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An iterative dynamic programming/convex optimization procedure for optimal sizing and energy management of PHEVs

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Abstract: This paper proposes a time-efficient method for sub-optimal design of a plug-in hybrid electric vehicle with a parallel powertrain topology. The method finds the optimal design of the vehicle by iteratively using dynamic programming (DP) and convex optimization to minimize sum of operational and component costs over a given driving cycle. In particular, DP is used to optimize energy management, gear shifting and engine on-off for given component sizes, and convex optimization is used to optimize energy management and component sizes using the gear shifting and engine on-off strategies obtained by DP. Next, DP is re-optimized with the component sizes obtained by convex optimization, and the procedure is repeated until the component sizes converge. The result of this iterative method is compared by using DP on a grid of possible component sizes. It is shown that the iterative method gives a result very close to the global optimum in a comparably short time.

Keywords: Hybrid electric vehicles, Optimal sizing, Driving cycles, Convex optimization, Dynamic programming, Gear shifting, Engine on-off.

1. INTRODUCTION

A hybrid electric vehicle (HEV) is a type of hybrid vehicle that has an electric propulsion system in addition to a conventional internal combustion engine (ICE). HEVs can reduce the fuel consumption by downsizing the engine, recovering braking energy, having extra power control freedom by the two power sources, and stopping the engine when idle. Plug-in hybrid electric vehicles (PHEV)s are the next generation of hybrid vehicles that have the ability to store energy from the electrical grid using large capacity batteries. PHEVs may drive short trips entirely on stored electrical energy, thus decreasing the vehicle's dependency on petroleum.

The total cost of ownership of the PHEV depends directly on the size of the powertrain components, and also on the way the vehicle is operated, given a certain driving cycle. This is because energy management [1] affects the design, therefore, the control strategy should ideally be part of the optimal design process, to exclude its influence on component sizing. Hence, the problem of optimizing the total vehicle cost should be approached by simultaneous optimization of both energy management and component sizes. Moreover, the standard driving cycles that are commonly used are too short to reflect life time driving behavior of a driver, therefore, a long driving cycle is needed which includes different driving situations.

The cost function comprises two parts; the first part reflects the cost of key components of the vehicle, namely battery, electric motor (EM), and ICE, and the second part reflects the operational cost of fuel and electricity. This is a dynamic, non-convex, nonlinear and mixed integer problem. The problem has been solved using different methods, but the most typically used methods based on well developed optimization theories are convex optimization and dynamic programming (DP).

Convex optimization is used to solve this complex problem by [Murgovski et al., 2012] and [Pourabdollah et al., 2013]. Convex problems have a unique optimum, and can be solved fast and reliably. Both component sizes and the complete *control trajectory* of the continuous variables, i.e. the torque split between ICE and EM can be included as optimization variables. Convex optimization method can find the optimal sizes of the battery, electric motor, and ICE for a PHEV. The problem with convex optimization is that it cannot handle integer variables and therefore, the integer variables of this problem, i.e., gear shifting and engine on-off should either determined based on heuristics prior to the optimization, or be given as an input and this strategy can only solve a sub-problem.

Methods based on mixed integer programming are not suitable in this type of problem due to the large number of the integer variables. The number of the integer variables depends directly on the size of the driving cycle which in this case is considered to be relatively large.

On the other hand, Bellman's principle (Bellman [1957]) is also used to solve this highly nonlinear problem. DP can find the optimal energy management which minimize the fuel cost for given component sizes. Therefore, to find the optimal component sizes, DP has to be used on a grid of possible component sizes which is growing computationally ([Li et al., 2012], [Ebbesen et al., 2012], and [Ravey et al., 2012]). The main disadvantage of using DP is that its computational complexity grows exponentially with the number of states. This problem has one state, the battery state of charge, but it has also three design parameters. Each design parameter requires an outer

[1] the control strategy that determines the power split between ICE and additional energy source at every time instant, also including the gear shifting and engine on-off

loops in DP which in term of computational complexity can be considered as an extra state.

The idea in this paper is to use a combination of convex optimization and DP to avoid the drawbacks of the two methods. First, DP is used to find the optimal gear shifting and engine on-off, for an initial component sizes. The optimal engine on-off and gear shifting are given to convex optimization as an input to get the sub-optimal component sizes. The component sizes are then used as an input to run DP again. This procedure is continued until the cost and sizes converge to the optimal value. The result of this method is compared to the optimal results obtained by using DP over a grid of component sizes. It is shown that the component sizes obtained by this iterative method converge to optimal value in a relatively short time.

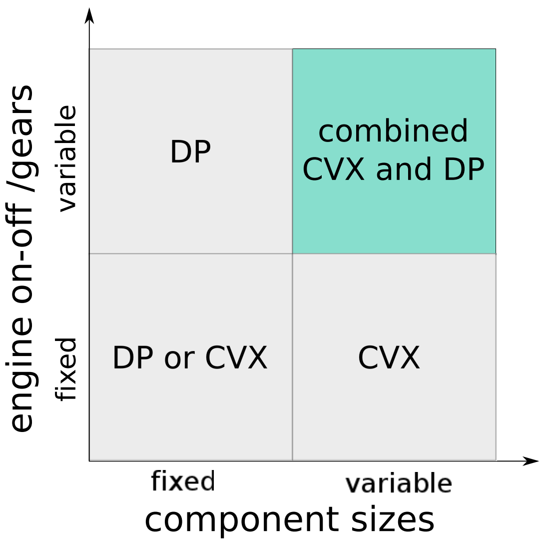

Fig. 1. Characteristics and features of DP, convex optimization and combined optimization methods.

The rest of this paper is organized as follows. An overall picture of the optimization problem, the driving cycle and the model of the powertrain and its components are presented in Section 2; A brief explanation of the optimization methods, i.e., convex optimization, DP, and the proposed method are provided in Section 3. Illustrative results from the study are shown in Section 4. Finally conclusions are drawn in Section 5.

2. PROBLEM FORMULATION AND MODELING

In this section, the problem formulation and modeling details are introduced. The study is concerned with an optimization problem of finding a cost-effective vehicle that minimizes the energy consumption. The problem constitutes an objective function and constraints, where the objective function is a weighted sum of operational costs over the driving cycle J_{op} and component costs J_{comp}. The operational cost includes the consumed fossil fuel and electrical energy, and the components cost is the sum of the costs of battery, EM, and ICE. The problem can be stated as:

$$\min_{u,s} J = \sum_{k=1}^{N} J_{op}(u(k), s) + J_{comp}(s) \tag{1}$$

subject to:

$$x(k+1) = f(x(k), u(k), s)$$
$$x \in X, u \in U, s \in S,$$

where x and s are the state variable vector and the scaling factor for components. The control input variable u is defined as:

$$u = [u_d u_c] \tag{2}$$

where u_c includes the continuous control inputs of length N and u_d includes the integer control inputs of size N. These variables are explained in more detail at the end of this section.

2.1 Driving cycle

In the optimization problem, we try to find the optimal component sizes and the energy management variables over a given driving pattern. Therefore, designing a vehicle with optimal component sizes requires knowledge about the lifetime driving pattern of the vehicle. However, since it is impossible to predict the precise lifetime driving cycle, and also computational resources are limited, here we use a long driving cycle of length N [2] that represents a real-life driving cycle. We assume that the vehicle is driven on a horizontal road and has the possibility to charge from the grid overnight.

The long driving cycle used in the optimization can reflect real-life driving, but might not include extreme situations that require high performance. Acceleration requirement is considered as an important vehicle attribute by many drivers, and are hence added in the constraints. Acceleration as a function of speed on a flat road is used to make a so called performance cycle, which is then appended to the driving cycle. The performance cycle includes speeds from zero to maximum speed, increasing according to the accelerations interpolated from the curve as explained in [Pourabdollah et al., 2013]. The driving cycle used in the simulations is shown in Section 4. We assume that the battery has the possibility to be charged with constant power from the grid at charging occasions, where the car is parked for 8 hours.

2.2 Modeling

In this section the models of the powertrain and its components are presented. Since same models are used both for DP and convex optimization, they all guarantee convexity [3]. Quasi-static models are therefore approximated with nonlinear convex functions and some variable change is also used. The accuracy of these approximations are high and are discussed in detail in [Murgovski et al., 2012].

Powertrain The studied PHEV, depicted in Fig. 2, is a parallel powertrain, where both the ICE and EM are mechanically linked to the drive train and can propel the wheels. Having the velocity $v(k)$, the acceleration $a(k)$, and zero road slope at discrete time instants k from the driving cycle, the required traction force $F_t(k)$ can be calculated as

$$F_t(k) = \frac{c_d A_f \rho v(k)^2}{2} + m_{tot} g c_r + m_{tot} a(k), \tag{3}$$

where F_t is the longitudinal force from the drive line and m_{tot}, A_f, c_d, ρ, g, c_r, and a are total vehicle mass, frontal area, air drag coefficient, air density, gravitational acceleration, rolling resistance coefficient, and acceleration, respectively. The total vehicle mass, m_{tot}, is the sum of the masses of the glider,

[2] provided by ETC Battery and FuelCells Sweden AB

[3] A convex function satisfies $f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda) f(y)$ for all $x, y \in \mathbf{R}$ with $0 \leq \lambda \leq 1$ (see [Boyd et al., 2009]).

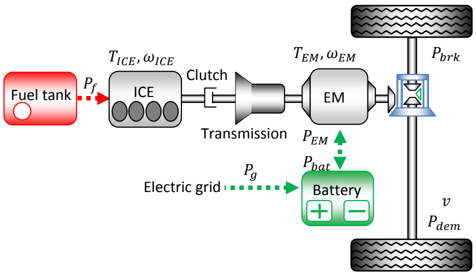

Fig. 2. Parallel PHEV configuration (solid lines: mechanical link, and dashed lines: electrical links).

Table 1. Vehicle parameters

parameter	value
baseline mass (m)	1600 kg
glider mass (m_g)	1280 kg
frontal area (Af)	2.37 m^2
rolling resistance (c_r)	0.009
aerodynamic drag coefficient (c_d)	0.33
air density (ρ)	1.293 kg/m^3
wheel radius (r_w)	0.3 m
ratio of the final gear (r_{fg})	4.2
EM reduction gear (r_{EM})	2

battery, EM, and ICE. The component masses are assumed to be linear functions of sizes as

$$m_j = m_{j,slope} s_j \tag{4}$$

for $j \in \{bat, EM, ICE\}$.

The powertrain model is described by power balance equations, given as

$$P_{dem}(k) + P_{brk}(k) = \tag{5}$$
$$T_{EM}(k)\omega_{EM}(k) + e_{on}(k)T_{ICE}(k)\omega_{ICE}(k)\eta(k) \tag{6}$$
$$\omega_{EM}(k)T_{EM}(k) + P_{EM,loss}(k) + P_{aux}(k) = P_{bat}(k) + P_g(k)\eta_g, \tag{7}$$

where $P_{dem}(k)$ is calculated as $P_{dem} = F_t v$ using (3). P_{brk} is the power dissipated at the friction brakes; T_{EM}, ω_{EM} and $P_{EM,loss}$ are torque, speed and power losses of the EM; P_{bat}, P_{ICE}, and P_{aux} are battery power, mechanical power of the ICE and electrical power used by auxiliary devices; $\eta(k)$ and η_g are transmission and charger efficiencies, which are assumed to be constant. P_g is the charger power and it is assumed to be constant. For simplicity, the rotational inertias are neglected in the models. The vehicle's parameters are given in Table 1.

At each time instant on the driving cycle, the angular speed of the EM and ICE are calculated as

$$\omega_{EM}(k) = r_{EM}\frac{r_{fg}}{r_w}v(k), \tag{8}$$

$$\omega_{ICE}(k) = r_{\gamma_{ICE}}(\gamma_{ICE}(k))\frac{r_{fg}}{r_w}v(k), \tag{9}$$

where r_w, r_{fg}, r_{EM}, and $r_{\gamma_{ICE}}(\gamma_{ICE}(k))$ are wheel radius, ratio of the final gear (differential), EM reduction gear, and ratio of the transmission gear of ICE, respectively [Guzzella et al., 2007]. The model does not allow any slip in the clutch; therefore, the vehicle is propelled by the EM at very low speeds.

Battery The battery consists of s_{bat} identical cells, each modeled as an open circuit voltage V_{oc} in series with a constant internal resistance, R. The open circuit voltage is approximated to be constant in the allowed state of charge (SoC) operating region. The terminal power, P_{bat}, and the stored energy of the battery, E_b, are calculated as

$$P_{bat}(k) = s_{bat}(V_{oc}i(k) - Ri^2(k)) = V_{oc}\tilde{i}(k) - R\frac{\tilde{i}^2(k)}{s_{bat}} \tag{10}$$

$$E_b(k+1) = E_b(k) - h(k)\, V_{oc}\, \tilde{i}(k). \tag{11}$$

The cell current $i(k) \in [i_{min}, i_{max}]$ is chosen to be positive when discharging. The variable change $\tilde{i}(k) = s_{bat}i(k)$ is introduced to preserve the problem convexity [Murgovski et al., 2012]. During the available parking periods, it is assumed that the vehicle is charged with constant current and power. Then, it is assumed, without loss of generality, that the whole charging energy enters the battery in one extra long sample, $\Delta t_c(k)$, at the parking occasions.

EM The EM model with its power electronics is described by a power loss map, $P_{EM,loss,base}$, where the losses are measured at steady-state for different torque-speed combinations. The power losses for each EM speed are approximated by a second-order polynomial in torque. To vary the size of the EM, the torque limits and losses are scaled linearly by scaling factor s_{EM}. In this way the losses of the scaled EM are calculated at each time instant as

$$P_{EM,loss}(k) = c_1(k)\frac{T_{EM}^2(k)}{s_{EM}} + c_2(k)T_{EM}(k) + c_3(k)s_{EM}. \tag{12}$$

where the coefficients $c_1 \geq 0$, c_2 and c_3 functions of ω_{EM} and are calculated using least squares method for a number of grid points of ω_{EM}. For speed values not belonging to the grid nodes, the coefficients are obtained by linear interpolation ([Murgovski et al., 2012], [Pourabdollah et al., 2013]).

ICE The ICE fuel power, P_f, is a function of the engine torque and speed, and is derived from a map obtained from engine experiments at steady state. The fuel power represented as Willans lines ([Heywood, 1988], [Pachernegg, 1969]), approximates the ICE model by affine relations which is a second-order polynomial in T_{ICE}, parameterized in engine speed. Assuming that torque and losses scale linearly with a scaling factor s_{ICE}, the fuel power is calculated as

$$P_f(k) = b_1(k)\frac{T_{ICE}^2(k)}{s_{ICE}} + b_2(k)T_{ICE}(k) + b_3(k)s_{ICE}e_{on}, \tag{13}$$

where the coefficients $b_1 \geq 0$, b_2, and b_3 are functions of ω_{ICE} and hence time dependent. These coefficients are calculated in a similar way as c_1, c_2, and c_3 for the EM. The engine on-off variable e_{on} is introduced to remove the idling losses b_3 in (13), when the ICE is off.

Problem Formulation The variables in (1) are given in more detail this section. The operational cost in (1) includes the consumed fossil fuel and electrical energy as

$$J_{op}(k) = \frac{\rho_f}{\rho_{LHV}}P_f(k)h(k) + \frac{\rho_{el}}{1000 \cdot 3600}P_g(k)h(k), \tag{14}$$

where ρ_{LHV} is the lower heating value of gasoline and the fuel power, P_f, and charger power, P_g, are converted to an equivalent cost in EUR using energy prices ρ_f for gasoline and ρ_{el} for electricity, which are explained in more details later. The sampling interval $h(k)$ is equal to 1 s when driving and changes at charging times where we consider that charging happens in one sampling interval ([Pourabdollah et al., 2013]).

The components cost in (1) is the sum of the costs of battery, EM, and ICE; the remaining cost of the vehicle is assumed to be independent of sizing and is therefore excluded from the problem. The components cost is calculated as the depreciation over the vehicle life-time, i.e., the proportion of the components cost given by the ratio between the length of the driving cycle, d, and the lifetime driving distance of the vehicle. Including a yearly interest rate of $p_c = 5\%$, the components cost is given by

$$cost_{comp} = \frac{d}{sy_v}\left(1 + p_c\frac{y_v + 1}{2}\right)(cost_{bat} + cost_{EM} + cost_{ICE}), \tag{15}$$

where y_v is the vehicle lifetime, and s is the average traveled distance of the vehicle in one year. For each component, the cost model is an affine function

$$cost_j = cost_{j,init} + cost_{j,slope}\, s_j, \tag{16}$$

for $j \in \{bat, EM, ICE\}$, where s_j is a component scaling parameter used to scale the size of the components. The cost and mass functions are calculated from a baseline EM of 35 kW power, and a baseline ICE with a displacement size equal to 1.6 L, and power equal to 65 kW. The battery is energy optimized with a cell capacity of Q = 159 Wh.

The state variable, x, in (1) is the energy in the battery given in (11). The continuous input variable, u_c is the torque split between the ICE torque and the EM torque. The integer input variable, u_d consists of the engine on-off variable, e_{on} and the gears, γ.

3. OPTIMIZATION METHODS

The dynamic programming and convex optimization methods used in this paper are explained in more details in this section. The proposed method which uses the DP and convex optimization in iteration is introduced at the end of this section.

3.1 Dynamic programing

Dynamic programming is a method to solve optimal control problems based on the Bellman's principle of optimality ([Bellman, 1957]). In automotive applications, DP is used by many authors to find the optimal energy management and gear shifting which minimizes fuel consumption, while satisfying the constraints on the SoC level and the powertrain models ([Lin et al., 2003], [Hofman et al., 2004]). The dynamic programming algorithm proceeds backward in time from $N - 1$ to 0. At each time instant, the optimal torque split and gear is the one that minimizes a cost. The cost at final time $J_N(x_N)$ here is assumed to be zero, because there is no constraint on the final state of charge. For given component sizes, DP finds the optimal energy management, gear shifting and engine on-off at every time instant.

The computation time of DP increases exponentially with the number of states and is an issue despite the efforts that has been done to reduce the burdens ([Johannesson et al., 2009], [Sundström et al., 2013]). In order to solve the problem numerically, the dynamic states and the control inputs are discretized both in time and value. To get an accurate result for costs and gears, the number of grid points used for the state of charge (SoC) and torque split need to be sufficiently high. But increasing the number of grid points raises the computational time dramatically. To show this, DP is used to find the optimal solution over the driving cycle introduced in Section 2.1 with different number of grid points and given component sizes. As

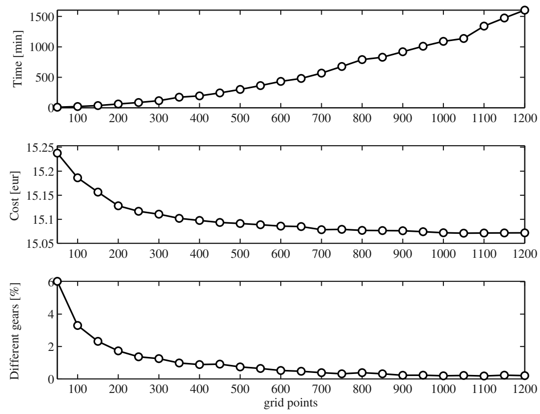

Fig. 3. The computational time, total cost , and percentage of the time that a different gear is selected as a function of grid points.

shown in Fig. 3, by increasing the grid points the accuracy increases but at the same time the computational time rises drastically. Moreover, the number of grid points also effects the gear selection. To show this, the percentage of time instants that the gear is selected differently compared to the gears selected using 1200 grid points is illustrated in Fig. 3. Using the results in Fig. 3, we choose 400 grid points in the simulations to get an acceptable accuracy and computational time.

3.2 Convex optimization

Convex optimization is also used to solve the problem of finding the optimal design and energy management. We first need to define the convex powertrain and component models, in addition with the cost models and the performance requirements, to solve a convex problem. For a vehicle model and a given driving cycle, the gears and engine on-off need to be decided a-priori and given to the convex optimization to preserve convexity. Modeling of the powertrain and its components to guarantee the convexity is the main step of the optimization method. Once the problem is defined as a convex optimization problem, it can effectively be solved by using solvers, in a relatively short time. We use a tool called CVX ([Grant et al., 2010]) to automatically translate the problem to a form required by a publicly available solver, e.g. Sedumi. The constraints in the convex problem are in forms of powertrain and component models, introduced in section 2.2 and the maximum component ratings.

The decision variables of the optimization problem include, firstly, the component scaling parameters s_{bat}, s_{EM}, and s_{ICE}, which are all dimensionless scaling parameters for battery, EM, and ICE. The second group consists of optimization variables which are related to the energy management and are determined for every time instant. These variables are EM torque $T_{EM}(k)$, ICE torque, $T_{ICE}(k)$, battery current, $\tilde{i}(k)$, battery state of energy, $E_b(k)$, grid power, $P_g(k)$, and braking power, $P_{brk}(k)$.

3.3 The combined convex optimization and DP method

The main idea introduced in this paper is to combine the optimal results from DP and convex optimization, by using the two methods alternately, to find the optimal component sizes and energy management of a PHEV. In order to do this, we start with DP using initial component sizes. DP provides the optimal energy management and integer variables, namely

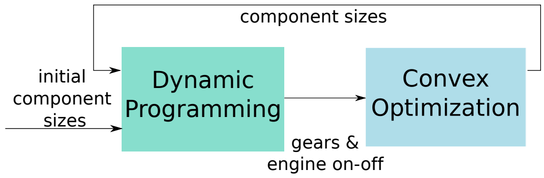

Fig. 4. The combined convex optimization and DP method. Dynamic programming finds the optimal energy management, gear and engine on-off for given component sizes, whereas convex optimization finds the optimal energy management and component sizes for given gear and engine on-off.

gears and engine on-off for the given component sizes. The integer variables are then given to the convex optimization to find the component sizes. This iteration is continued until the cost and component sizes converge as shown in Fig 4.

4. RESULTS

In this section, the results of the iterative optimization of energy management and component sizing are given. The optimization is performed over a 176 km long (more than 10000 seconds) real life driving cycle, followed by a performance cycle, including 4 occasions where the car has possibility to charge the battery with constant grid power for 8 hours, shown in Fig. 5.

The iteration starts with DP, given initial component sizes. These initial inputs can have strong impact on the final results. If not chosen correctly, they can result in a local optimum from which the iterative procedure is not able to escape. There are two main situations that can lead the iteration to local optimal solutions. The first situation is if the engine is oversized in the first iteration. In this way, to improve the efficiency, the gears may be chosen so that the ICE torque becomes very close to the maximum value. Using this gear shifting in convex optimization puts a high demand on ICE torque and hence results in a large ICE. In order to avoid this problem, the maximum ICE torque is limited to 97% of its value. The limit is relaxed if DP gives an infeasible solution, which means that the ICE can not be decreased further. This manipulation is justified since most torque demanding operating points are from the performance cycle. At these points, DP without restriction chooses a higher gear to increase efficiency. If the maximum torque at these points is limited, a higher gear will be selected, which demands lower engine torque and therefore size with lower efficiency.

Similarly, when the initial battery size is bigger than the optimal value. In this way, in the first iteration of DP, the vehicle is propelled most of the time by the cheap energy from the battery and the engine is turned off in order to reduce the losses. Giving this optimal engine on-off to the convex optimization, the battery stays oversized because the energy is provided often by the battery, as the engine is turned off. In this way, the iterative optimization never gets a chance to result in a smaller battery size even if it is optimal.

To avoid this problem, a minimum feasible battery size is chosen as the initial value. Here, the minimum allowed battery is 2.39 kWh. This is because at speeds lower than the idling speed the ICE is not used and the power is provided by the battery. The result of the optimization in the first round is shown in Fig. 5.

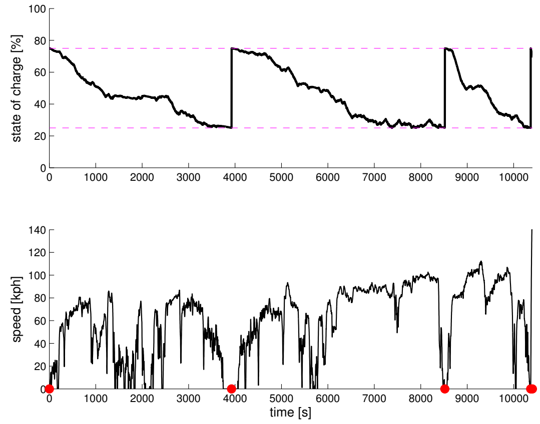

Fig. 5. The optimal SoC (upper figure) and the speed profile of the driving cycle (lower figure). The charging occasions are also shown by red dots.

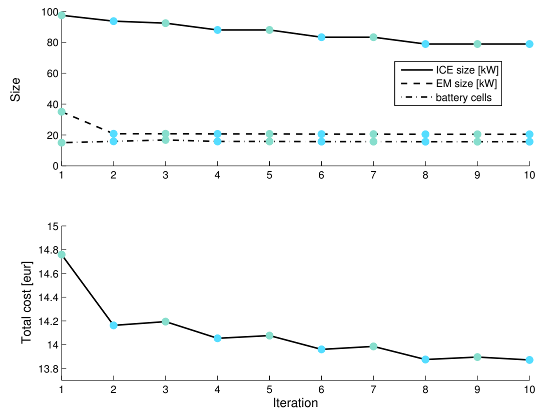

Fig. 6. Number of battery cells, size of EM and ICE (upper figure) and the total cost(lower figure), over 10 iteration of convex optimization and DP starting with minimum battery size.

The procedure of using convex optimization and DP is continued until the cost and component sizes converge. In Fig. 6 the results of 10 iterations are given. As we can see, the optimal sizes of the motor and the battery converge after the first iteration. The cost and ICE size continue converging, but with a slower rate until the last iteration. The total cost decreases from € 14.76 in the first iteration to € 13.77 in the last, which is 7% difference.

The phenomena of prematurely converging to a local optimum is shown in Fig. 7 when the iteration started with a large initial battery size.

To evaluate the accuracy of the proposed method, we use DP over a grid of component sizes shown in 8.

As seen in the figure, the optimal values obtained by DP and iterative method, shown by red circle and star, are very close. The total cost obtained by the iterative method is € 13.87 which is even lower than the value from DP, € 13.9. This indicates that the optimal value is not on the grid sizes, and to find the correct value, DP needs to be run over smaller griding around the current optimal point. However, this increases the computational time even more. We have to bear in mind that the computational time to find the optimal value using the iterative method took around 13 hours whereas DP took around 1051

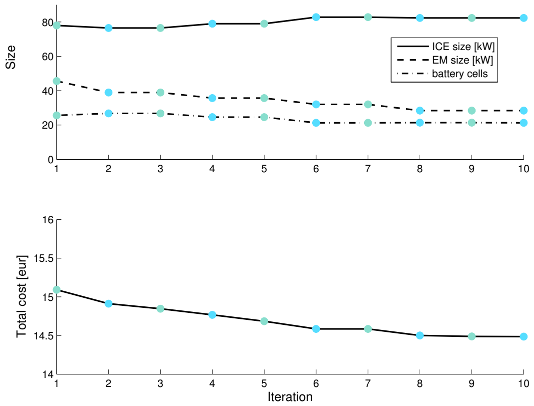

Fig. 7. The size of ICE and EM, number of battery cells,(upper figure) and the total cost (lower figure), over 10 iterations of convex optimization and DP starting with a large initial battery size.

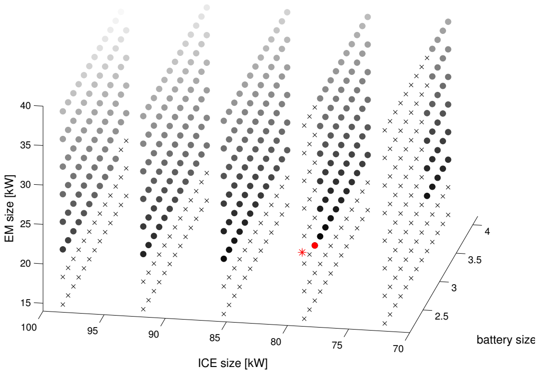

Fig. 8. Total cost over different sizes of battery, ICE and EM. Darker color shows more cost efficient solutions and the infeasible solutions are shown by cross. The optimal sizes obtained by DP and iterative method are shown by red circle and star.

hours. DP Iterations are run over 13*5*8 component sizes. For the selected 167 minutes long driving cycle, each iteration takes around 2 hours of calculations on a Intel core 2, 2.67GHz processor and 8GB memory.

5. CONCLUSION

In this paper, we used an iterative method of dynamic programming and convex optimization to find close to optimal component sizes, gear shifting, engine on-off and energy management. The results show the cost and sizes converge to the optimal value with a good accuracy after few iterations. For comparison, DP is used on a grid of component sizes to find the optimal design. The method does not guarantee global optima but when taking some initiations into consideration, the iterative method can find the component sizes close to the optimal in much shorter computational time compared to DP. In the future work, more investigations need to be done to ensure that the method results in global optimal solution in various senarios. Moreover, since using less grid points for the states and control inputs in DP results in lower computational time, the effect of this can be studied on the convergence of the solution.

6. ACKNOWLEDGMENT

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