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H_∞ Filter Design for State Estimation and Unknown Inputs Reconstruction of a Class of Nonlinear systems

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***Abstract*— We consider a novel method to design a H_∞ filter for a class of nonlinear systems subject to unknown inputs. First, we rewrite the system dynamics as a descriptor system. Then, we design a robust H_∞ reduced-order filter to estimate both state variables and unknown inputs at the same time. Based on a Lyapunov functional, we derive a sufficient condition for existence of the designed filter which requires solving a nonlinear matrix inequality. The achieved condition is further formulated in terms of a linear matrix inequality (LMI) that is straightforward to solve by popular methods. Finally, the proposed filter is illustrated with an example.**

I. INTRODUCTION

Observer design for nonlinear systems is a popular problem in control theory that has been studied in many angles. Moreover, state estimation of nonlinear system in the presence of unknown inputs is one of the fascinating relevant topics in the modern control theory. See for instance [1]–[6] and references therein.

However, to the best of our knowledge, the idea of using descriptor systems to estimate the state variables together with unknown inputs is still an open question and it needs more attention. Briefly, observer design for descriptor systems has been investigated in different aspects. In this field, we can refer our readers to [7]–[10] where the authors consider a variety of nonlinear methods on the descriptor systems with Lipschitz nonlinearities.

On the other hand, the problem of the state estimation for descriptor systems in presence of noise has also been the subject of several studies in the past decades. There are two popular methods in this field as Kalman filtering and the H_∞ approach. In Kalman filtering, briefly, the system and the measurement noises are assumed to be Gaussian with known statistics [11] while for arbitrary type of noise with bounded energy, H_∞ filtering can guarantee a noise attenuation level. [12]–[14] and references therein are recent researches that have been done in filtering for descriptor systems.

Inspired by [14], where the authors designed an H_∞ filter for a class of nonlinear singular systems, in this article we try to rewrite our problem as a descriptor system and use the features of the descriptor systems to estimate the state variables of the system together with the unknown inputs, simultaneously. To achieve this objective, a sufficient condition for existence of the designed filter is derived, which requires solving a nonlinear matrix inequality. In order to facilitate the filter design, the obtained condition is formulated in terms of a linear matrix inequality (LMI) that can be easily solved by well-known algorithms in this area.

The rest of this paper is organized as follows. In Section 2, we introduce the class of nonlinear systems with unknown inputs. In Section 3, we propose a new method to design a reduced-order filter for the systems under study. To validate the proposed nonlinear filter, an example is shown in Section 4. Conclusions are presented in Section 5.

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II. PRELIMINARIES AND PROBLEM STATEMENT

Consider the following nonlinear system subject to unknown inputs:

$$\begin{aligned}\dot{x} =& Ax+Bu+Df(x)+D_1\omega+\sum_{i=1}^{h}F_i\nu_i,\\ y =& Cx+D_2\omega+\sum_{i=1}^{h}G_i\nu_i\end{aligned}\tag{1}$$

where $x \in R^n$, $u \in R^m$ and $y \in R^p$ are the state vector, known input and output vector, respectively. For $i = 1,...,h$ $(h \le p)$, $\nu_i(t) \in R$ are unknown inputs that can affect both actuators and sensor. Matrices A, B, C, D, D_1, D_2, F_i (for $i = 1,...,h$) and G_i (for $i = 1,...,h$) are real and with appropriate dimensions. The function f is nonlinear. ω is exogenous disturbance which belongs to $L_2[0,\infty)$. The aim is to design a nonlinear filter such that it can estimate the state vector x together with the unknown inputs ν_i (for $i = 1,...,h$) asymptotically. To specify the class of nonlinear systems under study, we use the following assumption.

Assumption 1. The function $f(x)$ is nonlinear, and satisfies a Lipschitz constraint as follows:

$$\|f(\alpha)-f(\beta)\| \le \gamma\|\alpha-\beta\| \tag{2}$$

where $\gamma > 0$ is the Lipschitz constant and $\|x\| = \sqrt{x^T x}$.

III. MAIN RESULTS

Assume that,

$$\zeta = \begin{bmatrix}x^T & \nu_1 & \nu_2 & \dots & \nu_h\end{bmatrix}^T \tag{3}$$

then, the augmented system dynamics are,

$$\begin{aligned}\eta\dot{\zeta} =& \bar{A}\zeta+Bu+D\bar{f}(\zeta)+\ D_1\omega\\ y =& \bar{C}\zeta+\ D_2\omega\end{aligned}\tag{4}$$

where,

$$\eta = \begin{bmatrix} I_{n\times n} & 0_{n\times h} \end{bmatrix},$$
$$\bar{A} = \begin{bmatrix} A & F_1 & F_2 & \dots & F_h \end{bmatrix},$$
$$\bar{C} = \begin{bmatrix} C & G_1 & G_2 & \dots & G_h \end{bmatrix} \quad (5)$$

and,

$$\bar{f}(\zeta) = f(\eta\zeta); \quad \bar{f} : R^{n+h} \to R^q \quad (6)$$

We can make the following assumption which will be used in the sequel of the paper.

Assumption 2 *rank* $\begin{bmatrix} \eta \\ \bar{C} \end{bmatrix} = n+h.$

This assumption is necessary for the pair $(\eta, \bar{C})$ to be observable [15].

Now, the aim is to design a filter such that it can estimate ζ asymptotically.

Consider the following reduced-order filter for the augmented system (4),

$$\dot{z} = Nz + Ly + Gu + MD\bar{f}(\hat{\zeta})$$
$$\hat{\zeta} = Jz + Ey \quad (7)$$

where vector $z \in R^{q_1}$ and $\hat{\zeta}$ is the estimation of ζ. The matrices N, L, G, M, J, and E must be determined such that the error dynamics $e = \hat{\zeta} - \zeta$ converge to zero asymptotically.

By defining the error between z and $M\eta\zeta$,

$$\varepsilon = z - M\eta\zeta \quad (8)$$

the error dynamics will be,

$$\begin{aligned}\dot{\varepsilon} =& N\varepsilon + (NM\eta + L\bar{C} - M\bar{A})\zeta \\ &+ (G - MB)u + MD(\bar{f}(\hat{\zeta}) - \bar{f}(\zeta)) \\ &+ (LD_2 - MD_1)\omega \end{aligned} \quad (9)$$

on the other hand,

$$\begin{aligned}\hat{\zeta} &= Jz + Ey \\ &= J\varepsilon + JM\eta\zeta + E\bar{C}\zeta + ED_2\omega \\ &= J\varepsilon + (JM\eta + E\bar{C})\zeta + ED_2\omega \end{aligned} \quad (10)$$

if there exists a matrix M such that,

$$G = MB, \ NM\eta + L\bar{C} - M\bar{A} = 0, \ JM\eta + E\bar{C} = I \quad (11)$$

or,

$$G = MB, \quad (12)$$
$$\begin{bmatrix} N & L \\ J & E \end{bmatrix} \begin{bmatrix} M\eta \\ \bar{C} \end{bmatrix} = \begin{bmatrix} M\bar{A} \\ I \end{bmatrix} \quad (13)$$

then (9) and (10) become,

$$\begin{aligned}\dot{\varepsilon} &= N\varepsilon + MD(\bar{f}(\hat{\zeta}) - \bar{f}(\zeta)) + (LD_2 - MD_1)\omega \\ e &= J\varepsilon + ED_2\omega \end{aligned} \quad (14)$$

We will now formulate and solve an H_∞ filter design problem with the following conditions:

(1) The filter error (14) with $\omega = 0$ is stable.

(2) Under zero initial condition, the induced L_2 norm of the operator from ω to e is less than μ , i.e. $\|e\|_2 < \mu\|\omega\|_2$.

A. Stability Analysis

Since $e = J\varepsilon$ for $\omega = 0$, obviously the asymptotic stability of ε is sufficient condition for $lim_{t\to\infty} e(t) = 0$. The following theorem gives the conditions for stability of the dynamics of $e(t)$.

Theorem 1 *Under Assumption 1, for* $\omega = 0$ *and given a scalar* $\gamma > 0$*, if there exists real matrices N, J, M, and* $P > 0$ *with appropriate dimensions such that the inequalities below are satisfied,*

$$\Gamma - I < 0, \quad \begin{bmatrix} N^T P + PN + \gamma^2 J^T J & PMD \\ D^T M^T P & -\Gamma \end{bmatrix} < 0 \quad (15)$$

then the state estimation error (14) produced by filter (7) tends to zero asymptotically.

Proof. First, consider a Lyapunov functional as:

$$V = \varepsilon^T P \varepsilon \quad (16)$$

where P is a positive definite matrix. From (14), Assumption 1, and by taking the derivative of $V(t)$ along the trajectory of (14) for $\omega = 0$, we have

$$\begin{aligned}\dot{V} =& \dot{\varepsilon}^T P\varepsilon + \varepsilon^T P\dot{\varepsilon} \\ =& [N\varepsilon + MD\Delta\bar{f}]^T P\varepsilon + \varepsilon^T P[N\varepsilon + MD\Delta\bar{f}] \\ =& \varepsilon^T (N^T P + PN)\varepsilon \\ &+ \Delta\bar{f}^T D^T M^T P\varepsilon + \varepsilon^T PMD\Delta\bar{f} \\ &+ \Delta\bar{f}^T \Gamma\Delta\bar{f} - \Delta\bar{f}^T \Gamma\Delta\bar{f} \end{aligned} \quad (17)$$

for $\Gamma < I$,

$$\begin{aligned}\dot{V} \le& \varepsilon^T (N^T P + PN)\varepsilon \\ &+ \Delta\bar{f}^T D^T M^T P\varepsilon + \varepsilon^T PMD\Delta\bar{f} \\ &+ \Delta\bar{f}^T \Delta\bar{f} - \Delta\bar{f}^T \Gamma\Delta\bar{f} \end{aligned} \quad (18)$$

since,

$$\begin{aligned}\Delta\bar{f}^T \Delta\bar{f} =& \Delta\bar{f}(\zeta)^T \Delta\bar{f}(\zeta) \\ =& \Delta f(\bar{\eta}\zeta)^T \Delta f(\bar{\eta}\zeta) \\ =& \Delta f(x)^T \Delta f(x) \\ \le& \gamma^2 e^T e \end{aligned} \quad (19)$$

so,

$$\dot{V} \le \begin{bmatrix} \varepsilon \\ \Delta\bar{f} \end{bmatrix}^T \begin{bmatrix} N^T P + PN + \gamma^2 J^T J & PMD \\ D^T M^T P & -\Gamma \end{bmatrix} \begin{bmatrix} \varepsilon \\ \Delta\bar{f} \end{bmatrix} \quad (20)$$

It should be noted that if (15) is satisfied then the state estimation error (14) tends to zero asymptotically for any initial value $\varepsilon(0)$. □

B. H_∞ Design

The following theorem gives the sufficient conditions for (14) to be stable for $\omega = 0$ and $\|e\|_2 < \mu\|\omega\|_2$ for $\omega \neq 0$.

Theorem 2 *Consider the system (1) together with the nonlinear filter (7). Under Assumption 1, given admissible Lipschitz constant* γ *and disturbance tuning parameter* μ*,* *there exist matrices N, L, M, J, E, and $P > 0$ with appropriate dimensions such that the following inequalities have a feasible solution:*

$$\Gamma - I < 0, \quad \Phi = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12}^T & -\Gamma & 0 \\ a_{13}^T & 0 & a_{33} \end{bmatrix} < 0 \tag{21}$$

with,

$$\begin{aligned} a_{11} &= N^T P + PN + (1+\gamma^2) J^T J, \\ a_{12} &= PMD, \\ a_{13} &= P(LD_2 - MD_1) + (1+\gamma^2) J^T E D_2, \\ a_{33} &= (1+\gamma^2) D_2^T E^T E D_2 - \mu^2 I \end{aligned} \tag{22}$$

Then the state estimation error (14) produced by filter (7) tends to zero asymptotically for $\omega = 0$ and $\|e\|_2 < \mu \|\omega\|_2$ for $\omega \neq 0$.

Proof. From Theorem 1, we know that system (14) is asymptotically stable if (15) is valid. Now let $\omega \neq 0$,

$$\begin{aligned} \dot{V} =& \dot{\varepsilon}^T P \varepsilon + \varepsilon^T P \dot{\varepsilon} \\ =& [N\varepsilon + MD\Delta\bar{f} + (LD_2 - MD_1)\omega]^T P \varepsilon \\ &+ \varepsilon^T P [N\varepsilon + MD\Delta\bar{f} + (LD_2 - MD_1)\omega] \end{aligned} \tag{23}$$

by adding and subtracting the right side of (23) by $\Delta\bar{f}^T \Gamma \Delta\bar{f}$,

$$\begin{aligned} \dot{V} =& \varepsilon^T (N^T P + PN) \varepsilon \\ &+ \varepsilon^T PMD\Delta\bar{f} + \Delta\bar{f}^T D^T M^T P \varepsilon \\ &+ \omega^T (LD_2 - MD_1)^T P \varepsilon \\ &+ \varepsilon^T P (LD_2 - MD_1)^T \omega \\ &+ \Delta\bar{f}^T \Gamma \Delta\bar{f} - \Delta\bar{f}^T \Gamma \Delta\bar{f} \\ \leq& \varepsilon^T (N^T P + PN) \varepsilon \\ &+ \Delta\bar{f}^T D^T M^T P \varepsilon + \varepsilon^T PMD\Delta\bar{f} \\ &+ \omega^T (LD_2 - MD_1)^T P \varepsilon \\ &+ \varepsilon^T P (LD_2 - MD_1) \omega \\ &+ \Delta\bar{f}^T \Delta\bar{f} - \Delta\bar{f}^T \Gamma \Delta\bar{f} \end{aligned} \tag{24}$$

from (19),

$$\begin{aligned} \dot{V} \leq& \varepsilon^T (N^T P + PN) \varepsilon \\ &+ \Delta\bar{f}^T D^T M^T P \varepsilon + \varepsilon^T PMD\Delta\bar{f} \\ &+ \omega^T (LD_2 - MD_1)^T P \varepsilon \\ &+ \varepsilon^T (LD_2 - MD_1)^T P \omega \\ &+ \gamma^2 e^T e - \Delta\bar{f}^T \Gamma \Delta\bar{f} \end{aligned} \tag{25}$$

Since,

$$\begin{aligned} e^T e =& \varepsilon^T J^T J \varepsilon + \omega^T D_2^T E^T E D_2 \omega + \varepsilon^T J^T E D_2 \omega \\ & \omega^T D_2^T E^T J \varepsilon \end{aligned} \tag{26}$$

By letting $\lambda = \begin{bmatrix} \varepsilon & \Delta\bar{f} & \omega \end{bmatrix}$, the following inequality is obtained through (25) and (26)

$$\dot{V} + e^T e - \mu^2 \omega^T \omega \leq \lambda^T \Phi \lambda \tag{27}$$

Clearly, if $\Phi < 0$, then

$$\dot{V} \leq \mu^2 \omega^T \omega - e^T e \tag{28}$$

Integrating of both sides of this inequality from zero to infinity yields

$$V(\infty) - V(0) \leq \mu^2 \|\omega(t)\|_2^2 - \|e(t)\|_2^2 \tag{29}$$

for zero initial values, we obtain

$$V(\infty) \leq \mu^2 \|\omega(t)\|_2^2 - \|e(t)\|_2^2 \tag{30}$$

which leads to

$$\|e(t)\|_2^2 < \mu^2 \|\omega(t)\|_2^2 \tag{31}$$

and this completes the proof. □

According to [14], (13) has a solution if only if,

$$rank \begin{bmatrix} M\eta \\ \bar{C} \\ M\bar{A} \\ I \end{bmatrix} = rank \begin{bmatrix} M\eta \\ \bar{C} \end{bmatrix} = n \tag{32}$$

let R be any arbitrary full row rank matrix such that,

$$rank \begin{bmatrix} R \\ \bar{C} \end{bmatrix} = rank \begin{bmatrix} M\eta \\ \bar{C} \end{bmatrix} = n \tag{33}$$

then there always exist matrices M and K such that,

$$M\eta = R - K\bar{C} \tag{34}$$

or,

$$\begin{bmatrix} M & K \end{bmatrix} \begin{bmatrix} \eta \\ \bar{C} \end{bmatrix} = R \tag{35}$$

Now from Assumption 2, equation (35) always has a solution given by,

$$\begin{bmatrix} M & K \end{bmatrix} = R \begin{bmatrix} \eta \\ \bar{C} \end{bmatrix}^+ \tag{36}$$

where $[.]^+$ represents any generalized inverse operator satisfying $[.][.]^+[.] = [.]$. Thus,

$$M = R \begin{bmatrix} \eta \\ \bar{C} \end{bmatrix}^+ \begin{bmatrix} I \\ 0 \end{bmatrix} \quad and \quad K = R \begin{bmatrix} \eta \\ \bar{C} \end{bmatrix}^+ \begin{bmatrix} 0 \\ I \end{bmatrix} \tag{37}$$

Also, the general solution for (13) is given by,

$$\begin{bmatrix} N & L \\ J & E \end{bmatrix} = \begin{bmatrix} M\bar{A} \\ I \end{bmatrix} \begin{bmatrix} M\eta \\ \bar{C} \end{bmatrix}^+ + \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} (I - \begin{bmatrix} M\eta \\ \bar{C} \end{bmatrix} \begin{bmatrix} M\eta \\ \bar{C} \end{bmatrix}^+) \tag{38}$$

where $[Y_1^T \; Y_2^T]^T$ is an arbitrary matrix of appropriate dimension that will be identified later on. In order to facilitate the filter design, we convert the provided nonlinear inequalities in (21) to an LMI problem. For this end, let's define,

$$\alpha_1 = \begin{bmatrix} M\eta \\ \bar{C} \end{bmatrix}^+ \begin{bmatrix} I \\ 0 \end{bmatrix} \tag{39a}$$

$$\alpha_2 = \begin{bmatrix} M\eta \\ \bar{C} \end{bmatrix}^+ \begin{bmatrix} 0 \\ I \end{bmatrix} \tag{39b}$$

$$\beta_1 = (I - \begin{bmatrix} M\eta \\ \bar{C} \end{bmatrix} \begin{bmatrix} M\eta \\ \bar{C} \end{bmatrix}^+) \begin{bmatrix} I \\ 0 \end{bmatrix} \tag{39c}$$

$$\beta_2 = (I - \begin{bmatrix} M\eta \\ \bar{C} \end{bmatrix} \begin{bmatrix} M\eta \\ \bar{C} \end{bmatrix}^+) \begin{bmatrix} 0 \\ I \end{bmatrix} \tag{39d}$$

so,

$$N = M\bar{A}\alpha_1 + Y_1\beta_1 \quad (40a)$$
$$L = M\bar{A}\alpha_2 + Y_1\beta_2 \quad (40b)$$
$$G = MB \quad (40c)$$
$$J = \alpha_1 + Y_2\beta_1 \quad (40d)$$
$$E = \alpha_2 + Y_2\beta_2 \quad (40e)$$

Theorem 3 *Consider the system (1) together with the nonlinear filter (7). Under Assumption 1 and Assumption 2 and for a disturbance tuning parameter* μ*, assume that there exists real matrices* $\bar{Y}_1$*,* Y_2*, and* $P > 0$ *with appropriate dimensions, such that the following LMIs have a feasible solution:*

$$\Gamma - I < 0, \quad \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{12}^T & -\Gamma & 0 & 0 \\ a_{13}^T & 0 & a_{33} & a_{34} \\ a_{14}^T & 0 & a_{34}^T & a_{44} \end{bmatrix} < 0 \quad (41)$$

where,

$$a_{11} = \alpha_1^T\bar{A}^TM^TP + \beta_1^T\bar{Y}_1^T + PM\bar{A}\alpha_1 + \bar{Y}_1\beta_1,$$
$$a_{12} = PMD,$$
$$a_{13} = PM\bar{A}\alpha_2D_2 + \bar{Y}_1\beta_2D_2 - PMD_1,$$
$$a_{14} = \alpha_1^T + \beta_1^TY_2^T,$$
$$a_{33} = -\mu^2I$$
$$a_{34} = D_2^T\alpha_2^T + D_2^T\beta^TY_2^T$$
$$a_{44} = -\frac{1}{1+\gamma^2}I \quad (42)$$

and $Y_1 = P^{-1}\bar{Y}_1$*. Then the state estimation error (14) produced by filter (7) tends to zero asymptotically for* $\omega = 0$ *and* $\|e\|_2 < \mu\|\omega\|_2$ *for* $\omega \neq 0$*.*

Proof. From Theorem 1, we know that system (14) is asymptotically stable if (15) is valid. Now let $\omega \neq 0$. If we substitute (40) into (15), we will obtain a new matrix new submatrices M, Y_1, Y_2, and P. In other words, the problem of finding N, L, M, J, E, and $P > 0$ in Theorem 2 is now equivalent to the problem of solving (37) for M, (21) for Y_1, Y_2, and P. Since there is a multiplication between two unknown matrices Y_1 and P, it is not yet an LMI in the variable Y_1 and P. Define a new variable as,

$$\bar{Y}_1 = PY_1 \quad (43)$$

Now, using Schur complement, (21) is rewritten as (41). Thus we can use the proof of Theorem 2 to show that state estimation error (14) tends to zero asymptotically and $\|e\|_2 < \mu\|\omega\|_2$. □

Based on Theorem 3, the reduced-order estimator algorithm for state estimation and unknown inputs reconstruction in the class of nonlinear systems under study can be summarized in the following steps,

1) Find a Lipschitz constant γ_0 satisfying Assumption 1
2) Fix the order q_1 of the filter and choose a full row rank matrix R such that (33) is satisfied, then compute matrix M by (37)
3) Compute α_1, α_2, β_1 and β_2 by (39)
4) Solve the LMIs defined by (41) for $\bar{Y}_1$, Y_2, and P
5) Compute $Y_1 = P^{-1}\bar{Y}_1$
6) Using Y_1 and Y_2, compute the filter gains as (40)

IV. NUMERICAL EXAMPLE

In this section, we show that the proposed filter is effective on a simulation example. Consider system (1) with parameters,

$$A = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -3 & -1 \\ 0 & 1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},$$
$$D = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},$$
$$F_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}^T, \quad F_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}^T,$$
$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$
$$G_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (44)$$

and a nonlinear function,

$$f(x) = 0.4sin(x_1) + 0.45cos(x_3) \quad (45)$$

Since there are two different unknown inputs, by choosing augmented states as,

$$\zeta = \begin{bmatrix} x_1 & x_2 & x_3 & \nu_1 & \nu_2 \end{bmatrix}^T \quad (46)$$

we follow our proposed algorithm to estimate both the real states and unknown inputs at the same time. According to Assumption 2, the augmented system is observable. For the arbitrary matrix R (full row rank) as,

$$R = \begin{bmatrix} 1 & 3 & 2 & 0 & -1 \\ -1 & 2 & 0 & 1 & -2 \\ -3 & 2 & 3 & 0 & 4 \end{bmatrix}, \quad (47)$$

from (37), M is computed as follows:

$$M = \begin{bmatrix} 1 & 4 & 2 \\ -2 & 4 & 0 \\ -3 & -2 & 3 \end{bmatrix} \quad (48)$$

Now, using the Matlab LMI toolbox, for $\gamma_0 = 0.45$ and $\mu = 2.5$, we can solve the LMI defined in Theorem 3 with respect to P, Y_1 and Y_2. One feasible solution is found as,

$$P = \begin{bmatrix} 0.114 & -0.054 & 0.07 \\ -0.054 & 0.194 & 0.065 \\ 0.070 & 0.065 & 0.361 \end{bmatrix},$$
$$Y_1 = \begin{bmatrix} 2937.4 & 3 & -1036.2 & -380.2 & -3327.2 \\ 1133.8 & -347.1 & -416.4 & -63.9 & -1129.7 \\ -960.6 & -106.6 & 411.8 & -6.3 & 510.6 \end{bmatrix},$$
$$Y_2 = \begin{bmatrix} 97.9 & 9 & 220.4 & -159.8 & -236.5 \\ 9 & -40.8 & 153.5 & -17.9 & 65.8 \\ 220.4 & 153.5 & -65.3 & -35.1 & -127.8 \\ -159.8 & -17.9 & -35.1 & 67.5 & 276.7 \\ -236.5 & 65.7 & -127.8 & 276.7 & 0.5 \end{bmatrix},$$

Therefore, according to (40), the H_∞ observer dynamics are as follows:

$$\dot{z}(t)=\begin{bmatrix}-4 & 0.5 & 0\\ -1.3 & -2.9 & -0.47\\ 1.1 & 0.63 & -2.05\end{bmatrix}z(t)+\begin{bmatrix}3 & 5\\ -2 & 2\\ 0 & -5\end{bmatrix}y(t)$$
$$+\begin{bmatrix}1\\ -2\\ -3\end{bmatrix}u(t)+\begin{bmatrix}4\\ 4\\ -2\end{bmatrix}(0.4sin(\hat{x}_1(t))+0.45cos(\hat{x}_3(t)))$$
$$\hat{\zeta}(t)=\begin{bmatrix}0.176 & -0.235 & -0.118\\ 0.088 & 0.132 & -0.059\\ 0.235 & -0.147 & 0.176\\ -0.176 & 0.235 & 0.118\\ -0.088 & -0.132 & 0.059\end{bmatrix}z(t)+\begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\\ 1 & 0\\ 0 & 1\end{bmatrix}y(t) \tag{49}$$

In order to simulate the designed filter, the following assumptions are made:

$$\begin{aligned} x(0)&=\begin{bmatrix}3 & 2 & -2\end{bmatrix}^T,\\ z(0)&=\begin{bmatrix}0 & 0 & 0\end{bmatrix}^T,\\ u(t)&=0.1 \end{aligned}\tag{50}$$

Also we assume that the exogenous disturbance $\omega(t)$ is a random signal with amplitude 0.5, frequency 0.01 Hertz, and bias 0.1 as depicted in Fig. 1.

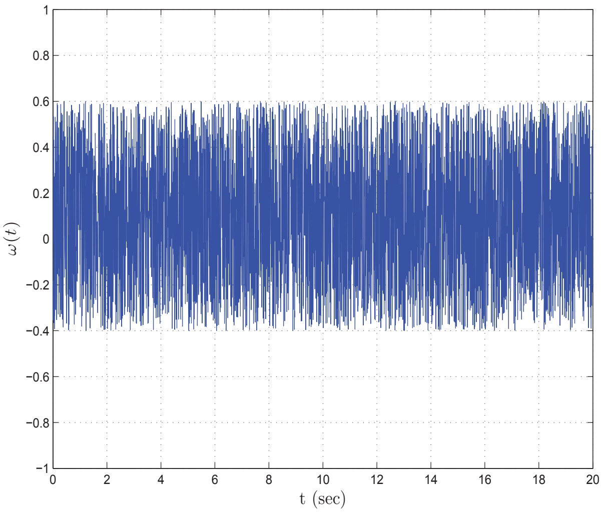

Fig. 1. Exogenous Disturbance $\omega(t)$

After simulating the designed filter described as (49), results are presented in Fig. 2, Fig. 3 and Fig. 4. The Fig. 2 shows the real and estimated state variables for initial conditions given by (50) simultaneously. The estimated unknown inputs are shown in Fig. 3 and Fig. 4, respectively. Based on Fig. 2, Fig. 3 and Fig. 4, one can see that the filter performs as expected.

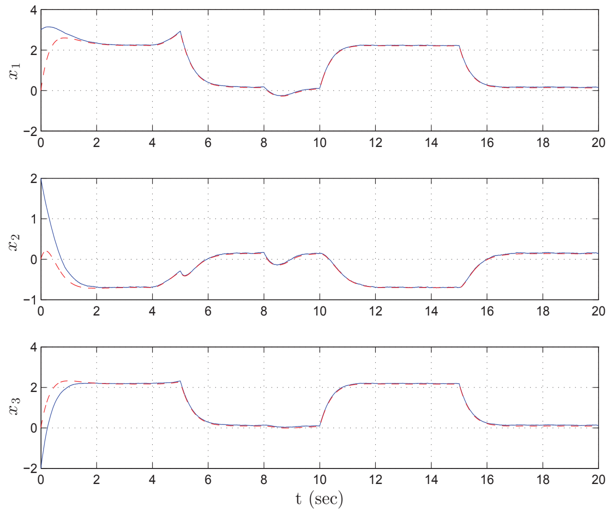

Fig. 2. Real (solid) and Estimated (dashed) State Variables

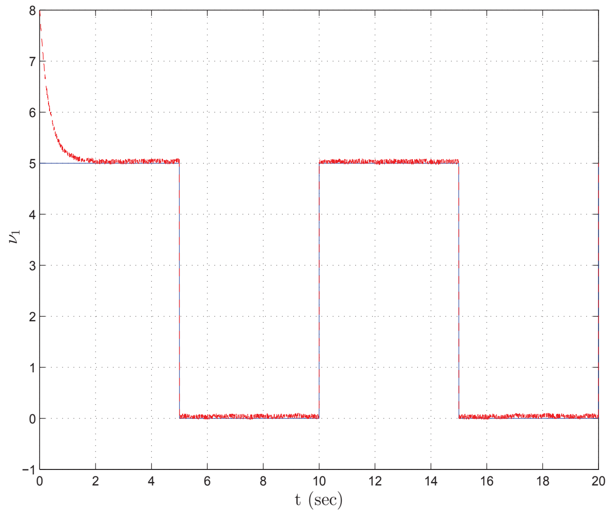

Fig. 3. Real (solid) and Estimated (dashed) Unknown Input v_1

V. CONCLUSIONS

In this paper, we have presented a new and efficient approach to design a H_∞ filter for a class of nonlinear system with unknown inputs. The designed filter is able to estimate both states and unknown inputs, simultaneously. In the proposed method, first we rewrite the system dynamics as a descriptor system and design an H_∞ reduced-order filter for the new system dynamics. Then, we derive a sufficient condition for existence of the designed filter which requires solving a nonlinear matrix inequality. In order to facilitate the proposed filter design, the obtained condition is formulated in terms of LMIs that can be solved by well-known algorithms easily. Finally, the proposed filter is validated by an example and simulation results are shown.

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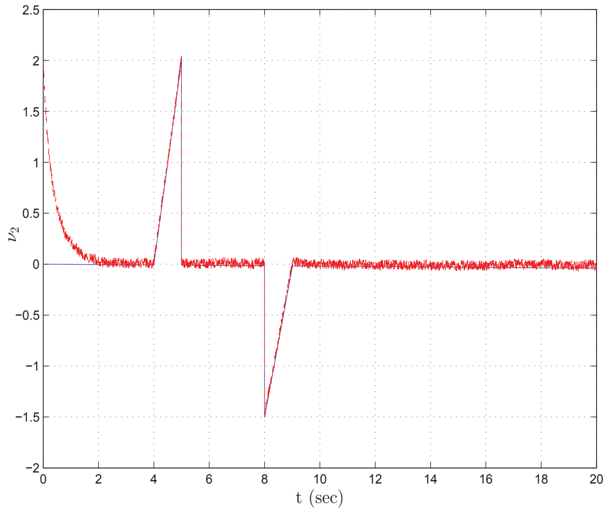

Fig. 4. Real (solid) and Estimated (dashed) Unknown Input v_2

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