitude and the extracted desired attitude, defined as: $\tilde{\mathbf{Q}}_i = \left((\eta_{d_i}\mathbf{q}_i - \eta_i\mathbf{q}_{d_i} - \mathbf{S}(\mathbf{q}_{d_i})\mathbf{q}_i)^\top,\ \eta_i\eta_{d_i} - \mathbf{q}_i^\top\mathbf{q}_{d_i}\right)^\top$, and satisfies the following dynamics

$$\dot{\tilde{\mathbf{Q}}}_i = \frac{1}{2}\mathbf{T}(\tilde{\mathbf{Q}}_i)\tilde{\boldsymbol{\omega}}_i, \quad \tilde{\boldsymbol{\omega}}_i = \boldsymbol{\omega}_i - \mathbf{R}(\tilde{\mathbf{Q}}_i)\ \boldsymbol{\omega}_{d_i}, \tag{21}$$

where $\tilde{\boldsymbol{\omega}}_i$ is the angular velocity tracking error vector, $\mathbf{R}(\tilde{\mathbf{Q}}_i) = \mathbf{R}(\mathbf{Q}_i)\mathbf{R}(\mathbf{Q}_{d_i})^\top$, and $\boldsymbol{\omega}_{d_i}$ is the desired angular velocity for the i^{th} vehicle, which is related to

[1] The definitions of input-to-state stability (ISS) and input-to-output stability (IOS) of multiple inputs multiple outputs systems can be found in [Sontag, 2008].

$\mathbf{Q}_{d_i}$ by: $\boldsymbol{\omega}_{d_i} = 2\mathbf{T}(\mathbf{Q}_{d_i})^\top \dot{\mathbf{Q}}_{d_i}$. Note that attitude tracking is achieved when $\mathbf{Q}_i$ coincides with $\mathbf{Q}_{d_i}$, or $\tilde{\mathbf{Q}}_i = (0,0,0,\pm 1)^\top$. Also, since $\mathbf{Q}_{d_i}$ in (11) is time-varying, the desired angular velocity and its time derivative can be parameterised as [Abdessameud and Tayebi, 2013]

$$\boldsymbol{\omega}_{d_i} = \Xi(\mathbf{F}_i)\dot{\mathbf{F}}_i, \quad (22)$$
$$\dot{\boldsymbol{\omega}}_{d_i} = \bar{\Xi}(\mathbf{F}_i, \dot{\mathbf{F}}_i)\dot{\mathbf{F}}_i + \Xi(\mathbf{F}_i)\ddot{\mathbf{F}}_i, \quad (23)$$

where the matrices $\Xi(\mathbf{F}_i)$ and $\bar{\Xi}(\mathbf{F}_i, \dot{\mathbf{F}}_i)$ can be easily derived. In addition, in view of (13), we have

$$\dot{\mathbf{F}}_i = -K_i^p \mathbf{h}(\boldsymbol{\theta}_i)\dot{\boldsymbol{\theta}}_i - K_i^d \mathbf{h}(\dot{\boldsymbol{\theta}}_i)\ddot{\boldsymbol{\theta}}_i, \quad (24)$$
$$\ddot{\mathbf{F}}_i = -K_i^p \dot{\mathbf{h}}(\boldsymbol{\theta}_i)\dot{\boldsymbol{\theta}}_i - \left(K_i^p \mathbf{h}(\boldsymbol{\theta}_i) + K_i^d \dot{\mathbf{h}}(\dot{\boldsymbol{\theta}}_i)\right)\ddot{\boldsymbol{\theta}}_i - K_i^d \mathbf{h}(\dot{\boldsymbol{\theta}}_i)(\dot{\mathbf{F}}_i - \dot{\boldsymbol{\phi}}_i), \quad (25)$$

where $\mathbf{h}(\varrho) := \mathrm{diag}\left(\frac{\partial\sigma(x)}{\partial x}, \frac{\partial\sigma(y)}{\partial y}, \frac{\partial\sigma(z)}{\partial z}\right)$, for $\varrho = (x,y,z)^\top \in \mathbb{R}^3$, and $\dot{\mathbf{h}}(\cdot)$ is the time-derivative of $\mathbf{h}(\cdot)$.

We propose the following individual torque input

$$\boldsymbol{\Gamma}_i = \mathbf{H}_i + \mathbf{J}_i\dot{\boldsymbol{\beta}}_i - k_i^q\tilde{\mathbf{q}}_i - k_i^\Omega(\tilde{\boldsymbol{\omega}}_i - \boldsymbol{\beta}_i), \quad (26)$$
$$\boldsymbol{\beta}_i = -k_i^\beta\tilde{\mathbf{q}}_i + \frac{\mathcal{T}_i}{k_i^q m_i}\boldsymbol{\Pi}_i^\top(\dot{\boldsymbol{\xi}}_i - \boldsymbol{\zeta}_i), \quad (27)$$

where $\mathbf{H}_i = \mathbf{S}(\boldsymbol{\omega}_i)\mathbf{J}_i\boldsymbol{\omega}_i - \mathbf{J}_i\mathbf{S}(\tilde{\boldsymbol{\omega}}_i)\mathbf{R}(\tilde{\mathbf{Q}}_i)\boldsymbol{\omega}_{d_i} + \mathbf{J}_i\mathbf{R}(\tilde{\mathbf{Q}}_i)\dot{\boldsymbol{\omega}}_{d_i}$, k_i^q, k_i^Ω and k_i^β are positive scalar gains, $\boldsymbol{\xi}_i$ and $\boldsymbol{\zeta}_i$ are defined in (16) and (17), respectively, $\tilde{\mathbf{q}}_i$ is the vector part of $\tilde{\mathbf{Q}}_i$, $\boldsymbol{\omega}_{d_i}$ is given in (22)-(25), and $\boldsymbol{\Pi}_i$ satisfies [Abdessameud and Tayebi, 2010a]

$$\left(\mathbf{R}(\mathbf{Q}_i)^\top - \mathbf{R}(\mathbf{Q}_{d_i})^\top\right)\hat{z} = \boldsymbol{\Pi}_i\tilde{\mathbf{q}}_i. \quad (28)$$

It should be noted, from (23) and (25), that due to the underactuation of the vehicles, the successive two time-derivatives of the intermediary control input $\mathbf{F}_i$ are used in the attitude tracking control law (26). Since the input $\mathbf{F}_i$ is designed using the auxiliary variables $\boldsymbol{\theta}_i$, $\boldsymbol{\zeta}_i$ and $\boldsymbol{\psi}_i$, the signals $\dot{\mathbf{F}}_i$ and $\ddot{\mathbf{F}}_i$, with

$$\dot{\boldsymbol{\phi}}_i = -k_i^r(\boldsymbol{\phi}_i - \dot{\boldsymbol{\zeta}}_i - \frac{\mathcal{T}_i}{m_i}\boldsymbol{\Pi}_i\tilde{\mathbf{q}}_i) - L_i\dot{\boldsymbol{\zeta}}_i - \lambda_i\left(\kappa_i(\mathbf{v}_i - \dot{\boldsymbol{\theta}}_i) - \dot{\psi}_i\right),$$

can be explicitly computed using available signals, and are well defined. This simplifies the torque input design knowing that the received positions from neighbors can be discontinuous due to the irregular communication delays. Another advantage from the introduction of the auxiliary systems (13) and (17)-(18) can be seen from the facts that the upper bound of the input thrust $\mathcal{T}_i$ can be determined *a priori* and condition (12) can be satisfied with a suitable choice of the control gains independently from the interconnection topology between the vehicles. Furthermore, as will become clear later, the system (17)-(18) will play an important role in achieving our objectives under easily checkable sufficient conditions on the delays.

4. CONVERGENCE ANALYSIS

Our result is given in the following theorem.

Theorem 2. Consider the network of n thrust-propelled vehicles described by (1), where the interconnection between the vehicles is described by the directed communication graph $\mathcal{G}$ and suppose Assumption 1 holds. Let the thrust input for each vehicle be defined in (10) using the intermediary input (13) with (15)-(18) under restriction (19), and the torque input be given in (26)-(27). Let the control gains for each vehicle that receives information from at least one other vehicle, *i.e.,* having $\kappa_i \neq 0$, satisfy [2]

$$\sum_{j=1}^n \frac{a_{ij}}{\mu_i}\left(1 + \Upsilon_{ij} + 2\cdot\Delta_{ij}^\tau\right) < 1, \quad (29)$$

where $\mu_i = -\max(\mathcal{R}e(\mu_{i,1}), \mathcal{R}e(\mu_{i,2}))$ and $\mu_{i,1}$, $\mu_{i,2}$ are the roots of $p^2 + L_ip + \lambda_i\kappa_i = 0$. Then starting from any initial conditions, the signals $\mathbf{v}_i$, $\sum_{j=1}^n a_{ij}(\mathbf{p}_i - \mathbf{p}_j(t - \tau_{ij}(t)))$ and $\tilde{\boldsymbol{\omega}}_i$ are uniformly bounded and $\mathbf{v}_i(t) \to 0$, $\sum_{j=1}^n a_{ij}(\mathbf{p}_i(t) - \mathbf{p}_j(t - \tau_{ij}(t)) - \mathbf{d}_{ij}) \to 0$, $\tilde{\mathbf{q}}_i(t) \to 0$, and $\tilde{\boldsymbol{\omega}}_i(t) \to 0$, as $t \to +\infty$ for all $i \in \mathcal{N}$. Furthermore, if the directed communication graph contains a spanning tree and $\tau^*(t)$ in Assumption 1, point i), satisfies $\limsup_{t\to+\infty} \tau^*(t) < \infty$, then $\sum_{j=1}^n a_{ij}(\mathbf{p}_i - \mathbf{p}_j)$ is uniformly bounded and $(\mathbf{p}_i(t) - \mathbf{p}_j(t)) \to \mathbf{d}_{ij}$, as $t \to +\infty$ for all $i, j \in \mathcal{N}$. □

Proof.

let us first define the error variables $\mathbf{r}_i = (\dot{\boldsymbol{\xi}}_i - \boldsymbol{\zeta}_i)$ and $\boldsymbol{\Omega}_i = (\tilde{\boldsymbol{\omega}}_i - \boldsymbol{\beta}_i)$, which, using (26)-(27), satisfy

$$\dot{\mathbf{r}}_i = -k_i^r\mathbf{r}_i - \frac{\mathcal{T}_i}{m_i}\boldsymbol{\Pi}_i\tilde{\mathbf{q}}_i, \quad (30)$$
$$\mathbf{J}_i\dot{\boldsymbol{\Omega}}_i = -k_i^q\tilde{\mathbf{q}}_i - k_i^\Omega\boldsymbol{\Omega}_i. \quad (31)$$

We can show from (30)-(31), in view of (21) and (27), that $\mathbf{r}_i$, $\boldsymbol{\Omega}_i$, $\tilde{\mathbf{q}}_i \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, and $\dot{\mathbf{r}}_i$, $\boldsymbol{\beta}_i$, $\dot{\tilde{\mathbf{q}}}_i$, $\dot{\boldsymbol{\Omega}}_i \in \mathcal{L}_\infty$. This can be verified using the Lyapunov-like function: $V = \sum_{i=1}^n\left(\frac{1}{2}\mathbf{r}_i^\top\mathbf{r}_i + \frac{1}{2}\boldsymbol{\Omega}_i^\top\mathbf{J}_i\boldsymbol{\Omega}_i + k_i^q\bar{\mathbf{Q}}_i^\top\bar{\mathbf{Q}}_i\right)$, where $\bar{\mathbf{Q}}_i = (\tilde{\mathbf{q}}_i^\top, (1 - \tilde{\eta}_i))^\top$. Then, by virtue of Barbălat Lemma, one can conclude that $\mathbf{r}_i(t) \to 0$, $\boldsymbol{\Omega}_i(t) \to 0$, $\tilde{\mathbf{q}}_i(t) \to 0$, $\boldsymbol{\beta}_i(t) \to 0$ and $\tilde{\boldsymbol{\omega}}_i(t) \to 0$, as $t \to +\infty$ for $i \in \mathcal{N}$.

Next, let $\tilde{\boldsymbol{\xi}}_i = \kappa_i\boldsymbol{\xi}_i - \sum_{j=1}^n a_{ij}\mathbf{d}_{ij} - \boldsymbol{\psi}_i$, $\tilde{\boldsymbol{\psi}}_i = \boldsymbol{\psi}_i - \sum_{j=1}^n a_{ij}\boldsymbol{\xi}_j(t - \tau_{ij}^s(t))$, where $\tau_{ij}^s(t)$ is defined in Assumption 1. The dynamic systems (17)-(18), for $i \in \mathcal{N}$, can be shown to be equivalent to

$$\dot{\boldsymbol{\zeta}}_i = -L_i\boldsymbol{\zeta}_i - \lambda_i\tilde{\boldsymbol{\xi}}_i, \quad (32)$$
$$\dot{\tilde{\boldsymbol{\xi}}}_i = \kappa_i\boldsymbol{\zeta}_i + \tilde{\boldsymbol{\psi}}_i + \boldsymbol{\nu}_{i_1}, \quad (33)$$
$$\dot{\tilde{\boldsymbol{\psi}}}_i = -\tilde{\boldsymbol{\psi}}_i - \boldsymbol{\nu}_{i_1} + \boldsymbol{\nu}_{i_2}, \quad (34)$$

for $i \in \mathcal{N}$, where

$$\boldsymbol{\nu}_{i_1} = \sum_{j=1}^n a_{ij}\left(\mathbf{r}_i - \boldsymbol{\xi}_j(t - \tau_{ij}(t)) + \boldsymbol{\xi}_j(t - \tau_{ij}^s(t))\right), \quad (35)$$
$$\boldsymbol{\nu}_{i_2} = \sum_{j=1}^n a_{ij}\left(\mathbf{r}_i - \left(1 - \frac{d\tau_{ij}^s(t)}{dt}\right)\dot{\boldsymbol{\xi}}_j(t - \tau_{ij}^s)\right). \quad (36)$$

Since the communication graph is directed and is assumed to contain a spanning tree, there *might* exist at most one system, denoted by l, that does not receive information from any other system in the team. In this case, system (32)-(34) reduces to

$$\dot{\boldsymbol{\zeta}}_l = -L_l\boldsymbol{\zeta}_l + \lambda_l\boldsymbol{\psi}_l, \quad \dot{\boldsymbol{\psi}}_l = -\boldsymbol{\psi}_l, \quad (37)$$

[2] Condition (29) is not imposed on the vehicles that do not receive information from any other vehicle in the network.

with $\kappa_l = 0$, $\tilde{\boldsymbol{\xi}}_l = -\boldsymbol{\psi}_l = -\tilde{\boldsymbol{\psi}}_l$, $\boldsymbol{\nu}_{l_1} = \boldsymbol{\nu}_{l_2} = 0$.

Now, following similar steps as in the proof of [Abdessameud et al., 2014, Theorem 2], we can show that the system (32)-(34), for $i \in \mathcal{N}$, is ISS with respect to its inputs $\boldsymbol{\nu}_{i_1}$ and $\boldsymbol{\nu}_{i_2}$. We can also verify that the overall system consisting of all the systems (32)-(36), for $i \in \mathcal{N}$, and having n outputs, given by $y_i = \boldsymbol{\zeta}_i$, $i \in \mathcal{N}$, and $2n$ inputs, ordered as: $u_{2i} := \boldsymbol{\nu}_{i_1}$, $u_{2i-1} := \boldsymbol{\nu}_{i_2}$, for $i \in \mathcal{N}$, is IOS with IOS gain matrix $\Gamma^0 := \{\gamma^0_{ij}\} \in \mathbb{R}^{n\times 2n}$ given as follows:

$$\gamma^0_{il} = \begin{cases} 1/\mu_i & \text{if } l = 2i-1,\ i \in \mathcal{N},\ \kappa_i \neq 0, \\ 2/\mu_i & \text{if } l = 2i,\ i \in \mathcal{N},\ \kappa_i \neq 0, \\ 0 & \text{otherwise.} \end{cases}$$

On the other hand, using Assumption 1 and $\dot{\boldsymbol{\xi}}_i = (\mathbf{r}_i + \boldsymbol{\zeta}_i)$, the following estimates of the above defined inputs $|u_{2i-1}|$ and $|u_{2i}|$, $i \in \mathcal{N}$, can be derived as

$$\begin{aligned} |u_{2i}(t)| &\leq \sum_{j=1}^{n} a_{ij}\Delta^{\tau}_{ij} \cdot \left(\sup_{\sigma\in[t_1,t_2]} |y_j(\sigma)| \right) + \delta_{2i}(t), \\ |u_{2i-1}(t)| &\leq \sum_{j=1}^{n} a_{ij}\left(1 + \Upsilon_{ij}\right) \left| y_j\left(t - \tau^s_{ij}(t)\right)\right| + \delta_{2i-1}(t), \end{aligned}$$

where, we used the notation $t_1 := (t - \max\{\tau_{ij}(t), \tau^s_{ij}(t)\})$, $t_2 := (t - \min\{\tau_{ij}(t), \tau^s_{ij}(t)\})$ and

$$\begin{aligned} \delta_{2i} &= \kappa_i|\mathbf{r}_i| + \sum_{j=1}^{n} a_{ij}\Delta^{\tau}_{ij} \cdot \left(\sup_{\sigma\in[t_1,t_2]} |\mathbf{r}_i(\sigma)| \right), \\ \delta_{2i-1} &= \kappa_i|\mathbf{r}_i| + \sum_{j=1}^{n} a_{ij}\left(1 + \Upsilon_{ij}\right) \left|\mathbf{r}_i\left(t - \tau^s_{ij}(t)\right)\right|. \end{aligned} \quad (38)$$

Consequently, one can conclude that the input vectors u_j, $j \in \{1, \ldots, 2n\}$, satisfy the conditions of Theorem 1 with the elements of the interconnection matrix $\mathcal{M} := \{\mu_{lj}\} \in \mathbb{R}^{2n\times n}$ are obtained as

$$\mu_{lj} = \begin{cases} a_{ij}\left(1 + \Upsilon_{ij}\right) & \text{if } l = 2i-1,\ j \in \mathcal{N},\ i \in \mathcal{N}, \\ a_{ij} \cdot \Delta^{\tau}_{ij} & \text{if } l = 2i,\ j \in \mathcal{N},\ i \in \mathcal{N}, \end{cases}$$

and where $\delta_j(t)$, $j \in \{1, \ldots, 2n\}$, given in (38), are uniformly bounded and satisfy $\delta_j(t) \to 0$, as $t \to +\infty$, since Assumption 1 implies that $t_1 \to +\infty$ and $t_2 \to +\infty$ as $t \to +\infty$. As a result, the elements of the gain matrix $\Gamma := \Gamma^0 \cdot \mathcal{M} = \{\bar{\gamma}_{ij}\} \in \mathbb{R}^{n\times n}$ in Theorem 1 are obtained as follows

$$\bar{\gamma}_{ij} := \sum_{l=1}^{n} \gamma^0_{il} \cdot \mu_{lj} = \begin{cases} \dfrac{a_{ij}}{\mu_i}\left(1 + \Upsilon_{ij} + 2\cdot\Delta^{\tau}_{ij}\right), & \text{if } \kappa_i \neq 0, \\ 0 & \text{otherwise.} \end{cases}$$

Note that the diagonal elements of Γ are all zeros, *i.e.,* $\bar{\gamma}_{ii} = 0$ for all $i \in \mathcal{N}$, and one can apply Geršgorin disc theorem [Horn and Johnson, 1985] to verify that $\rho(\Gamma) < 1$ if $\sum_{j=1}^{n} \bar{\gamma}_{ij} < 1$ for all $i \in \mathcal{N}$, which is satisfied by (29). As a result, the conditions of Theorem 1 are verified, and one can conclude that $\boldsymbol{\zeta}_i$, $\boldsymbol{\nu}_{i_1}$ and $\boldsymbol{\nu}_{i_2}$, for $i \in \mathcal{N}$, are uniformly bounded and $\boldsymbol{\zeta}_i(t) \to 0$, $\boldsymbol{\nu}_{i_1}(t) \to 0$, $\boldsymbol{\nu}_{i_2}(t) \to 0$ as $t \to +\infty$, for $i \in \mathcal{N}$. This, with the ISS property of (32)-(34), imply that $\tilde{\boldsymbol{\xi}}_i$, $\tilde{\boldsymbol{\psi}}_i$ are uniformly bounded and $\tilde{\boldsymbol{\xi}}_i(t) \to 0$, $\tilde{\boldsymbol{\psi}}_i(t) \to 0$, $\dot{\boldsymbol{\xi}}_i(t) \to 0$ as $t \to +\infty$, for $i \in \mathcal{N}$. As a result, we conclude that $\sum_{j=1}^{n} a_{ij}(\boldsymbol{\xi}_i - \boldsymbol{\xi}_j(t - \tau_{ij}(t)))$ is uniformly bounded and $\sum_{j=1}^{n} a_{ij}(\boldsymbol{\xi}_i - \boldsymbol{\xi}_j(t - \tau_{ij}(t)) - \mathbf{d}_{ij}) \to 0$ as $t \to +\infty$, for $i \in \mathcal{N}$.

Now, one can see that the dynamics of the auxiliary vector $\boldsymbol{\theta}_i$ in (13) can be rewritten, in view of (13), as: $\ddot{\boldsymbol{\theta}}_i = -K^p_i\chi(\boldsymbol{\theta}_i) - K^d_i\chi(\dot{\boldsymbol{\theta}}_i) - \boldsymbol{\phi}_i$, where $\boldsymbol{\phi}_i$ is given in (15), which, in view of the above results, is uniformly bounded and satisfies $\boldsymbol{\phi}_i(t) \to 0$, for $i \in \mathcal{N}$, as $t \to +\infty$. Therefore, the result in [Abdessameud and Tayebi, 2013, Lemma 2.9], leads to the conclusion that $\boldsymbol{\theta}_i$, $\dot{\boldsymbol{\theta}}_i$ are uniformly bounded and $\boldsymbol{\theta}_i(t) \to 0$, $\dot{\boldsymbol{\theta}}_i(t) \to 0$, as $t \to +\infty$ for $i \in \mathcal{N}$. Therefore, one can conclude, that $\mathbf{v}_i$, $\sum_{j=1}^{n} a_{ij}(\mathbf{p}_i - \mathbf{p}_j(t - \tau_{ij}(t)))$ are uniformly bounded and $\mathbf{v}_i(t) \to 0$ and $\sum_{j=1}^{n} a_{ij}(\mathbf{p}_i - \mathbf{p}_j(t - \tau_{ij}(t)) - \mathbf{d}_{ij}) \to 0$, as $t \to +\infty$ for $i \in \mathcal{N}$.

Furthermore, since $\mathbf{v}_i(t) \to 0$, as $t \to +\infty$, we can verify, using parts i) and iv) of Assumption 1 and $\limsup_{t\to+\infty} \tau^*(t) < \infty$, that

$$(\mathbf{p}_j - \mathbf{p}_j(t - \tau_{ij}(t))) := \int_{t-\tau_{ij}(t)}^{t} \mathbf{v}_j(s)ds \to 0,$$

as $t \to +\infty$. Consequently, we conclude that $\sum_{j=1}^{n} a_{ij}(\mathbf{p}_i - \mathbf{p}_j)$ is uniformly bounded and $\sum_{j=1}^{n} a_{ij}(\mathbf{p}_i(t) - \mathbf{p}_j(t) - \mathbf{d}_{ij}) \to 0$ as $t \to +\infty$, for $i \in \mathcal{N}$, which is equivalent to

$$(\mathbf{L} \otimes \mathbf{I}_3)\bar{\mathbf{P}}(t) \to 0,$$

as $t \to +\infty$, where $\mathbf{L}$ is the Laplacian matrix of the communication graph $\mathcal{G}$, $\otimes$ is the Kronecker product, $\bar{\mathbf{P}} \in \mathbb{R}^{3n}$ is the vector containing all $\bar{\mathbf{p}}_i := (\mathbf{p}_i - \mathbf{d}_i)$, for $i \in \mathcal{N}$, and the constant vector $\mathbf{d}_i$ can be seen as the desired position of the i-th vehicle with respect to the center of the formation, and satisfies $\mathbf{d}_{ij} = (\mathbf{d}_i - \mathbf{d}_j)$. With the condition that the communication graph contains a spanning tree, we know that $(\mathbf{L} \otimes \mathbf{I}_m)\bar{\mathbf{P}} = 0$ implies that $\bar{\mathbf{p}}_1 = \ldots = \bar{\mathbf{p}}_n$ [Ren and Beard, 2005]. As a result, $(\mathbf{p}_i(t) - \mathbf{p}_j(t)) \to \mathbf{d}_{ij}$ as $t \to +\infty$ for all $i, j \in \mathcal{N}$. □

Remark 1. Note that condition (29) is met if the control gains satisfy $\frac{\mu_i}{\kappa_i} > \max_{j\in\mathcal{N}_i}(1 + \Upsilon_{ij} + 2\Delta^{\tau}_{ij})$, with $\mathcal{N}_i = \{j : (j,i) \in \mathcal{E}\}$, from which it is clear that the control gains can be easily selected to achieve our control objectives.

5. SIMULATION RESULTS

In this section, we provide simulation results to demonstrate the effectiveness of the proposed control schemes. We consider a group of four VTOL aircraft modeled as in (1), with $m_i = 3$ kg, $\mathbf{J}_i = \mathrm{diag}(0.13, 0.13, 0.04)$ kg.m^2, for $i \in \mathcal{N} := \{1, \ldots, 4\}$. The control objective is to guarantee that the four aircraft maintain a pre-defined formation pattern, described by a square parallel to the universal $x - y$ plane. The information flow between aircraft is represented by the directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{E}, \mathcal{A})$ that contains a spanning tree with: $\mathcal{E} = \{(1,2), (1,4), (2,3), (3,1)\}$, and the adjacency matrix $\mathcal{A} = \{a_{ij}\}$, with $a_{ij} = 0.5$ for $(i,j) \in \mathcal{E}$ and zero otherwise. The function $\sigma(\cdot) = \tanh(\cdot)$, with $\sigma_b = 1$, is considered in (14).

We implement the control law in Theorem 2, with the control gains $K^p_i = \mathrm{diag}(k^p_i)$, $K^d_i = \mathrm{diag}(k^d_i)$, $(k^p_i, k^d_i, k^r_i, \lambda_i, k^\beta_i, k^q_i, k^\Omega_i) = (2, 6, 1, 1, 50, 20, 20)$, for $i \in \mathcal{N}$, $L_i = \sqrt{\lambda_i\kappa_i}$, where κ_i are obtained from the adjacency matrix $\mathcal{A}$. The time-varying communication delays are taken as: $\tau_{ij}(t) = \tilde{\tau}_{ij}\left(1 - \cos(0.25t + 1) + 0.25r(t)\right)$ sec, with $\tilde{\tau}_{1i} = 0.1$, $\tilde{\tau}_{2i} = 0.15$, $\tilde{\tau}_{3i} = \tilde{\tau}_{4i} = 0.2$, for $i \in \mathcal{N}$, and $r(t) \in [0,1]$ is a uniform random function. Note that the selected control gains satisfy condition (29). The obtained results are given in Fig. 1 and Fig. 2, which illustrate the aircraft linear velocities $\mathbf{v}_i$ and relative positions $\mathbf{p}_{1i} = (\mathbf{p}_1 - \mathbf{p}_i + \mathbf{d}_{1i})$, for $i = 2, 3, 4$. It can be seen from these figures that all aircraft stabilize to the desired formation in the presence of unknown irregular communication delays.

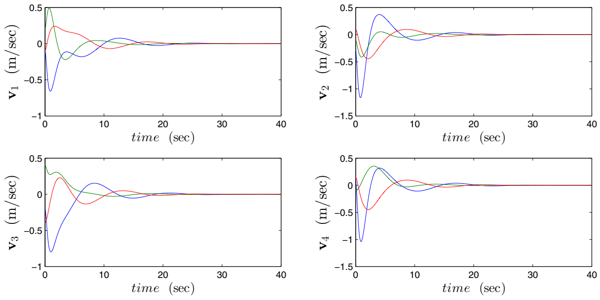

Fig. 1. Linear velocity vectors.

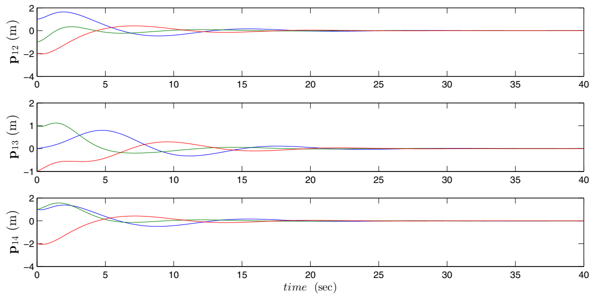

Fig. 2. Relative positions.

6. CONCLUDING REMARKS

We proposed a coordinated control scheme for multiple thrust-propelled vehicles in the presence of time-varying communication delays. To deal with the underactuation of the systems, we considered a design method presented in Abdessameud and Tayebi [2013] that enables an almost separate design for the translational and rotational dynamics of the systems. Based on the small-gain framework, we showed that the proposed distributed control scheme solves the formation control problem under the weak assumption that the directed communication graph contains a spanning tree. In addition, the control objective is achieved under sufficient conditions on the communication process – conditions that can be easily satisfied with an appropriate choice of the control gains.

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