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Optimal Sensor Trajectories for Mobile Underwater Target Positioning with Noisy Range Measurements

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Abstract: There is considerable interest in reducing the number of sensors/beacons involved in underwater positioning/navigation systems since this has the potential to drastically reduce the costs and the time spent in deploying, calibrating, and recovering acoustic equipment at sea. Motivated by these considerations, we address the problem of single underwater target positioning based on acoustic range measurements between the target and a moving sensor at the sea surface. In particular, the goal of the present work is to compute optimal geometric trajectories for the surface sensor that will, in a well defined sense, maximize the range-related information available for underwater target positioning and tracking. To this effect, the Fisher Information Matrix and the maximization of its determinant are used to determine the sensor trajectory that yields the most accurate positioning of the target, while the latter describes a preplanned trajectory. It is shown that the optimal trajectory depends on the velocity of the sensor, the velocity and trajectory of the target, the sampling time between measurements, the measurement error model, and the number of measurements used to compute the FIM. Simulation examples illustrate the key results derived.

Keywords: Autonomous underwater vehicles, Fisher information matrix, Cramer-Rao Bound, optimization, positioning, single tracker.

1. INTRODUCTION

In the last decade there has been a surge of interest worldwide in the development of marine technologies, in general, and in autonomous underwater vehicles (AUVs), in particular, for ocean exploration and exploitation. The interest in these vehicles resides in the important fact that they are capable of roaming the oceans freely, collecting relevant data at an unprecedented scale without the need for explicit human interaction. In fact, for reasons that have to do with autonomy, flexibility, and the new trend in miniaturization, AUVs are steadily emerging as excellent tools to execute many demanding tasks at sea that include pipeline inspection, seabed surveying, and archaeological research, to name but a few.

Central to the operation of some classes of AUVs is the availability of reliable underwater positioning systems capable of positioning one or more vehicles simultaneously, based on information received on-board a support ship or an autonomous surface vehicle. The info thus obtained can be used to follow the state of progress of a particular mission or, if reliable acoustic modems are available, to relay it as a navigation aid to the navigation systems existent on-board the AUV. For a review on underwater positioning systems and related work on this area the reader is referred to Moreno-Salinas et al. (2013) and references therein.

A large body of work described in the literature exploits the geometric configuration of a number of sensors in order to estimate the position of a target from range or bearings measurements. These measurements are obtained at different locations to determine the target position. In this paper an alternative approach is adopted: a single mobile sensor measures the successive ranges to an underwater target and, by exploiting its own spatial diversity, acquires information to compute the position of the latter. Our objective is to compute the optimal motion or trajectory of a single sensor that will, in a well defined sense, maximize the range-related information available for underwater target positioning and tracking. To this effect, we assume that the range measurements are corrupted by white Gaussian noise, the variance of which depends on the distance between the two objects that exchange range data. It is interesting to remark that in spite of the importance and relevance of the optimal sensor placement problem, the topic is far from being studied exhaustively when a single sensor is used.

The rationale for this study stems from the fact that, from a practical standpoint, there is considerable interest in reducing the number of beacons involved in acoustic navigation/positioning systems, as they usually involve deployment, calibration and recovery time which is costly and time consuming. For these reasons, the concept of underwater localization using ranges to a single beacon/transponder has received increasing attention in the marine robotics community. The challenge of reducing the number of sensors involved in underwater acoustic systems has been addressed previously in the literature in the different, yet related context of underwater navigation (in contrast with positioning, which is the core problem considered in this paper). In fact, there is a vast number of references that tackle the underwater navigation problem by assuming that only ranges from a moving vehicle to a single beacon/transponder installed at a known position are available (single beacon navigation). For a review on early work in this area and relevant references that address the problem from diverse perspectives the reader is referred to Alcocer (2009).

The dual of the above problem is that of tracking an underwater target with a single range measuring device. We recall that an important question in underwater target positioning with sensor networks is that of finding the minimum number of beacons that can be used to perform an underwater target positioning task. In a practical situation, because the target is known to be beneath the sea surface, only 3 non colinear range measurements are needed. Instead of a static surface sensor network, one may envision a surface vehicle that, by moving along adequate trajectories, exploits its spatial diversity while measuring ranges to the underwater platform in order to determine the position of the latter, as we do in the present paper. An early reference to this problem can be found in Been et al. (1991) where target motion analysis (TMA) with respect to an unknown marine platform using sonar measurements is discussed. Other previous results in this challenging area go back to the work of Passerieux & Capel (1998) where optimal control theory is used to determine the course of a constant speed observer. Other interesting references on the subject are Dandach et al. (2009) that presents a continuous time adaptive localization algorithm for a mobile agent where only distances are used to estimate the localization of an static source, Fallon et al. (2010) that describes the experimental implementation of an online algorithm for cooperative localization of AUVs supported by an autonomous surface craft, Batista et al. (2011) where the problem of navigation and source localization based on range measurements to a source is tackled, and Arrichiello et al. (2011) where the problem of observability of the relative motion of two AUVs equipped with velocity and depth sensors, and inter-vehicle ranging devices, is studied. In Scherbatyuk & Dubrovin (2012) a number of algorithms to position an AUV based on range measurements obtained with a single acoustic sensor at the ocean surface are also described. Finally, in Moreno-Salinas et al. (2013b) the authors study algorithms to position a static underwater target using range measurements assuming constant covariance of the measurement noise.

Motivated by this circle of ideas, in this paper we seek to characterize, from a theoretical standpoint, the optimal trajectories that a single range sensor must execute, in order to maximize the accuracy with which a target can be tracked and localized. From a practical standpoint, this will provide guidelines as to how one should operate in practical scenarios. The key contributions of the present paper are threefold: i) a methodology is proposed to compute the optimal trajectory that a single sensor must execute once the number of measurements is fixed, ii) a general solution is obtained numerically for positioning and tracking of a mobile underwater target describing straight lines and circumferences, and iii) a distance-dependent measurement error is considered to include the important fact that measurement errors may grow in a nonlinear manner with the distance between the sensor and the target.

The paper is organized as follows. Section 2 describes the computation of the Fisher Information Matrix (FIM) for the problem at hand when a noise model with distance-dependent covariance is considered. In Section 3 the positioning problem is formulated and the assumptions made for the computation of its optimal trajectories are established. Section 4 contains the description of the algorithm adopted to compute the trajectories. Simulation examples are included for different target trajectories. Finally, Section 5 contains the conclusions.

2. FISHER INFORMATION MATRIX WITH DISTANCE-DEPENDENT MEASUREMENT NOISE

There is a wide range of error sources that can affect underwater range measurements: depth-dependent speed of propagation of sound in the water, physical propagation barriers, ambient noise, and degrading signal-to-noise ratio as the distance between the two objects increases, to name but a few. For analytical tractability, it is usually assumed that the measurement errors are corrupted by Gaussian, zero mean, additive noise with constant covariance. To better capture physical reality, in this paper we assume that the measurement noise is modelled by a zero-mean Gaussian process where the covariance depends on the distance between the two objects that exchange range data. Other references where similar measurement error descriptions can be found are Jourdan & Roy (2008) and Moreno-Salinas et al. (2013). Stated mathematically,

$$\omega = (I + \eta\delta(r^{\gamma})) \cdot \omega_0 \tag{1}$$

where ω is the measurement noise, ω_0 is a zero mean Gaussian process $N(0, \Sigma_0)$ with $\Sigma_0 = \sigma^2 \cdot I$, I is the identity matrix, r is range, and η and γ are the modelling parameters for the distance-dependent noise component. In the above, δ is the operator *diag*, that converts a vector into a square diagonal matrix whose diagonal components are the array elements. With these assumptions, the measurement noise covariance is given by

$$\Sigma = \sigma^2 \left(I + \eta\delta(r^{\gamma})\right)^2 \tag{2}$$

Let $q = [q_{ix}, q_{iy}, q_{iz}]^T$ be the position of an arbitrary target at sampling time i, $p_i = [p_{ix}, p_{iy}, p_{iz}]; i = 1, 2, .., n$, the position of the acoustic ranging sensor also at sampling time i, and ω_i the corresponding measurement noise. Further let r_i be the distance between the target position q_i and the $i-th$ sensor position at the time at which the acoustic reply from the target is received by the sensor, see Section 3 for further details. With this notation, the measurement model adopted is given by

$$\hat{r}_i(q) = \|(p_i - q_i)\| + \omega_i = r_i + w_i \quad (3)$$

To compute the optimal trajectory for the ranging sensor, a numerical algorithm that optimizes an indicator based on an appropriately defined Fisher Information Matrix is computed. Stated in simple terms, the FIM captures the amount of information that measured data provide about an unknown parameter (or vector of parameters) to be estimated. Under known assumptions, the FIM is the inverse of the Cramer-Rao Bound matrix (abbv. CRB), which lower bounds the covariance of the estimation error that can possibly be obtained with any unbiased estimator, see Van Trees (2001). Thus, "minimizing the CRB" may yield (by proper estimator selection) a decrease of uncertainty in the parameter estimation. We therefore focus on the computation of the FIM. In particular, we maximize the FIM determinant to determine the optimal acoustic sensor trajectory that maximizes the expected positioning accuracy. Following standard procedures, the FIM for a moving target q_i, with $i = 1, \cdots, n$, where i represents the different points at which the ranges are measured, is computed from the likelihood function,

$$\mathcal{P}_q = \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma|^{\frac{1}{2}}} \exp\left\{-\frac{1}{2}\left(y - h(q)\right)^T \Sigma^{-1} \left(y - h(q)\right)\right\} \quad (4)$$

where $y = [r_1, r_2, ..., r_n]^T$ consists of n measured ranges, $h(q) = r(q)$ are the actual ranges, $q = [q_1, q_2, ..., q_n]$ are the successive target positions, and n is the number of range measurements with which the FIM is computed. Notice the important fact that if one knows in advance the type of trajectory that the target follows as well as its speed, but not its initial position, we can define the different target positions as a function of the initial position $q_1 = q_0$, the target speed V_t, and its orientation $\alpha_t(t)$. Thus, $q_i = q_0 + f_i(V_t, \alpha_t(t))$ and we may estimate the initial target position q_0 from the n measured ranges. Thus, taking the logarithm of (4), computing its derivative with respect to q_0, and computing its expected value, the FIM becomes:

$$FIM = C\delta(r)^{-1}\delta\left(r^{\gamma-1}\right)\Sigma^{-1}\delta\left(r^{\gamma-1}\right)\delta(r)^{-1}C^T \quad (5)$$

where $C = (q1_n^T - p) \in \Re^{3xn}$, $1_n \in \Re^{nx1}$ is a vector of 1s, and p is the vector of sensor positions, the latter being defined in $\Re^{3xn}$. Once the FIM is computed, then the CRB becomes $CRB = FIM^{-1}$. In this context, the optimal sensor trajectory is obtained by maximizing a quantity related to the determinant of the FIM. In a practical situation, the remaining $n-1$ target positions may be obtained with the estimate of q_0 and the known target trajectory.

3. PROBLEM FORMULATION

For a given moving target tracking problem, the optimal sensor trajectory will depend strongly on the constraints imposed by the task (e.g. maximum number of measurements used for the computation of the FIM, target trajectory, or the type of sensor that can be used) and the environment (e.g. ambient noise). It is clear that an inadequate sensor trajectory may yield large positioning errors, thus, it is important to define the constraints and assumptions considered in this paper to solve the problem at hand. The problem framework is defined as follows:

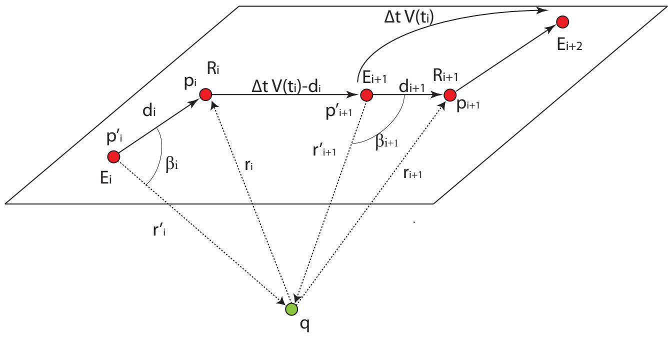

Fig. 1. Problem Setup for a Static Target

- The unmanned surface vehicle (USV) that carries the acoustic ranging sensor must localize a single moving target that describes straight lines and circumferences with constant speed $V_t(t) = V_t$.
- The initial USV position is arbitrary for it does not condition the final optimal solution but only conditions the time to reach it.
- The target position is estimated with a fixed number of measurements n, i.e., the FIM is computed using n range measurements.
- The acoustic signals are emitted at constant intervals of time Δt and there exists a delay between the emission by the pinger on board the USV and the reply from the target. Therefore, the reception of the acoustic reply takes place at a different point from the emission point, see Fig 1.
- The sensor, or USV, moves with constant speed $V(t) = V$ that is always larger than that of the target.
- It is considered that the measured ranges r_k used to define the FIM correspond to the time for the acoustic signal to travel from the target back to the sensor.

Some of the above issues are illustrated in Figure 1. Notice how the sensor (red points) emits the acoustic signal at time E_i and the reply from the target (green points) is received by the sensor at time R_i, with d_i being the distance between the two above points. The distance d_i depends on the velocity of sound in the water, the sensor speed V, target speed V_t, and on the range distances r'_i and r_i associated with the times that it takes for the acoustic signal to travel from the emitter to the receiver underwater and back to the emitter, respectively. The emission point E_i defines the point p'_i, the reception point R_i defines the $i-th$ measurement point p_i, and the range distance measured for the FIM computation is considered to be r_i, i.e., the distance between the target position q_i and the sensor position p_i. Thus, with the above notation $p_i = R_i$ and q_i corresponds to the target position at the moment of the reply by the underwater target. In this theoretical setting it is considered that r'_i and r_i, and therefore p'_i and p_i, are known, so we can define analytically the distance d_i that separates the emission and reception points. Let c_s be the speed of sound in the water. Then,

$$\frac{d_i}{V} = \frac{r'_i}{c_s} + \frac{r_i}{c_s} \quad (6)$$

Moreover, if β is the angle defined by r'_i and d_i, from the theorem of cosines it follows that $r_i^2 = {r'}_i^2 + d_i^2 + 2d_i r'_i \cos(\beta)$ with

$$\beta = \arccos\left(\frac{\langle (q - p'_i)(p'_i - p_{i-1})\rangle}{r'_i \cdot (\Delta t V(t_i) - d_{i-1})}\right), \qquad (7)$$

where <> denotes the inner product operation, see Fig. 1. It follows from (6) that

$$\frac{d_k}{V} - \frac{r'_i}{c_s} = \frac{\sqrt{{r'}_i^2 + d_i^2 + 2d_i r'_i \cos(\beta)}}{c_s} \qquad (8)$$

Taking the square of both sides and rewriting the above equation yields

$$d_i = \frac{2r'_i}{c_s}\left(\frac{\cos(\beta)}{c_s} - \frac{1}{V}\right)\left(\frac{1}{c_s^2} - \frac{1}{V^2}\right)^{-1} \qquad (9)$$

so that the new measurement points may be explicitly defined considering only the past known trajectory information and the orientation angles α_i that the surface sensor must take at the R_i (also p_i) points.

In this work we study the (ideal and seemingly artificial) situation where the successive positions or trajectory of the target are known in advance in order to characterize and fully understand the types of optimal solutions that the sensor should follow in this ideal case. In a practical and real situation, these points or trajectory are only known with uncertainty, and this uncertainty must be explicitly considered. In this case, an iterative process can be envisioned in which an initial estimate of the target trajectory is used to compute the corresponding optimal sensor trajectory, and once the mission unfolds the information acquired by the sensor can in turn be used to refine the underwater target trajectory, after which the cycle repeats itself. Clearly, having the means to generate, for an assumed trajectory of the target, the corresponding optimal trajectory of the surface ranging device (sensor) is also advantageous in this case. See Moreno-Salinas et al. (2013) for a discussion of this circle of ideas in the case of positioning with sensor networks.

4. OPTIMAL TRAJECTORY COMPUTATION

In this section we describe the numerical algorithm to compute the trajectory that a moving surface sensor must follow in order to maximize the accuracy with which a moving underwater target describing different trajectories can be localized. The computation of the optimal trajectory is done recursively by maximizing the FIM determinant for a given number of measurements points, which the sensor must track. Therefore, the resultant optimal trajectory to be followed by the sensor is composed of partial trajectories of n points, so that for each n range measurements the positioning accuracy of the underwater target is maximized, and the new target position estimates may be obtained every $n \cdot \Delta t$ seconds. In a practical situation we would estimate the initial target position of the corresponding partial trajectory, and then the posterior target positions would be defined given the prior knowledge about the target's motion.

Once the mission is running and an initial estimation of the target position is available, possibly with a large error, it is necessary to determine the trajectory that the single tracker must follow in order to maximize the positioning accuracy. For given values of the sensor speed and sampling time, it is easy to derive the analytical expression that provides the next optimal points because the new FIM determinant will only have as unknown parameters the angles α_i, which define the successive heading angles of the sensor for the optimal trajectory. Since the sensor speed V and the sampling time Δt are known, the sensor positions for which the corresponding ranges are measured can be written as $p_{i+1} = p_i + [\xi\cos(\alpha_{i+1}),\ \xi\sin(\alpha_{i+1}),\ q_z]$ with $\xi = (V\Delta t - d_i + d_{i+1})$; and d_i, d_{i+1} defined as in Section 3. Thus, the problem to solve can be defined as that of finding

$$\alpha_i^* = \arg\max_{\alpha_i} |FIM| \qquad (10)$$

for $i = 1, \cdots, n$. The solution may be computed analytically from the derivatives of the FIM determinant with respect to the angles α_i, $i = 2, \cdots, n$, that determine the distance and relative orientation of two consecutive measurements. It is clear, by considering that the initial sensor position is known (i.e, the last measurement point of the previous trajectory of n points), that we have $n-1$ variables α_i, $i = 2, \cdots, n$, and $n-1$ derivatives with respect to these angles α_i, so that a system of equations with the same number of equations and unknowns is obtained. The complexity of this approach resides in the fact that the process to obtain the solution of this equation system is complex and tedious. Moreover, we must resort to numerical methods to solve it. Therefore, these derivatives are used for a gradient optimization algorithm and can be computed by decomposing the determinant in terms of its adjoints, yielding

$$\frac{\partial|FIM|}{\partial\alpha_i} = \sum_{j,k}^{3}(-1)^{j+k}|Adj_{j,k}(FIM)| \cdot \frac{\partial FIM(j,k)}{\partial\alpha_i} \qquad (11)$$

where $|Adj_{j,k}(FIM)|$ is the determinant of the adjoint matrix of the FIM with respect to the element (j,k). Making the equations obtained from (11) equal to 0 we can find the angles that make the FIM determinant maximum. It can be seen that although (11) depends only on the angles α_i and an analytical solution may be defined, the computation of the optimal solution is not immediate. Therefore, the optimal solution is obtained with a gradient optimization algorithm using the Armijo rule. The sensor trajectory is recomputed and the FIM updated each new n range measurements.

At this point it is interesting to comment that if the target is static and the values of V, Δt and n are the optimal ones for a given target depth so that the maximum theoretical FIM determinant can be obtained, the same solution defined in Moreno-Salinas et al. (2011) for surface sensor networks is recovered.

4.1 Simulation Examples

Some examples of optimal sensor trajectories are now studied addressing three different scenarios. The range measurements with which the FIM is computed in each scenario are $n = 5$ and $n = 8$, both with constant and distance-dependent covariance, for comparison purposes, but the procedure would be very similar for any number of measurements. For each iteration of the algorithm explained above, the first point of the new trajectory is the last one in the previous iteration, so that for each iteration the next $n-1$ measurement points are computed. Moreover, because there is no guarantee that the cost function in (10) is convex, Monte Carlo simulations will be carried out together with a gradient optimization algorithm to define the optimal sensor trajectory. The orientation angles that can be taken by the sensor's heading throughout its motion are limited, i.e., we consider that the vehicle orientation can change by a maximum angle of 45 deg between two consecutive measurement points. For all the examples, 100 trajectories of n points are computed, and a constant speed $V = 2\ m/s$ and a sampling time $\Delta t = 5\ s$ are considered, with a constant target depth of 50 m. In the distance-dependent examples, the added error parameters are set to $\eta = 0.01$ and $\gamma = 1$.

Example 1: Static target positioning. In this first example we consider a static target. In Figure 2(a) the trajectory followed by the sensor is shown for constant covariance error, with $n = 5$ (red) and $n = 8$ (green). Notice how the optimal trajectories are very similar to circumferences centered at the target position, with the radius depending on the number of range measurements used. The average FIM determinants obtained are $|FIM|_{avg1} = 4.3313 \cdot 10^4$ for $n = 5$, and $|FIM|_{avg2} = 8.7444 \cdot 10^5$ for $n = 8$, with the standard deviations $SD_1 = 3.402 \cdot 10^3$ and $SD_2 = 1.1437 \cdot 10^5$, respectively. These imply a deviation of about 10 percent with respect to the average value, and thus, the accuracies obtained are almost constant for the successive optimal partial trajectories. In Figure 2(b), the same problem is studied but in this case we consider a distance-dependent covariance error. Notice again how the optimal trajectories are very similar to circumferences, and how the radii of the latter increase with the number of points used to compute the FIM. The average determinants for this case are $|FIM|_{avg1} = 3.4395 \cdot 10^3$ for $n = 5$, and $|FIM|_{avg2} = 6.3654 \cdot 10^4$ for $n = 8$, with standard deviations $SD_1 = 355.7804$ and $SD_2 = 1.0706 \cdot 10^4$, respectively. Again, these standard deviations are less than the 10 percent of the average values.

Example 2: Straight line target tracking. This second example deals with a moving target that follows a straight line path. Figure 2(c) shows, for constant covariance, the trajectories followed by the sensor while it tracks the target, the latter advancing at a speed of $V_t = 0.2\ m/s$. Notice how these trajectories are spiral-like curves around the successive target positions. It is clear that these spirals are the deformation of the circumferences that would be obtained if the target were static, as in the previous example. The optimal FIM determinants obtained oscillate between a maximum and a minimum value due to the optimal spiral trajectory, that comes closer and further from the target following the target motion. The average FIM determinants obtained are $|FIM|_{avg1} = 3.5149 \cdot 10^4$ for $n = 5$, and $|FIM|_{avg2} = 7.6982 \cdot 10^5$ for $n = 8$, with the standard deviations $SD_1 = 8.8551 \cdot 10^3$ and $SD_2 = 2.0563 \cdot 10^5$, respectively. Notice how the average FIM determinants are very similar to those obtained for a static target. However, the standard deviations are larger, approximately 20 percent of the average values, due to the motion of the target. In Figure 2(d) the same situation is analysed for distance-dependent covariance error. In this case the average determinants are $|FIM|_{avg1} = 2.7687 \cdot 10^3$ for $n = 5$, and $|FIM|_{avg2} = 5.5492 \cdot 10^4$ for $n = 8$, with the standard deviations $SD_1 = 800.2237$ and $SD_2 = 1.5141 \cdot 10^4$, respectively. Again the FIM determinants are very similar to those obtained for a static target although the standard deviations are larger too.

Example 3: Circular target tracking. In this final example the target is considered to be moving following a circular trajectory with a speed $V_t = 0.4\ m/s$. Figure 2(e) shows the trajectories followed by the sensor while it tracks the target for the case of constant measurement noise covariance. Notice that the final trajectories are again spiral-like curves around the target trajectory. The average determinants are $|FIM|_{avg1} = 2.1037 \cdot 10^4$ for $n = 5$, and $|FIM|_{avg2} = 5.6331 \cdot 10^5$ for $n = 8$, with the standard deviations $SD_1 = 1.2243 \cdot 10^4$ and $SD_2 = 2.7349 \cdot 10^5$. Notice also how the FIM determinants oscillate due to the sensor spiral trajectories, although their average values are close to the optimal of the previous examples. The standard deviations are again larger due to the more complicated target trajectory and the larger target speed, which force the sensor to move back and forward to obtain adequate positioning accuracy. In Figure 2(f), the optimal trajectories are shown for distance-dependent covariance, and again the sensor describes spirals around the target path. The average FIM determinants for this case are $|FIM|_{avg1} = 2.0067 \cdot 10^3$ for $n = 5$, and $|FIM|_{avg2} = 4.7785 \cdot 10^4$ for $n = 8$, with the standard deviations $SD_1 = 1.1446 \cdot 10^3$ and $SD_2 = 1.9456 \cdot 10^4$, respectively, that are very similar to those of Examples 1 and 2.

It is important to stress that when the number of points used to compute the FIM increases, the trajectory described by the sensor is located further away from the target, i.e., larger spirals are obtained. Moreover, the accuracy obtained (i.e., FIM determinant) is larger too. It is clear that the accuracies obtained for the constant covariance cases are larger than those for distance-dependent covariance, since the measurement error does not increase with the distance. The accuracy, and the optimal trajectory, also depend on the target depth but this is not shown in detail due to space limitations. We can therefore conclude that the approach proposed yields an efficient method to compute the optimal trajectory that a sensor should track in order to maximize the precision with which the position of a moving target can be determined

5. CONCLUSIONS

In this paper, the problem of single underwater target positioning and tracking by a single surface sensor was studied. The analysis of optimal sensor trajectories exploited the spatial and temporal diversity of the measurements taken by the surface sensor. The problem considered was that of positioning a target moving along different trajectories. Three different scenarios were studied, i) a static target, ii) a moving target following a straight line trajectory, and iii) a moving target following a circular trajectory. The optimal trajectories were obtained by maximizing the determinant of an appropriate FIM for a given number

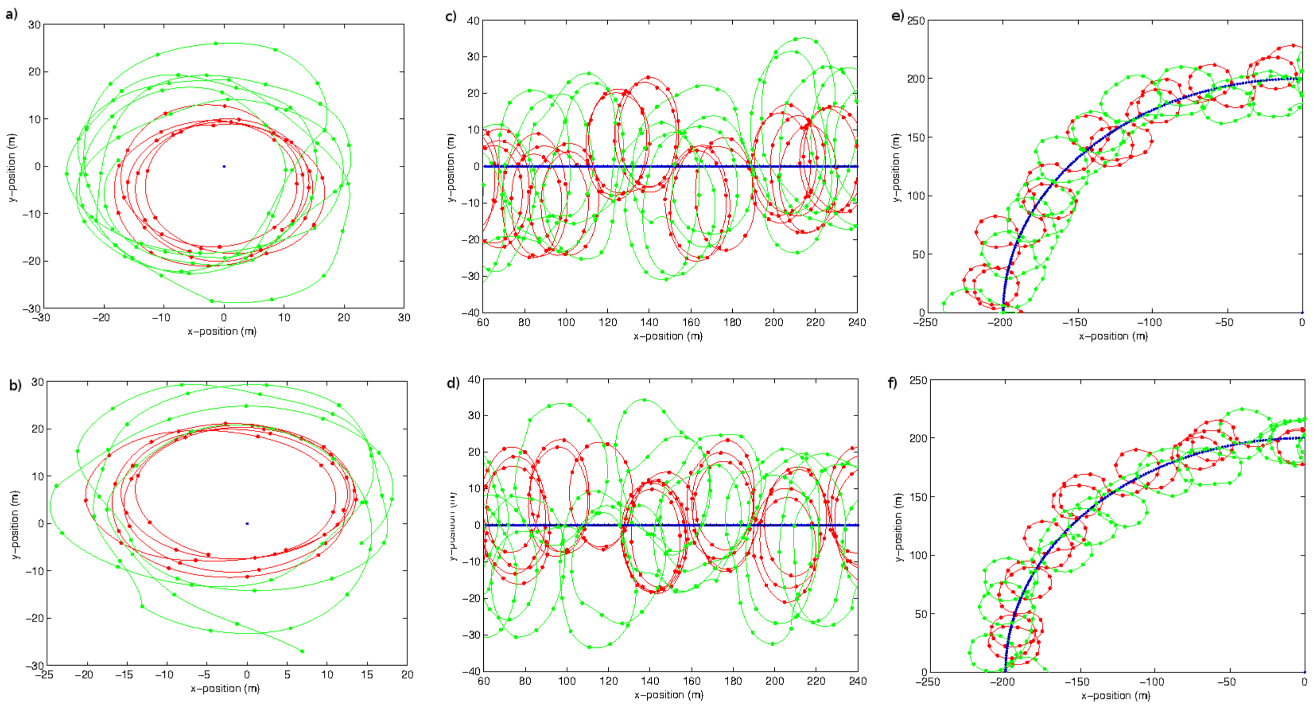

Fig. 2. Optimal sensor trajectories to position a static target, a) and b), a target following a straight line path, e) and f), and a target following a circular path, e) and f), assuming constant covariance (upper figures) and distance dependent covariance (lower figures) of the measurement noise; for $n = 5$ (red) and $n = 8$ (green).

of range measurements, i.e., the approach optimized the corresponding trajectory for the n range measurements considered for the FIM computation, and the whole trajectory was defined by joining the consecutive optimal trajectories of n points. The examples showed that the approach proposed holds good potential to be used in practice (in a moving-horizon type of approach) to compute the sensor trajectory that will yield optimal target localization.

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