Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Incident parameter scheduled freeway traffic control - a ramp meter approach

Azita Dabiri [*] **Balazs Kulcsar** [*]

[*] *Department of Signals and Systems, Chalmers University of Technology, SE-412 96, Gothenburg, Sweden (e-mail: (azitad,kulcsar)@chalmers.se)*

Abstract: A novel model based local ramp metering method is presented in the paper by means of incident scheduled freeway traffic control solution. First, second order macroscopic freeway model is used with appropriate incident parametrization to describe eventual and unattended traffic variation caused by off-nominal traffic conditions (e.g. accidents). These traffic anomalies are captured by adequate model parameters, i.e. incident parameters that can be on-line estimated Dabiri and Kulcsár (2013). The paper is motivated by incident scheduled ramp-meter solution to encounter real-time incident parameter information. The main idea is to use local freeway control solution triggered by available incident parameter values. The proposed approach is local in the sense of considering only non-coordinated ramp meter solution, first. Furthermore, we apply locally optimal (linearized) control solution to satisfy throughput maximization objective. The formal controller synthesis involves parameters to correct, compensate the effect of incidents. The proposed method is evaluated and compared to other existing approach by using simulation environment.

Keywords: traffic incident, ramp metering, robust control, linear matrix inequality

1. INTRODUCTION

During the past few years, advance freeway traffic control methodologies have been developed to meet the continuously increasing traffic demand related problems of both rural and metropolitan areas Papageorgiou et al. (1989), Hegyi et al. (2009), Luspay et al. (2011). Regular high-demand related congestions (morning or evening peak hours) or incurrent congestion generated by abnormal traffic conditions like traffic incidents (accidents, sudden change in weather or road conditions) are the most common reasons for capacity deficiency in freeways and therefore have been in the focus of traffic oriented research for several years. In order to balance the negative impact of traffic congestion on throughput, advanced freeway traffic control solutions based on accurate traffic models e.g. Papageorgiou et al. (1989) have been proposed. Amongst available modeling frameworks, macroscopic models aim at expressing the evolution of aggregated traffic variables such as mean speed, traffic density and traffic flow by averaging the behavior of individual vehicles. Due to its efficient computational complexity, this modeling framework, i.e. macroscopic, is devoted to traffic management solution and particularly traffic flow control approaches. One of the most attractive freeway traffic control algorithms is ramp metering, e.g. Papageorgiou and Kotsialos (2002). ALINEA Papageorgiou et al. (1997) has been developed for local as well as coordinated ramp control by means of feedback. Hegyi et al. (2005) applied predictive control method for flow control by the duality of ramp control and dynamic speed limit signs. Neural network based ramp metering is communicated in Zhang and Ritchie (1997).
While analyzing the causes of freeway congestions, traffic incidents have been considered as an important factor Kwon et al. (2006). The congestion created by incident then might be propagated backward for kilometers and block upstream off-ramps and on-ramps. Moreover, as the prevention point of view, effectiveness of ramp metering has been found to reduce the crash potential on freeway applications Leea et al. (2006).

In this paper, model-based incident corrective traffic control is targeted and hence we include information about off-nominal traffic conditions at the modeling step already, i.e. define incident parametrization in the nominal macroscopic model framework. From modeling point of view, a few number of existing macroscopic models take abnormal traffic conditions into consideration. In this regards, Wang et al. (2009) proposed an adaptive estimation of some core model parameters by means of Extended Kalman Filtering. In Sanwal et al. (1996), incident is defined as a partial lane blockage. Recently, in Dabiri and Kulcsár (2013), a two-parameter approach has been communicated to capture both driver and geometry related causes of eventual incidents. In Dabiri and Kulcsár (2013) analysis of the incident flow model as well as measured data based validation (by on-line parameter estimation) have been presented. These parameters are relative in the sense of quantifying the effect of incident relative to the nominal freeway model parameters. Online available traffic flow information can be used to schedule eventual control strategy. In other words, we can use the above referred incident parameters Dabiri and Kulcsár (2013) to schedule traffic controllers. Therefore, the resulting traffic model including incident parameters calls for incident corrective control strategies, since the inclusion of available incident related information might improve not only traffic safety but also capacity. In this paper, in order to guarantee capacity maximization under changing incident levels, we introduce scheduled robust optimization solution using ramp meter, which minimizes the effect of demand changes on predefined performance output. The purpose of the performance output is then to keep the traffic density close to the critical value in order to reach the maximum capacity. Instead of using the nonlinear freeway model with incident parameters, we apply a set of linearized model evaluated at the critical density points. The set of incident triggered linear models contain incident free and incident corrupted scenarios, scheduled by incident parameters. To ensure optimal induced $\mathcal{L}_2$ norm minimization Scherer and Weiland (2005), a scheduled controller is synthesized, not only for the incident free but also for the entire polytope of incident affected linear models. Since, incident parameters are real-time available, we propose to develop incident scheduled ramp metering on the basis of Linear Parameter Varying concepts, e.g. Shamma and Athans (1991); Luspay et al. (2012).

The first part of the paper details the incident inclusion in macroscopic model representation and presented in Section 2.1. The obtained and incident parametrized second order traffic flow model is linearized around the critical density point in Section 2.2. Derivation of local model information results in an incident parameter dependent set of linear time invariant models. In Section 2.3, we suggest to use these parameters as available scheduling parameters for the synthesis of robust controller. A robust state-feedback controller structure scheduled by incidents is designed to meet incident corrective traffic control objectives. The numerical solution is carried out by means of Semi-Definite Programing, by using Linear Matrix Inequalities Boyd et al. (1994). A simulation case study is provided to compare the novel method with other model-based traffic control alternative in Section 3.

2. PROBLEM FORMULATION

2.1 Incident corrupted nonlinear model

The advantage of macroscopic models lies in describing high level traffic behavior (disregarding low level, microscopic vehicle phenomena) for traffic management and control solutions. In macroscopic models, the studied variables are mean-valued ones formulated as aggregated variables. Traffic behavior can then be modeled via three main quantities as traffic density (ρ), space-mean speed (v) and traffic volume (q). Among the available macroscopic models in the literature, discretized Payne-Witham model (or METANET) Payne (1971); Whitham (1974); Papageorgiou et al. (1989) is well-known for its capability in modeling various traffic flow with high accuracy as well as its prosperity in real-time traffic applications Papageorgiou et al. (1989); Hegyi (2004). By using the latter framework, traffic incident inclusion has recently been proposed in Dabiri and Kulcsár (2013). Here, the incident effect is captured by two distinct, segment and time varying exogenous parameters $\alpha(k)$ and $\beta(k)$ called incident parameters. $\beta(k)$ describes the relative changes of equilibrium speed component when incident occurs, $\alpha(k)$ parameter represents the driver's reaction as a response to incidents in terms of headway selection. By using the results of Dabiri and Kulcsár (2013), we propose to include these incident parameters in the macroscopic freeway model for ramp meter purposes. Consequently, the following equations are obtained to describe the dynamical behavior of the traffic flow in segment i of a λ-lane freeway stretch in time step kT, where T is the sampling time. The first equation formulates the vehicle conservation law by means of traffic density dynamic as:

$$\rho_i(k+1) = \rho_i(k) + \frac{T}{L_i}\big(q_{i-1}(k) - q_i(k) + r_i(k) - f_i(k)\big) \quad (1)$$

where

$$q_i(k) = \lambda \rho_i(k) v_i(k), \quad (2)$$

with λ corresponds to number of lanes. $r_i(k)$ and $f_i(k)$ in (1) are the on- and off-ramp flows of segment i. The off-ramp flow can be assigned as a ϵ portion of flow exiting the previous segment $i-1$:

$$f_i(k) = \epsilon(k) q_{i-1}(k) \quad (3)$$

Evolvement of average traffic speed in segment i can be described as an empirical equation. Incident parameters are included in this momentum equation by:

$$\begin{aligned} v_i(k+1) = v_i(k) &+ \frac{T}{\tau}\Big(\beta_i(k) V_e\big((1+\alpha_i(k))\rho_i(k)\big) - v_i(k)\Big) \\ &+ \frac{T}{L_i} v_i(k)\big(v_{i-1}(k) - v_i(k)\big) \\ &- \beta_i(k)\big(1-\alpha_i(k)\big)\frac{\eta T}{\tau L_i}\frac{\rho_{i+1}(k) - \rho_i(k)}{\rho_i(k)+\kappa} \\ &- \frac{\delta T r_i(k) v_i(k)}{\lambda L_i(\rho_i(k)+\kappa)}, \end{aligned} \quad (4)$$

where $V_e(\rho)$ is called equilibrium speed and is defined as in equation (5). Equilibrium speed accounts for the density-dependent speed that drivers feel relax to choose as their preferred velocity and in this note, it's used in the following analytical form,

$$V_e(\rho_i(k)) = v_{free} exp\Big[-\frac{1}{a}\Big(\frac{\rho_i(k)}{\rho_{cr}}\Big)^a\Big], \quad (5)$$

with ρ_{cr} standing for the critical density which together with a and v_{free} is model parameter. Maximum throughput in freeway segment(s) is achieved when the density reaches the critical value. The last part in the freeway model describes the queue dynamics in on-ramps where $d_{o,i}(k)$ represents the flow demand that enters the ramp i while $l_i(k)$ denotes the queue length.

$$l_i(k+1) = l_i(k) + T\big(d_{o,i}(k) - r_i(k)\big). \quad (6)$$

Finally, τ, δ, κ, η, ϵ, are constant and nominal model parameters, that can be off-line identified Cremer and Papageorgiou (1981). For a given freeway stretch, Interconnection of required number of segments described as

(1) to (6) is used to describe the overall freeway system dynamics as:

$$x(k+1) = \phi\big(x(k), u(k), d(k), \alpha(k), \beta(k)\big), \quad (7)$$

where the system's states are the density and speed variables of all required n segments along with the queue length of the on-ramps:

$$x(k) = [\rho_1(k), v_1(k), \rho_2(k), v_2(k), \\ \dots \rho_n(k), v_n(k), l_1(k), l_2(k), \dots, l_n(k)]^T. \quad (8)$$

The freeway system dynamics have the input of on-ramp volumes as a controlled variable by:

$$u(k) = [r_1(k), r_2(k), \dots r_n(k)]^T. \quad (9)$$

Clearly, if there is no onramp in segment i, the corresponding ramp flow is zero. Demand in (7) which is denoted by $d(k)$ is in general the boundary condition as well as each on-ramps' demand given by:

$$d(k) = [q_0(k), v_0(k), \rho_{n+1}(k), d_{o,1}(k), d_{o,2}(k), \dots, d_{o,n}(k)]^T. \quad (10)$$

Finally, the dependency of ϕ on the segment-wise incident parameters are lumped into the vectors $\alpha(k)$ and $\beta(k)$.

Remark 1. If $\alpha_i(k) = 0$ and $\beta_i(k) = 1$ then, the segment in question is unaffected by incidents, i.e. incident-free case. $0 \le \beta_i(k) \le 1$ is a normalized incident parameter describing the effect of incident by means of relative degradation of the equilibrium speed, e.g. road capacity. $0 \le \alpha_i(k) \le 1$ represents a relative headway change of the average drivers.

2.2 Local incident parameter dependent models

Traffic model derived in the form of (7) of a nonlinear model requires complex, e.g. nonlinear control techniques. Alternatively, we can reduce the domain of validity of the previously introduced nonlinear model by only focusing on the important local traffic conditions. In this paper, for model-based controller design the aforementioned nonlinear system will be linearized around the selected nominal points which are the critical density and speed. Nominal values for u_i has been chosen as $(r_{i,min}+r_{i,max})/2$. Assuming that $v_0(k)$ and $\rho_{n+1}(k)$ are equal to $v_{free}(k)$ and $\rho_{cr}(k)$ respectively, the local disturbance vector can be defined as $\Delta w(k) = [\Delta q_0(k), \Delta d_{o,1}(k), \Delta d_{o,2}(k), \dots, \Delta d_{o,n}(k), 1]^T$ to generate the linearized system in the form of:

$$\Delta x(k+1) = A(\theta)\Delta x(k) + B\Delta u(k) + E(\theta)\Delta w(k). \quad (11)$$

A, B and E are matrices of appropriate sizes and Δ represent the deviation of the variable from the operating point. Since the linearization is not around the steady state, some constant terms will be generated which are taken into account by appending a constant 1 in the $\Delta w(k)$ vector (i.e. formulate the non-steady linearization part as a disturbance). Introduce, the following parameters $\theta(\alpha,\beta) = [\theta_1(\alpha,\beta), \theta_2(\alpha,\beta), \theta_3(\alpha,\beta)]$. Thus:

$$\theta_1(\alpha,\beta) = \beta(1+\alpha)^a exp\Big(\frac{-1}{a}(1+\alpha)^a\Big), \\ \theta_2(\alpha,\beta) = \beta(\alpha-1), \\ \theta_3(\alpha,\beta) = \beta exp\Big(\frac{-1}{a}(1+\alpha)^a\Big). \quad (12)$$

Such that A be defined as an affine form of:

$$A(\theta) = A_0 + \theta_1 A_1 + \theta_2 A_2. \quad (13)$$

In a similar manner, $E(\theta)$ can be written in the following affine form:

$$E(\theta) = E_0 + \theta_3 E_1. \quad (14)$$

Dependency of the matrices A and E on $\theta(\alpha,\beta)$ requires specific control design method. In the sequel, we assume that the α and β and consequently $\theta(\alpha,\beta)$ are real-time measured (or estimated) and hence we incorporate incident parametrization into locally optimal ramp meter solution. The controller is local in the sense that it does take into account linearized model-based solution to the originally nonlinear problem statement.

Remark 2. Note that the obtained class of system fits into the discrete-time model framework of affine linear parameter dependent systems. Affine parameter structure (i.e. linearity in parameter dependency) is proposed here to characterize later the locally optimal ramp law with finite number of inequalities, i.e. avoid approximations. Accordingly, direct model parametrization in α and β is a valid alternative for incident corrective traffic control solution.

2.3 Controller design

Incident parameter scheduled structure of the system dynamics in (11) necessitates a scheduled control strategy to include the corrective effect of incident parametrization. Assume to measure system states [1], the goal is to design a parameter dependent state feedback controller which not only makes the local system dynamics stable but also minimizes the effect of the demand disturbance over the performance output (locally defined as keeping the density in the critical value). As it is explained before, the throughput of the freeway is maximized if the density is kept at the critical value ρ_{cr}. Therefore the local performance output is defined as:

$$\Delta z(k) = C\Delta x(k), \quad (15)$$

where C selects the density of one or more freeway segments. Predefined performance specification can be well formulated for all incident parameter values and fits into the scheduled induced $\mathcal{L}_2$ norm minimization framework well as

$$\sup_{0<||\Delta w(k)||<\infty} \frac{||\Delta z(k)||}{||\Delta w(k)||} \le \gamma \quad (16)$$

[1] The proposed control policy is extendable to other controller structure, such as output feedback. The concept of incident parameter inclusion remains the same however.

with state feedback $u(k) = K(\theta)x(k) = \sum_{j=1}^{n_p} K_j\theta_j(k)x(k)$, with n_p standing for the number of incident-related parameters (θ). The closed-loop ramp metered system can be given by:

$$\begin{aligned}\Delta x(k+1) &= (A(\theta) + BK(\theta))\Delta x(k) + E(\theta)\Delta w(k), \\ &= A_{cl}(\theta)\Delta x(k) + E(\theta)\Delta w(k) \quad (17) \\ \Delta z(k) &= C\Delta x(k). \quad (18)\end{aligned}$$

Condition (16) is satisfied if the closed-loop system (17) is dissipative with supply function $s = \gamma^2||\Delta w(k)||^2 - ||z(k)||^2$ Scherer and Weiland (2005). Meaning that for a storage function $V(x(k))$:

$$V(x(k+1)) - V(x(k)) \leq s. \quad (19)$$

Choosing quadratic function $V(x(k)) = x(k)^T Px(k)$ with constant and symmetric positive definite P. After using (15), the relation (19) can be rewritten as:

$$\begin{bmatrix}\Delta x(k) \\ \Delta w(k)\end{bmatrix}^T H \begin{bmatrix}\Delta x(k) \\ \Delta w(k)\end{bmatrix} \prec 0, \quad (20)$$

where

$$H = \begin{bmatrix} A_{cl}^T(\theta)PA_{cl}(\theta) - P + C^T C & A_{cl}^T PE(\theta) \\ E^T(\theta)PA_{cl}(\theta) & E^T(\theta)PE(\theta) - \gamma^2 I_{nw}\end{bmatrix}. \quad (21)$$

equation (20) has to be valid for all Δx and Δw implying:

$$H \preceq 0. \quad (22)$$

After rearranging terms in H, we get:

$$\begin{bmatrix} P & 0 \\ 0 & \gamma^2 I_{nw}\end{bmatrix} - \quad (23)$$
$$\begin{bmatrix} A_{cl}^T(\theta)P & C^T \\ E^T(\theta)P & 0\end{bmatrix}\begin{bmatrix} P^{-1} & 0 \\ 0 & I_{nz}\end{bmatrix}\begin{bmatrix} PA_{cl}(\theta) & PE(\theta) \\ C(k) & 0\end{bmatrix} \succeq 0,$$

Apply the Schur-complement and the aforementioned inequality turns into:

$$\begin{bmatrix} P & 0 & A_{cl}^T(\theta)P & C^T \\ 0 & \gamma^2 I_{nw} & E^T(\theta)P & 0 \\ PA_{cl}(\theta) & PE(\theta) & P & 0 \\ C & 0 & 0 & I_{nz}\end{bmatrix} \succeq 0. \quad (24)$$

Note that (24) is nonlinear in the unknown variables. Pre and post multiplying the inequality with the term

$$\begin{bmatrix} P^{-1} & 0 & 0 & 0 \\ 0 & I_{nw} & 0 & 0 \\ 0 & 0 & P^{-1} & 0 \\ 0 & 0 & 0 & I_{nz}\end{bmatrix}, \quad (25)$$

and introducing $P^{-1} = Q$ and $K(\theta)Q = Y(\theta)$, results in the following LMI:

$$\begin{bmatrix} Q & 0 & Y(\theta)^T B^T + QA^T(\theta) & QC^T \\ 0 & \gamma^2 I_{nw} & E^T(\theta) & 0 \\ A(\theta)Q + BY(\theta) & E(\theta) & Q & 0 \\ CQ & 0 & 0 & I_{nz}\end{bmatrix} \succeq 0. \quad (26)$$

Finally, the robust local ramp meter control can be obtained as the results of the optimization problem:

$$\min_{\gamma, Y(\theta), Q} \gamma^2, \\ \text{subject to}(26) \quad (27)$$

The application of (27) represent a closed-loop robust control which guarantees the stability of the system. Since dependency of A and E on parameters are affine, it is only enough to satisfy the aforementioned conditions over a finite number of inequalities.

Remark 3. The proposed controller scheme is a scheduled and robust state-feedback solution. If incident parameter reconstruction is unavailable, the previous idea is still applicable by designing α and β independent, i.e. constant state-feedback gain solution robustly accounting for all incident parameters variation.

The obtained robust ramp meter law returns with a incident parameter scheduled control solutions aiming at locally reaching main stream throughput maximization.

3. CASE STUDY

In order to evaluate the proposed control strategy, a traffic scenario is selected to represent a hypothetical freeway stretch depicted in Fig. 1. The selected 3-lane freeway configuration consists of 3 segments, each with length of $L = 0.5km$. A one-lane onramp is connected to the second segment (segment 0 and 4 are depicted in the figure to illustrate the boundary conditions).

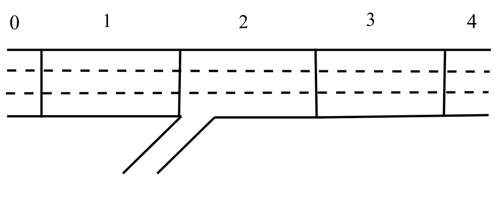

Fig. 1. Schematic representation of the freeway in the case study

Table 1. Nominal model parameters

$V_{free}(\frac{km}{h})$	$\rho_{cr}(\frac{veh}{kmlane})$	a	$\tau(s)$	$\kappa(\frac{veh}{kmlane})$	$\eta(\frac{km^2}{h})$
110	30	2.8	16	10	20

In Table 1, the nominal model parameters are summarized.

The traffic scenario is selected such that an incident happens at time $t = 35min$ which is modeled by a change in the incident parameters. For around 50 minutes, α and β are selected as 0.3 and 0.6 respectively before they go back to their nominal values which corresponds to resolved incident. Moreover, based on the selected profile, it is assumed that incident parameters reach their stationary value rapidly(see Fig. 2). The selected values correspond to closure of one lane due to incident Dabiri and Kulcsár (2013). Meanwhile, at $t = 50$, a change in the main stream traffic demand occurs. It starts to increase to 4500 from 3600 veh/h. The purpose of the case study is to evaluate the robustness of the proposed incident scheduled controller with respect to change in the demand. The optimal feedback gain scheduled by incident parameters has been obtained as a result of the minimization in (27). The achieved γ for the case study is 0.87. This accordingly means that the peak ratio of the 2-norm demand disturbance and performance is over bounded by $\gamma = 0.87$. Simulation results from the incident parameter dependent controller is compared to the performance of the scheduled LQR controller as well as the nominal LQR controller [2]. The gain of the scheduled LQR controller has been scheduled by incident parameters. Proper weights of the LQR cost function is chosen by trial and error method. However, controllers are designed on the basis of local state-space information, i.e. linear model, they are validated via the nonlinear simulation model.

Comparative simulation results of the three controllers along with no-control case are depicted in Fig. 3-4. As it is shown in the figures, the ramp meter can effectively balance the demand change and incurrent congestions caused by an incident. Figures also demonstrate that in comparison with the nominal LQR, controllers which are scheduled by incident parameters perform better when incident occurs at $t = 35$. Moreover, the proposed robust controller over-perform the LQR controller under traffic incident condition and mainstream demand changes. First, it suppresses the the effect of demand and incident over the main stream segment's capacity. Second, by taking into account incident parameter information, the controller inherently inject the available incident information in the closed-loop. On the other hand, although between $t = 35$ and $t = 50$, the scheduled-LQR controller takes the incident parameter information into account, it can not react properly when the change in the demand occurs at $t = 50$. Therefore, robust controller can over perform in rejecting the demand changes that is a natural property of the selected control design framework. Note that the performance of the LQR controllers highly depend on the chosen weights. For different incident effected traffic scenario, different weight settings may be required.

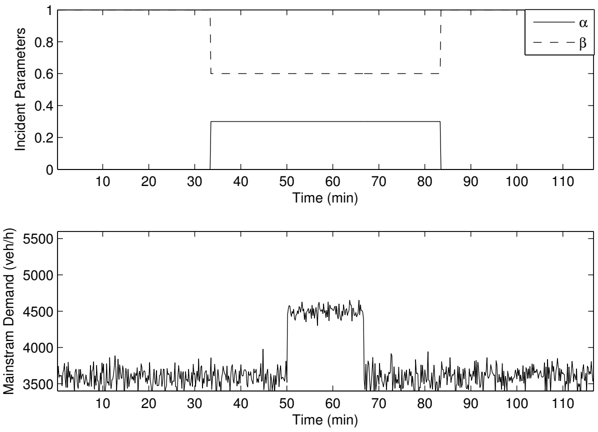

Fig. 2. Changes in incident parameters and mainstream demand

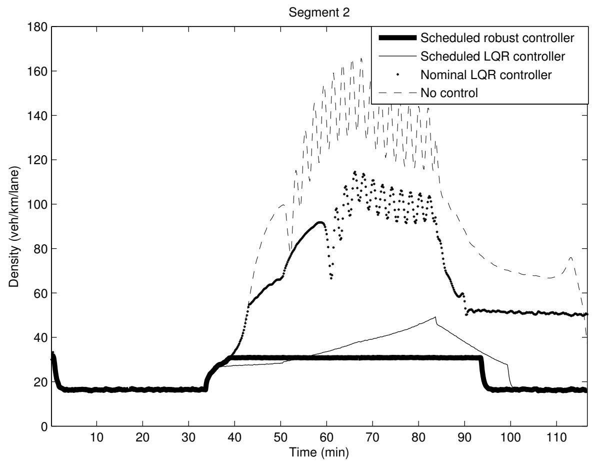

Fig. 3. Comparison of the traffic density in the second segment with different controller setup

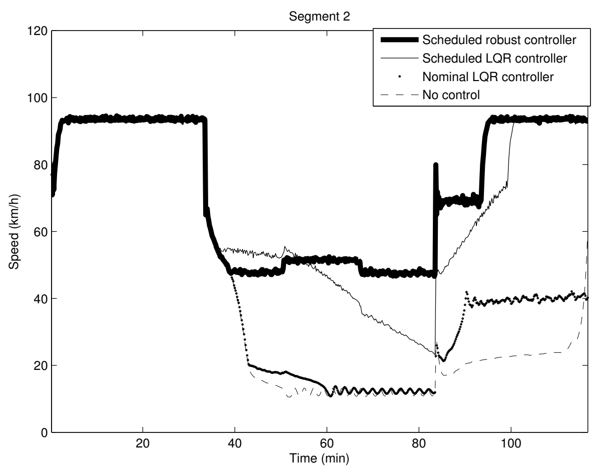

Fig. 4. Comparison of the traffic mean speed in the second segment with different controller setup

[2] Our intention is to compare the novel method to an already existing solution (similar to Papageorgiou et al. (1990)).

4. CONCLUSION

The paper reports an incident corrective control scenario for local ramp meter solution. The suggested traffic control technique is based on linear but incident scheduled models that have been used as a basis for incident scheduled robust controller design. The controller goal is to ensure local capacity maximization scheduled by incident parameters and aims at rejecting the effect of demand disturbance. Simulation scenario has been applied to underline the importance of incident scheduled ramp metering.

First, ramp-constraints are not directly considered in the methodology. Future work will focus on the use of different constrained controller structures as well as on coordinated incident tolerant control solutions. Furthermore, incident parameter estimation together with the robust controller will be addressed to encounter the case that incident parameters are not known or measured.

REFERENCES

Boyd, S., El Ghaoui, L., Feron, E., and Balakrishnan, V. (1994). *Linear Matrix Inequalities in System and Control Theory*, volume 15 of *Studies in Applied Mathematics*. SIAM.

Cremer, M. and Papageorgiou, M. (1981). Parameter identification for a traffic flow model. *Automatica*, 17(6), 837–843.

Dabiri, A. and Kulcsár, B. (2013). Freeway traffic incident reconstruction - a bi-parameter approach. *Submitted to Transportation Research Part C.*

Hegyi, A. (2004). *Model Predictive Control for Integrating Traffic Control Measures.* PhD dissertation, delft university of technology.

Hegyi, A., Bellemans, T., and DeSchutter, B. (2009). Freeway traffic management and control. In R.A. Meyers (ed.), *Encyclopedia of Complexity and Systems Science*, 3943–3964. Springer New York.

Hegyi, A., Schutter, B.D., and Hellendoorn, H. (2005). Model predictive control for optimal coordination of ramp metering and variable speed limits. *Transportation Research Part C*, 13(3), 185209.

Kwon, J., Mauch, M., and Varaiya, P. (2006). Components of congestion: Delay from incidents, special events, lane closures, weather, potential ramp metering gain, and excess demand. In *Proceedings of the 85th annual meeting of the Transportation Research.*

Leea, C., Hellingab, B., and Ozbay, K. (2006). Quantifying effects of ramp metering on freeway safety. *Accident Analysis and Prevention*, 38, 279288.

Luspay, T., Kulcsr, B., van Wingerden, J., Verhaegen, M., and Bokor, J. (2011). Linear parameter varying identification of freeway traffic models. *Control Systems Technology, IEEE Transactions on*, 19(1), 31–45.

Luspay, T., Péni, T., and Kulcsár, B. (2012). Constrained freeway traffic control via linear parameter varying paradigms. In J. Mohammadpour and C.W. Scherer (eds.), *Control of Linear Parameter Varying Systems with Applications*, 461–482. Springer US.

Papageorgiou, M., Blosseville, J., and Hadj-Salem, H. (1989). Macroscopic modelling of traffic flow on the boulevard peripherique in Paris. *Transportation research Part B*, 23, 29–47.

Papageorgiou, M., Blosseville, J., and Hadj-Salem, H. (1990). Modelling and real-time control of traffic flow on the southern part of boulevard peripherique in Paris: Part ii: Coordinated on-ramp metering. *Transportation Research Part A*, 24(5), 361370.

Papageorgiou, M., HadjSalem, H., and Middleham, F. (1997). ALINEA local ramp metering: summary of field study results. *Transportation Research Record*, 90–98.

Papageorgiou, M. and Kotsialos, A. (2002). Freeway ramp metering: an overview. volume 3, 271–281.

Payne, H.J. (1971). Models of freeway traffic and control. *Simulation councils proc. ser. Math. Models Publ. Sys.*, (1), 51–61.

Sanwal, K.K., Petty, K., Walrand, J., and Fawaz, Y. (1996). An extended macroscopic model for traffic flow. *Transportation Research Part B*, 30(1), 1–9.

Scherer, C. and Weiland, S. (2005). Linear matrix inequality in control. Lecture notes.

Shamma, J. and Athans, M. (1991). Guaranteed properties of gain scheduled control for linear parameter-varying plants. *Automatica*, 27(3), 559–564.

Wang, Y., Papageorgiou, M., Messmer, A., Coppola, P., Tzimitsi, A., and Nuzzolo, A. (2009). An adaptive freeway traffic state estimator. *Automatica*, 45, 10–24.

Whitham, G. (1974). *Linear and nonlinear waves.* John Wiley, New York.

Zhang, H.M. and Ritchie, S.G. (1997). freeway ramp metering using artificial neural networks. *Transportation Research Part C*, 5, 273286.

ACKNOWLEDGEMENTS

The authors express their thanks to Dr. Tamas Luspay for his valuable comments. The authors also acknowledge the support of Chalmers University of Technology through the Transport Area of Advance.