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Centralized Robust Multivariable Controller Design Using Optimization

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Abstract: The paper deals with the determination of controller parameters for multivariable systems by means of parameter optimization. In order to reduce the computational burden, the starting point in the optimization is determined through information provided by the feedback controller parameterized using the multivariable internal model controller (IMC) computed from the plant's p/q moment approximant and the original plant's multi-loop controller parameter magnitudes. Thereafter, controller parameter optimization proceeds to completion using the MATLAB optimization toolbox which is widely available. A potency of this technique is that all performance specifications and stipulated constraints are easily accommodated in the problem formulation thus facilitating complete problem solution in one go. The technique has been found to be effective for all plants considered so far and generally produces closed loop systems with favorable characteristics when compared to systems designed by similar methods.

Keywords: Centralized controller, robust stability and performance, optimization, moment approximant.

1. INTRODUCTION

Effective methods for designing simple controllers for multivariable plants which guarantee good performance and satisfy stipulated constraints is still a topic of on-going research. One reason for this is that the plants usually have distinctive complex characteristics and each must usually be separately analyzed. Parameter optimization has been used in many situations for feedback control system design. One such method is the method of inequalities (Zakian and Al-Naib, 1973 , Taiwo, 1980, Whidborne et al., 1995, Balachandran et al., 1997 and Zakian, 2005). This method works by solving a set of inequalities and has been found to work in many situations, although it is yet to be used to design centralized controllers for complex systems involving 4*4 or higher transfer function matrices. Recently, Escobar and Trierweler (2013) designed PID controllers for large process plants by an optimization technique based on frequency response approximation. His results are compared to those of the work reported here. Other methods of optimizing the performance of the closed loop systems are based on genetic algorithms and simulated annealing. Although such methods have the property of global convergence, they are usually computationally intensive and their applications to large multivariable plants are scanty. This paper proposes a new technique for centralized multivariable controller design. Here, the initial controller parameters in the optimization are specified by using data from the feedback controller parameterized using the p/q moment approximant of the plant model and the magnitudes of the multi-loop controller parameters giving good closed loop performance for the system involving the original plant. The optimization then proceeds and is deemed to have been completed when the nominal and robust performance of the feedback system meets the control objectives. In all cases so far considered, the centralized proportional plus integral (PI) controller has proved adequate, giving favourable performance in most situations. A merit of this method is that MATLAB optimization toolbox, which is widely available, has been used. Another advantage of this method is that various constraints such as bounds on structured singular values, internal variable magnitudes or their rates of change can be directly handled during optimization. The paper is arranged as follows. In section 2, the new method is described. Applications of the new method to large multivariable systems is considered in section 3. A discussion of the results and conclusions from the work are considered in section 4.

2. DESCRIPTION OF THE NEW METHOD

The new method computes the centralized feedback controller of simple structures (typically PI, proportional plus integral controller) using parameter optimization. In order to reduce computational burden, it is recommended that a good starting point for optimization be used. This is done by studying information from the feedback controller parameterized using the IMC controller computed for, typically, the 0/1 moment approximant of the plant as well as the magnitudes of the parameters of the multi-loop PI feedback controller designed for the original plant.

Computation of p/q moment approximant

If it is desired to use the new method to design a simple feedback controller for the plant then its p/q moment approximant should be computed. This is undertaken as follows: Expand the plant model G(s) (assumed asymptotically stable) in infinite series:

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$$G(s) = \sum_{i=0}^{\infty} G_i s^i \qquad (1)$$

Here, without loss of generality, we elect to express its reduced model R(s) in the right matrix fraction form:

$$R(s) = \left(\sum_{i=0}^{p} V_i s^i\right)\left(\sum_{i=0}^{q} T_i s^i\right)^{-1}, \quad (T_q = I) \qquad (2)$$

R(s) is a p/q moment approximant at s=0 if R(s) is asymptotically stable and

$$\sum_{i=0}^{q-1} G_{\mu-i} T_i = -G_{\mu-q} \quad (q \le \mu \le p+q) \qquad (3)$$

$$V_\mu = \sum_{i=0}^{\mu} G_{\mu-i} T_i \quad (0 \le \mu \le p) \qquad (4)$$

In (3) and (4), $G_\mu = 0$, $\mu < 0$. A unique solution exists and $R(s) = G(s) + 0(s^{p+q+1})$ (5)
where the notation means that the power series expansion on both sides exist and agree up to terms of degree (p+q) inclusive. However, if expansion about s=0 does not furnish a stable p/q moment approximant, Taiwo and Krebs (1995) have shown how, generically, a stable approximant may be obtained by resorting to matching moments about more than the single point s=0. All the controllers designed in this work were based on parameterizing a classical feedback controller from the IMC controller computed from the 0/1 moment approximant of the original complex plant. Consequently, for space economy, we limit discussion to this case in the sequel.

Assume that the PI controller is desired, then the 0/1 reduced model R(s) given by

$$R(s) = V_o(Is + T_o)^{-1} \qquad (6)$$

will be computed. In order to obtain the IMC controller $\overline{Q}$, invert (6), giving,

$$\overline{Q} = R^{-1} = (Is + T_o)V_o^{-1} \qquad (7)$$

also, $Q = \overline{Q}f$ where f is the filter given by

$$f = 1/(\lambda s + 1) \qquad (8)$$

The conventional feedback controller $C_o(s)$ is given by

$$C_o(s) = \overline{Q}f(I - fI)^{-1} \qquad (9)$$

For illustration purposes, suppose G(s) is 3*3 and $\overline{Q}f = (q_{ij})/(\lambda s + 1)$, (9) simplifies to

$$C_o(s) = \frac{1}{\lambda s}\begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix} \qquad (10)$$

where $q_{ij} = \hat{V}_{oij}s + \hat{G}_{oij}$ (11)

and $\hat{V}_{oij}$, $\hat{G}_{oij}$ respectively denotes the (i,j)th element of V_o^{-1} and G_o^{-1}.

The next issue is the choice of λ. One way to determine a preliminary value is to compute the multi-loop controllers for the original plant. The order of magnitude observed here should be used to estimate the starting value of λ in order to optimize the parameters of the feedback controller, C(s). Usually it is advisable to start with a λ which gives a closed loop stable system. A detailed exposition of this procedure will be given in the next Section. If it is desired to use a PID controller, then either a ½ or 0/2 moment approximant would be used for controller parameterization.

3. ILLUSTRATIVE EXAMPLES

3.1 Example 1: Depropanizer column

The process is a depropanizer column used to separate propane from the feed that comes from a de-ethanizer column (Wang, 2003) and its transfer function is given by

$$G(s) = \begin{pmatrix} \frac{-0.26978e^{-27s}}{97.5s+1} & \frac{1.978e^{-53.5s}}{118.5s+1} & \frac{0.07724e^{-56s}}{97.5s+1} \\ \frac{0.4881e^{-117s}}{56s+1} & \frac{-5.26e^{-26.5s}}{58.5s+1} & \frac{0.19996e^{-35s}}{51s+1} \\ \frac{0.6e^{-16.5s}}{40.5s+1} & \frac{5.5e^{-15.5s}}{19.5s+1} & \frac{-0.5e^{-17s}}{18s+1} \end{pmatrix} \qquad (12)$$

The following steps should be taken in designing a controller for the plant G(s):

Step 1: Assuming a centralized controller having PI elements is to be designed, then an 0/1 approximant of (12) should be computed and the controller C(s) should be calculated. In this example, V_0^{-1} and G_0^{-1} are given respectively as

$$V_0^{-1} = \begin{bmatrix} 567.308936 & 371.82068 & 161.84048 \\ 102.65397 & 31.06735 & 17.22488 \\ 1756.6934 & 738.70682 & 285.76937 \end{bmatrix} \qquad (13)$$

$$G_0^{-1} = \begin{bmatrix} 2.01784 & 1.86435 & 1.057307 \\ 0.480027 & 0.116762 & 0.12085 \\ 7.7017 & 3.5216 & 0.59812 \end{bmatrix} \qquad (14)$$

Step 2 is to estimate the value of λ. The computation of a multiloop PI controller for the plant will assist here. The relative gain array is given by

$$RGA = \begin{bmatrix} -0.5444 & 0.9495 & 0.595 \\ 0.91 & -0.6142 & 0.704 \\ 0.634 & 0.6647 & -0.299 \end{bmatrix} \qquad (15)$$

By simultaneously computing the Niederlinski index, it was found that the best control structure is as follows: $u_2 \to y_1$, $u_3 \to y_2$, and $u_1 \to y_3$, where u_i and y_i are respectively the controlled and manipulated variables. On computing the controller dictated by this structure, it was found that the values of the proportional and integral gains in (13) and (14) would have the same order of magnitude as those of the multi-loop controller elements if $\lambda \approx 1000$. Consequently, λ=1000 was used in (10). Hence with starting value

$$C_o(s) = \frac{1}{1000}\left(V_o^{-1} + {G_o^{-1}}/{s}\right) \qquad (16)$$

and the performance index chosen as the integral of the absolute error (IAE) for step responses to unit step changes in the reference at time zero and simultaneous disturbance inputs of size 0.1 at time 2000, MATLAB optimization toolbox function *fmin* was used to compute

$$C(s) = \begin{bmatrix} 0.6316 & 1.2807 & 0.2981 \\ 0.2757 & -0.5333 & 0.1389 \\ 3.4248 & -4.7539 & -0.1580 \end{bmatrix}$$

$$+\frac{1}{s}\begin{bmatrix} 0.0040 & 0.0181 & 0.0051 \\ 0.0026 & -0.005 & 0.0022 \\ 0.0313 & -0.0205 & -0.0391 \end{bmatrix} \qquad (17)$$

Although the IAE associated with this controller is acceptable, the value of a robust performance metric, the structured singular value is greater than unity. Further computation was therefore done and the computed set of parameters (17) were used as starting point with a constraint that $\mu_{RP} < 1$, where μ_{RP} denotes structured singular value for robust performance. Here the MATLAB optimization toolbox function *fmincon* was used. The final controller computed is

$$C(s) = \begin{bmatrix} 0.7351 & 1.0706 & 0.5431 \\ 0.2776 & -0.4192 & -0.0131 \\ 3.5645 & -3.6019 & -1.6202 \end{bmatrix}$$

$$+\frac{1}{s}\begin{bmatrix} 0.0047 & 0.0119 & 0.0134 \\ 0.0025 & -0.0029 & -0.0003 \\ 0.0308 & -0.0079 & -0.0528 \end{bmatrix} \qquad (18)$$

Note that to determine the robustness of the designed controllers the input uncertainty weight used by Garrido et al. (2012), $W_u = \frac{0.009s+0.15}{0.0045s+1}$ which permits up to 15% uncertainty at low frequencies and 200% uncertainty at high frequencies attaining 100% uncertainty at a frequency of about 15 rad/min, was used. The performance weight was also chosen as in Garrido et al. (2012), namely, $W_p = \frac{s/2.75+0.00075}{s}$.
The responses of the closed loop system with the controller are displayed in fig.1. The characteristics of the feedback systems are compared with those of the closed loop system designed by Garrido et al. (2012) (fig.2) in Table I. It is found that the overall characteristics of the system designed in this work are superior to those of Garrido et al. (2012). Note in this work that μ_{RS} and μ_{NP} respectively denotes structured singular value for robust stability and nominal performance.

Table 1: Performance and robustness indices for the depropanizer

	Proposed	Garrido et al.
IAE for a step in y_1	396.1	888.9
IAE for a step in y_2	307.2	811.8
IAE for a step in y_3	194.7	701.4
TOTAL IAE	898.0	2402.0
μ_{RP}	0.9735	0.7262
μ_{RS}	0.1504	0.0399
μ_{NP}	0.8131	0.6443

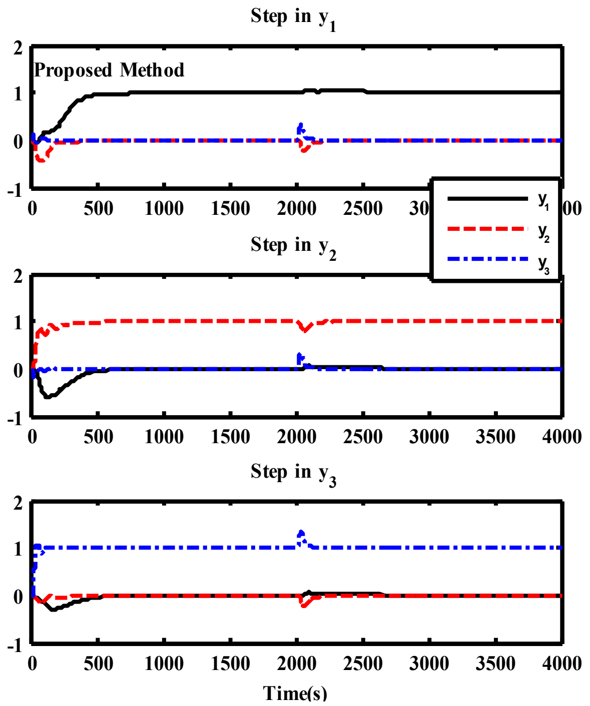

Fig.1: Depropanizer closed-loop response, this work

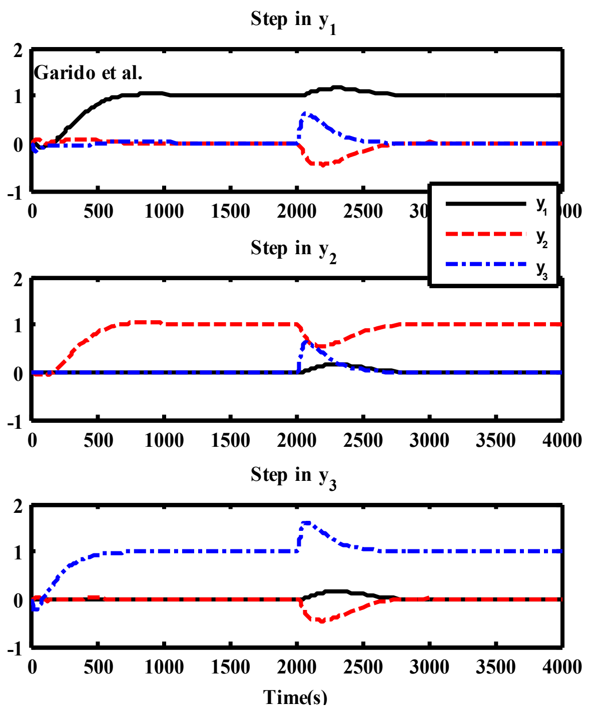

Fig.2: Depropanizer closed-loop response, Garrido et al.

3.2 Example 2: Heat integrated distillation column

Ding and Luyben (1990) presented the transfer function model for a Low-Purity heat integrated distillation column as

$$\begin{bmatrix} XB_1 \\ XD_2 \\ XS_2 \\ XB_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix}\begin{bmatrix} Q_1 \\ R_2 \\ S_2 \\ Q_2 \end{bmatrix} + \begin{bmatrix} g_{d_{11}} & g_{d_{12}} \\ g_{d_{21}} & g_{d_{22}} \\ g_{d_{31}} & g_{d_{32}} \\ g_{d_{41}} & g_{d_{42}} \end{bmatrix}\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix}$$

where, $g_{11} = \frac{-7.39e^{-s}}{(11s+1)(s+1)}$, $g_{12} = g_{13} = g_{14} = 0$

$$g_{21} = \frac{-0.11(200s+1)e^{-5s}}{(20s+1)^3},\ g_{22} = \frac{10.1e^{-s}}{(28s+1)(4s+1)}$$
$$g_{23} = \frac{1.18e^{-11s}}{(31s+1)(6s+1)},\ g_{24} = \frac{-18.3e^{-s}}{(28s+1)(5s+1)}$$
$$g_{31} = \frac{1.9e^{-2s}}{(4s+1)^2},\ g_{32} = \frac{1.7(200s+1)e^{-1.4s}}{(108s+1)(s+1)^2},$$
$$g_{33} = \frac{-3.15e^{-s}}{(3s+1)(0.3s+1)},\ g_{34} = \frac{-1.27(188s+1)e^{-s}}{(68s+1)(s+1)}$$
$$g_{41} = \frac{4.9e^{-1.6s}}{(40s+1)(3s+1)},\ g_{42} = \frac{-8.21e^{-2.5s}}{(24s+1)(3s+1)}$$
$$g_{43} = \frac{12e^{-s}}{(29s+1)(3s+1)},\ g_{44} = \frac{-19.4e^{-s}}{(26s+1)(3s+1)}$$
$$g_{d_{11}} = g_{d_{12}} = 0$$
$$g_{d_{21}} = \frac{2.42e^{-5s}}{(3s+1)(26s+1)^2},\ g_{d_{22}} = \frac{-2.47e^{-5s}}{(3s+1)(22s+1)^2}$$
$$g_{d_{31}} = \frac{0.592e^{-5s}}{(7s+1)^2},\ g_{d_{32}} = \frac{1.83e^{-6s}}{(25s+1)(2s+1)}$$
$$g_{d_{41}} = \frac{-1.51e^{-19s}}{(45s+1)(5s+1)^2},\ g_{d_{42}} = \frac{-4.52e^{-8s}}{(50s+1)(7s+1)^2}$$

Steps 1 and 2 are similar to those in the previous example except that RGA indicates a diagonal structure $D(s) = \mathrm{diag}(d_{11}, d_{22}, d_{33}, d_{44})$, which was tuned to give
$d_{11} = -0.259 - 0.0226/s$, $d_{22} = 0.248 + 0.008/s$
$d_{33} = -0.217 - 0.069/s$, $d_{44} = 0.142 + 0.005/s$.
Hence in choosing $C0(s)$, the values of these diagonal parameters were retained while every other element in V_0^{-1} and G_0^{-1} was divided by 10. In other words, $\lambda \cong 10$ was used for this example. Additionally, note that all the elements in row and column 1 of V_0^{-1} and G_0^{-1} apart from element (1,1) were zero. Upon applying the optimization toolbox in MATLAB, the eventual controller giving a system with acceptable servo and regulator characteristics is given by

$$C(s) = \begin{bmatrix} -0.3374 & 0 & 0 & 0 \\ 0 & 0.2854 & -0.0860 & 0.3356 \\ 0 & 0.3014 & -0.2001 & 0.2616 \\ 0 & -0.0237 & -0.0122 & 0.1962 \end{bmatrix}$$

$$+\frac{1}{s}\begin{bmatrix} -0.0285 & 0 & 0 & 0 \\ 0 & 0.0108 & 0.0825 & 0.0032 \\ 0 & 0.0072 & -0.0145 & 0.0096 \\ 0 & 0.0008 & 0.0442 & 0.0023 \end{bmatrix} \quad (19)$$

The adequacy of this controller in following reference changes and rejecting disturbance changes of 5% in Z_1 and Z_2 can be verified from figures 3 and 5 respectively. This is compared with controller $C_1(s)$, with PID elements, in Escobar and Trierweiler (2013) as shown in figs.4 and 5. The other attractive characteristics of the closed loop system are tabulated in Tables II and III. The uncertainty weight used for robustness analysis was chosen as $W_u = \frac{s+0.15}{0.5s+1}$. This permits up to 15% uncertainty at low frequency and 200% at high frequency attaining 100% uncertainty at a frequency of about 1rad/min. The performance weight was chosen as $W_p = \frac{s/2.25+0.04}{s}$. Here, we have specified peak sensitivity, M_s=2.25 (with an implication that $GM \geq 1.8$ and $PM \geq 25.68^0$) and zero steady state error. Note that the Escobar and Trierweller controller lacks performance robustness.

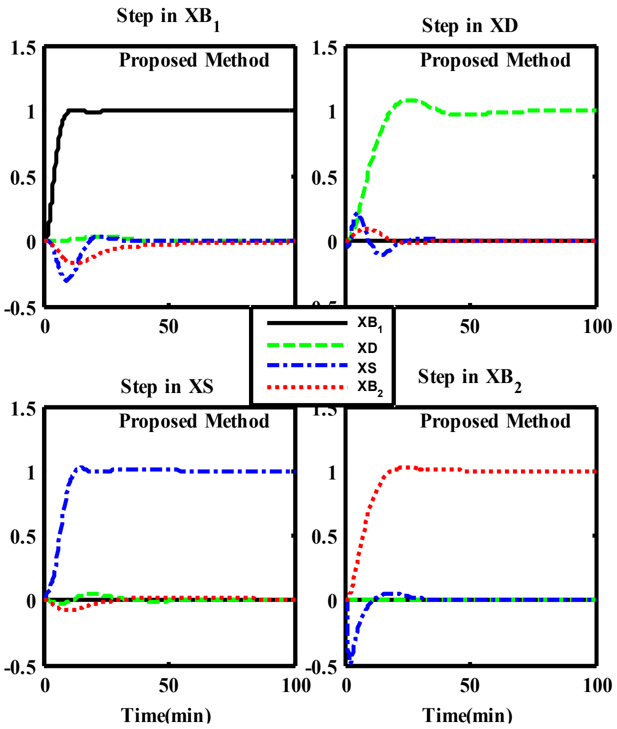

Fig.3: Servo performance for the Ding and Luyben column

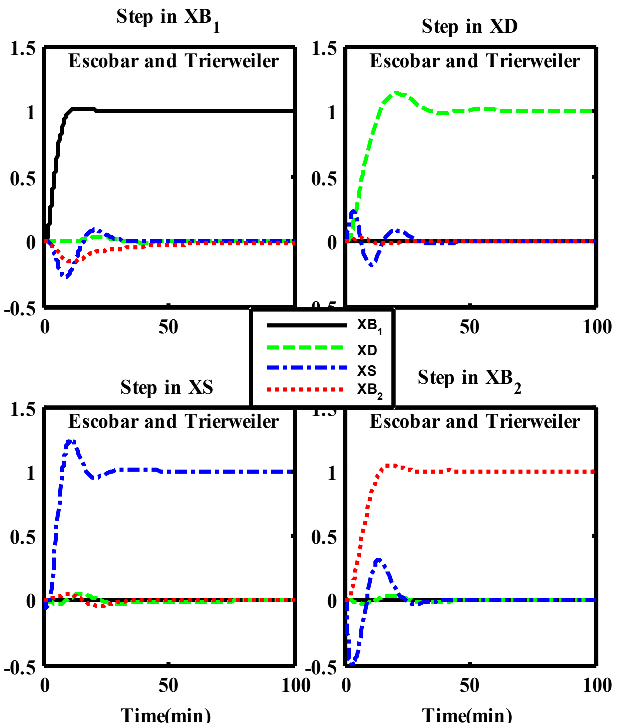

Fig.4: Servo performance for the Ding and Luyben column

Table III:Regulatory performance indices

	Proposed	Escobar & Trierweller
Total IAE for a step in Z_1	47.33	31.50
Total IAE for a step in Z_2	55.16	36.38

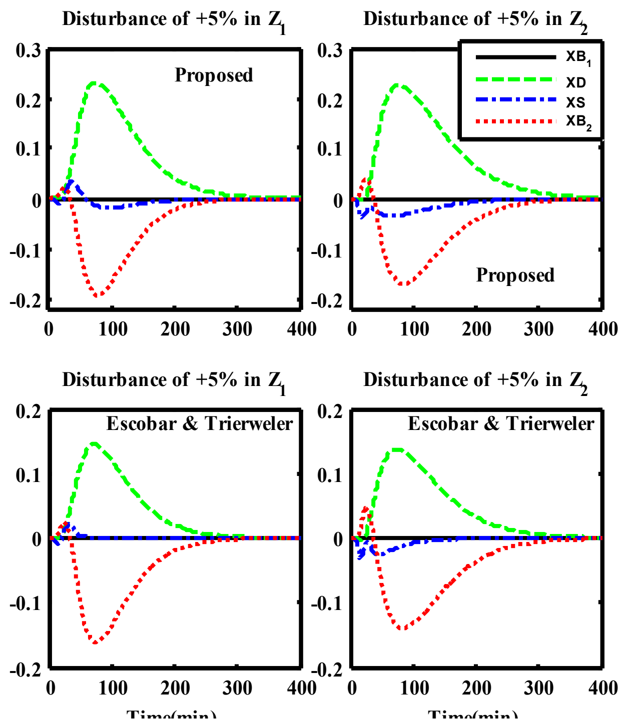

Fig.5: Regulatory performance for the Ding and Luyben column (z_1 and z_2 of size 5%).

Table II: Performance and robustness indices for the Ding and Luyben column

	Proposed	Escobar & Trierweler
IAE for a step in XB_1	13.24	12.99
IAE for a step in XD	14.94	12.65
IAE for a step in XS	10.24	9.67
IAE for a step in XB_4	11.26	13.90
TOTAL IAE	49.68	49.21
μ_{RP}	0.9648	1.8016
μ_{RS}	0.2144	0.2034
μ_{NP}	0.8749	1.1305

3.3 Example 3: Alatiqi distillation column

The Alatiqi column system used here is taken from Garrrido et al. (2012). It is a 4 by 4 system modelled by the transfer function matrix:

$$G = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix}, \text{ where}$$

$$g_{11} = \frac{2.22e^{-2.5s}}{(36s+1)(25s+1)},\ g_{12} = \frac{-2.94(7.9s+1)e^{-0.05s}}{(23.7s+1)^2}$$

$$g_{13} = \frac{0.017e^{-0.2s}}{(31.6s+1)(7s+1)},\ g_{14} = \frac{-0.64e^{-20s}}{(29s+1)^2},\ g_{21} = \frac{-2.33e^{-5s}}{(35s+1)^2},$$

$$g_{22} = \frac{3.46e^{-1.01s}}{32s+1},\ g_{23} = \frac{-0.51e^{-7.5s}}{(32s+1)^2},\ g_{24} = \frac{1.68e^{-2s}}{(28s+1)^2}$$

$$g_{31} = \frac{-1.06e^{-22s}}{(17s+1)^2},\ g_{32} = \frac{3.511e^{-13s}}{(12s+1)^2},\ g_{33} = \frac{4.41e^{-1.01s}}{16.2s+1},$$

$$g_{34} = \frac{-5.38e^{-0.5s}}{17s+1},\ g_{41} = \frac{5.73e^{-2.5s}}{(8s+1)(50s+1)},\ g_{42} = \frac{4.32(25s+1)e^{-0.01s}}{(50s+1)(5s+1)}$$

$$g_{43} = \frac{-1.25e^{-2.8s}}{(43.6s+1)(9s+1)},\ g_{44} = \frac{4.78e^{-1.15s}}{(48s+1)(5s+1)}$$

Steps 1 and 2 are similar to those of the preceding examples. A good starting point in this case was found by retaining the exact value of the diagonal controller for the multiloop structure. For the off-diagonal elements of the centralized controller, a value of λ of approximately 18 or greater was used, yielding

$$C_o(s) = \begin{bmatrix} 0.5200 & 0.7000 & -0.1500 & -0.3500 \\ -0.6000 & 0.4500 & -0.06500 & -0.1500 \\ 1.1500 & -0.37500 & 0.6000 & 0.5500 \\ 0.7500 & -0.3650 & -0.1500 & 0.5300 \end{bmatrix} + \frac{1}{s}\begin{bmatrix} 0.0100 & 0.0500 & 0.0040 & -0.0050 \\ 0.0100 & 0.0300 & 0.0015 & -0.0050 \\ 0.1000 & -0.0450 & 0.0350 & 0.0150 \\ 0.0800 & 0.0450 & 0.0100 & 0.0100 \end{bmatrix} \quad (20)$$

Upon using the *fmin* function, the controller in (21) was computed which gives a system with satisfactory nominal and robust performance.

$$C(s) = \begin{bmatrix} 0.8050 & 1.3115 & 0.1698 & -0.9105 \\ -0.1109 & 0.4034 & 0.0415 & 0.0936 \\ 1.5406 & 0.0430 & 1.8670 & 1.5954 \\ 1.3153 & 0.2395 & -0.0700 & 1.4410 \end{bmatrix} + \frac{1}{s}\begin{bmatrix} 0.0098 & 0.0521 & -0.0002 & -0.0119 \\ -0.0117 & 0.0266 & 0.0002 & -0.0137 \\ 0.0384 & 0.0553 & 0.1393 & 0.0221 \\ 0.0341 & 0.0538 & 0.0082 & 0.0302 \end{bmatrix} \quad (21)$$

The responses of the system with this controller are given in fig.(6) and some of the dynamic properties are listed in Table IV. Again, it is observed that the system obtained in this work is superior to the one in Garrido et al. (2012) displayed in fig.7. One observation is that we have used a more realistic uncertainty weight here rather than the one given by Garrido et al (2012). This is because among other considerations, we believe that uncertainty usually increases with frequency rather than the contrary impression given in their uncertainty weight. It is worth noting that our computed controller meets the required constraint $\mu_{RP}<1$ even when their original weight is used. The new uncertainty weight used for the robust analysis is given by $W_u = 0.15\frac{5s+1}{0.5s+1}$. However, the same performance weight as theirs is used, namely, $W_p = \frac{s/2.6+0.001}{s}$.

Table IV: Performance and robustness indices for the Alatiqi column

	Proposed	Garrido et al.
IAE for a step in y_1	59.95	47.62
IAE for a step in y_2	51.16	77.44
IAE for a step in y_3	8.67	14.32
IAE for a step in y_4	28.55	32.56
TOTAL IAE	148.3	172.0
μ_{RP}	0.9728	3.4457
μ_{RS}	0.2615	0.2890
μ_{NP}	0.7486	1.6601

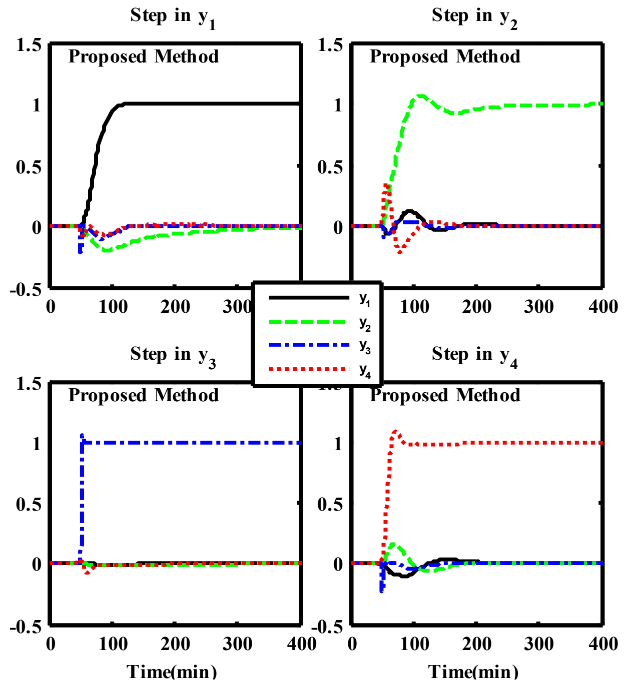

Fig.6:Servo responses for the Alatiqi column

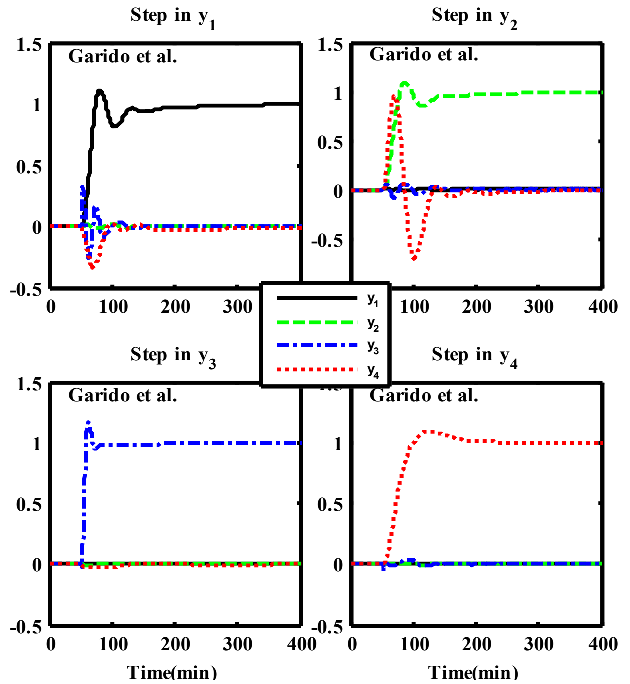

Fig.7: Servo responses for the Alatiqi column

4. DISCUSSION AND CONCLUSION

Simple centralized controllers have been computed for three benchmark process systems. It is gratifying that in all cases, PI elements suffice in producing closed-loop systems meeting desired performance and constraints. A novel technique for estimating the starting values of the controller parameters ensures that the starting closed-loop system is stable as well as substantially reduce computational burden. It also helps in determining the signs of the controller elements. Although optimization can be done in the Simulink environment, results in this work were obtained using the MATLAB environment and I_{MN} approximants (Zakian, 1975) (M=14, N=22) was used to compute system responses during optimization. The final results were confirmed by simulation using Simulink.

The design of simple feedback controllers through an initial IMC parameterization using a moment approximant is novel and has worked effectively for all the plants considered so far with many of the plants being matrices of orders 3 or 4. These are the highest transfer function matrix dimensions of plants we found in the current literature.

During controller parameter optimization, several performance indices were tried. It was found that either the integral of the squared error criterion or the integral of the absolute error criterion worked well for most examples. The integral of time multiplied by the absolute error criterion produced closed loop systems with relatively small settling times but large overshoots and large transient interactions.

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REFERENCES

Balachandran, R. and Chidambaram, M. (1997). Decentralized control of crude unit distillation towers, *Comput. Chem. Eng.*, 21, 783-786.

Escobar, M. and Trierweiler, J.O. (2013). Multivariable PID controller desing for chemical processes by frequency response approximation. *Chem. Eng. Sci.*, (88):1-15.

Garrido, J., Vazquez, F. and Morilla, F. (2012). Centralized multivariable control by simplified decoupling. *Journal of process control*, (22):1044-1062.

MATLAB optimization tool box, *mathwoks*.

Taiwo, O. (1980) Application of the method of inequalities to the multivariable control of binary distillation columns, *Chem. Eng. Scie.*, (35):847-858.

Taiwo, O. and Krebs, V. (1995) Multivariable system simplification using moment matching and optimization. *IEE Proc. Control Theory Application*, Vol.142 pp103-110.

Wang, Q.W.(2003) Decoupling control, lecture notes in control and information sciences,vol.285, springer-verlag.

Whidborne, J.F., Murad, G., Gu, D.W., and Postlewaite (1995) Robust control of an unknown plant, the IFAC '93 benchmark, *Int. J. Control*, (61):589-640.

Zakian, V. and Al-Naib, U.M.T. (1973). Design of dynamical and control systems by the method of inequalities, *Proc. IEE*, vol.120, pp1421-142.

Zakian, V.(1975). Properties of I_{MN} and J_{MN} approximants and applications to numerical inversion of Laplace transforms and initial value problems. *J. Math. Anal. Appl.*, 50, 191-222.

Zakian, V. (2005). Control Systems Design: A new Framework. *Springer*, London.